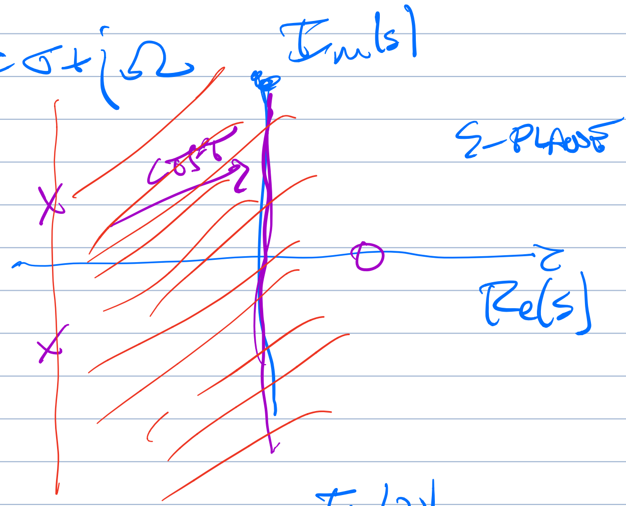
4/9/21 INTRO TO IIR FILTER DESIGN

LECTURE 20 A IIR DESIGN EXAMPLES

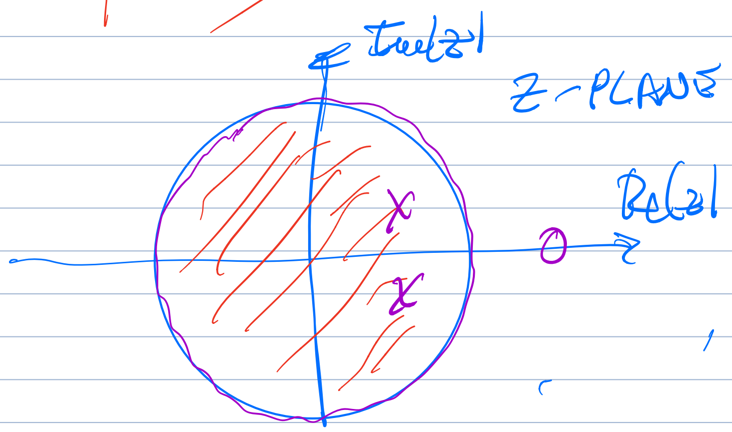
(OS4E 7.3.1)

$s = \sigma + j\Omega$



$$X(s) = \int_{-\infty}^{\infty} x(t)\, e^{-st}\, dt$$

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$



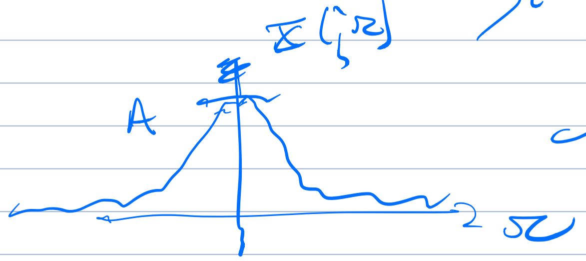
$$X(z) = \sum_{n=-\infty}^{\infty} x[n]\, z^{-n}$$

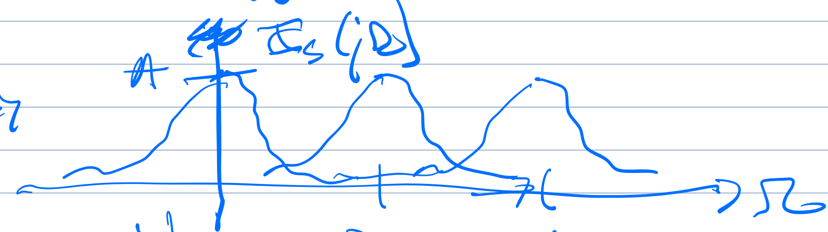
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

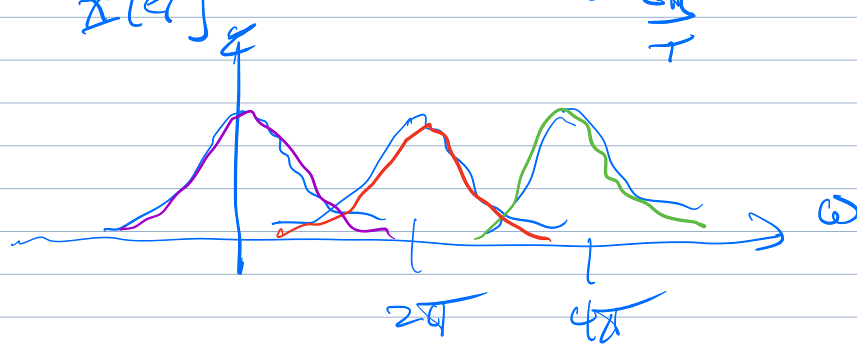
$$\boxed{\text{IMPULSE INVARIANCE}}$$

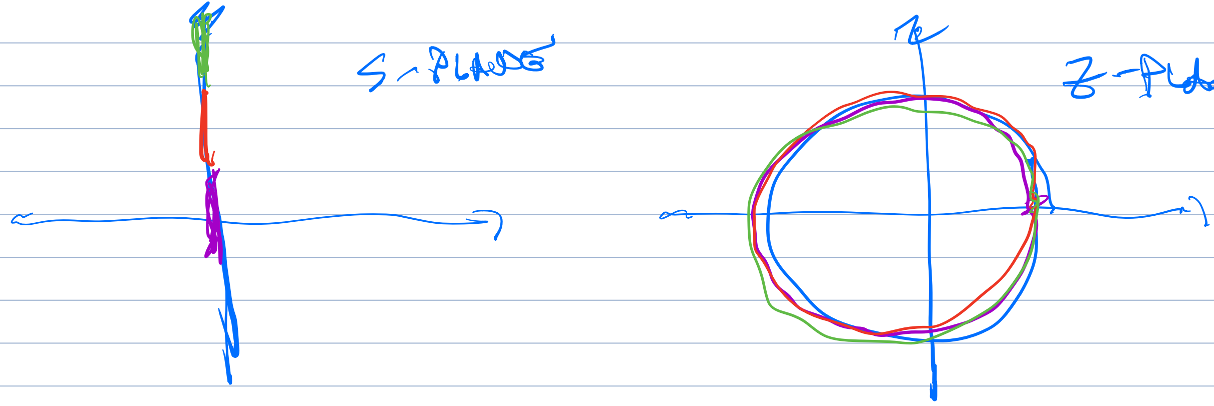
Let $h[n] = T\, h_c(t)$

$t = nT$









THE BILINEAR TRANSFORM

CONSIDER $$s = \frac{2}{T}\,\frac{1 - z^{-1}}{1 + z^{-1}}$$ BILINEAR TRANSFORM

$$s = \frac{2}{T}\,\frac{z-1}{z+1}$$

$$sTz + s = 2z - 2$$

$$z\,(sT - 2) = -s - 2$$

$$z = \frac{-2 - sT}{-2 + sT} = \frac{2 + sT}{2 - sT} = \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}}$$

Let $$H(s) = \frac{1}{s+a} \implies H(z) = \frac{1}{\frac{2}{T}\,\frac{1 - z^{-1}}{1 + z^{-1}} + a}$$

$s = \sigma + j\Omega$

CONSIDER $$z = \frac{1 + \frac{sT}{2}}{1 - \frac{sT}{2}} = \frac{1 + \frac{(\sigma + j\Omega)T}{2}}{1 - \frac{(\sigma + j\Omega)T}{2}}$$

$j\Omega$-AXIS : $\sigma = 0$

$$z = \frac{1 + \frac{j\Omega T}{2}}{1 - \frac{j\Omega T}{2}}$$

$$|z|^2 = z z^* = \frac{1+j\frac{\Omega T}{2}}{1-j\frac{\Omega T}{2}} \cdot \frac{1-j\frac{\Omega T}{2}}{1+j\frac{\Omega T}{2}} = 1$$

UNIT CIRCLE IN Z-PLANE!

$$z = \frac{1+\frac{sT}{2}}{1-\frac{sT}{2}} = \frac{1+\frac{(\sigma+j\Omega)T}{2}}{1-\frac{(\sigma+j\Omega)T}{2}}$$

IF $\sigma < 0$ $\qquad z = \dfrac{\left(1+\frac{\sigma T}{2}\right) + j\frac{\Omega T}{2}}{\left(1-\frac{\sigma T}{2}\right) - j\frac{\Omega T}{2}}$

$|z| < 1$

LHP S-PLANE $\Rightarrow$ INSIDE OF UC

RH S-PLANE $\Rightarrow$ OUTSIDE OF UC

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

Let $z = e^{j\omega}$

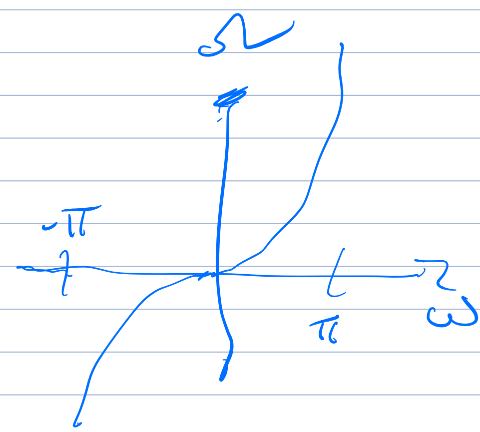
$s = j\Omega$

$$j\Omega = \frac{2}{T}\,\frac{1-e^{-j\omega}}{1+e^{-j\omega}}$$

$$j\Omega = \frac{2}{T}\,\frac{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}} - e^{-j\frac{\omega}{2}}\right)}{\cancel{e^{-j\frac{\omega}{2}}}\left(e^{j\frac{\omega}{2}} + e^{-j\frac{\omega}{2}}\right)} =$$

$$\cancel{j}\Omega = \frac{2}{T}\,\frac{2\cancel{j}\sin\left(\frac{\omega}{2}\right)}{2\cos\left(\frac{\omega}{2}\right)}$$

$$\boxed{\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)}$$



$$\frac{\Omega T}{2} = \tan\left(\frac{\omega}{2}\right)$$

$$\tan^{-1}\left(\frac{\Omega T}{2}\right) = \frac{\omega}{2}$$

$$\boxed{\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)}$$

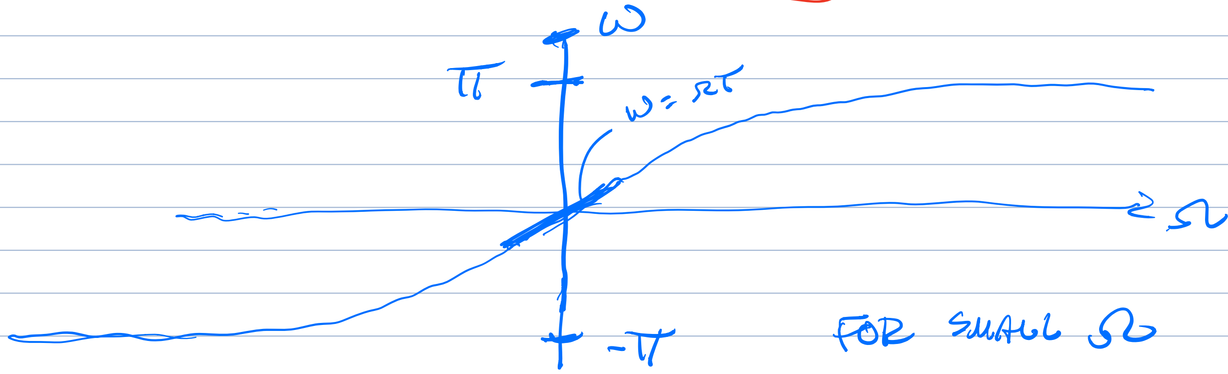
x SMALL

$\sin(x) \approx x$

$\cos(x) \approx 1$

$\tan(x) = x$

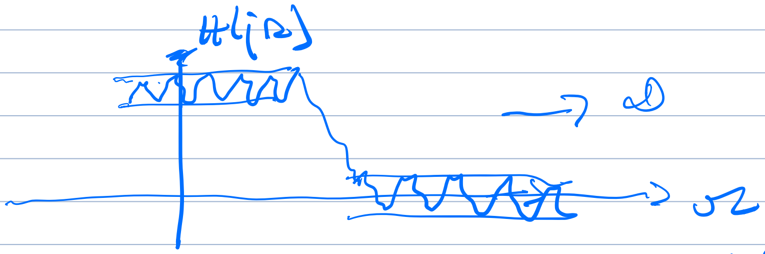
$\tan^{-1}(x) = x$



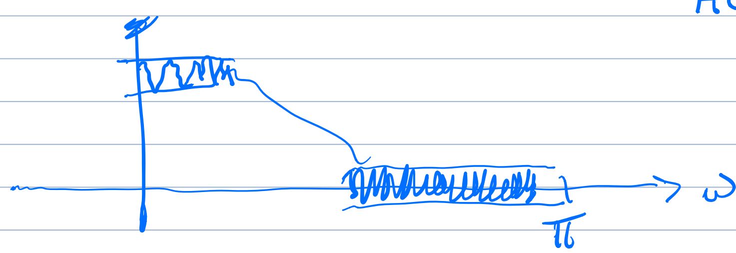
FOR SMALL Ω

$$\omega = \cancel{2}\cdot\frac{\Omega T}{\cancel{2}} = \Omega T$$

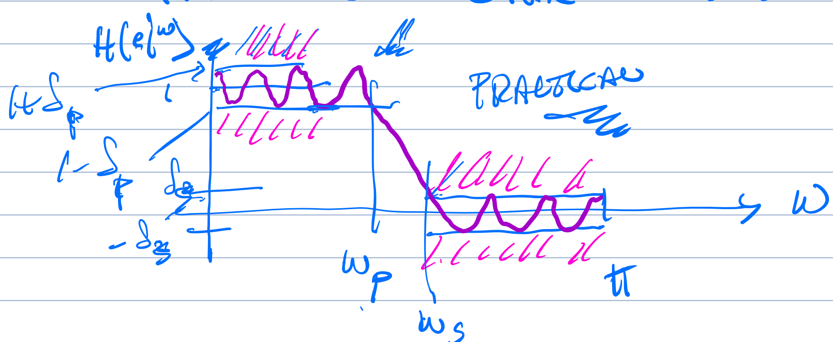
APPLICATION TO FILTER DESIGN

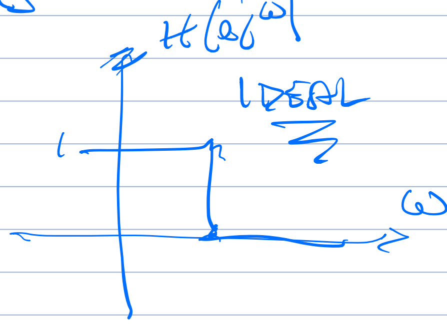


$$H(e^{j\omega}) = H(s)\Big|_{\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)}$$



PRACTICAL FILTER DESIGN SPECS





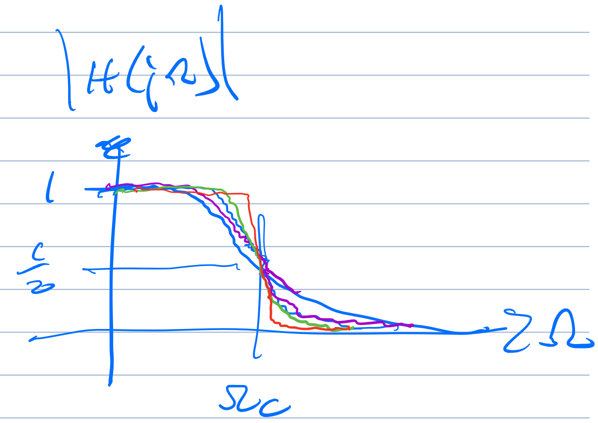
$\Delta\omega = \omega_s - \omega_p$ TRANSITION BW

IIR CT PROTOTYPE DESIGNS

- BUTTERWORTH
- CHEBYSCHEV I, II
- ELLIPTICAL

BUTTERWORTH FILTER

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$

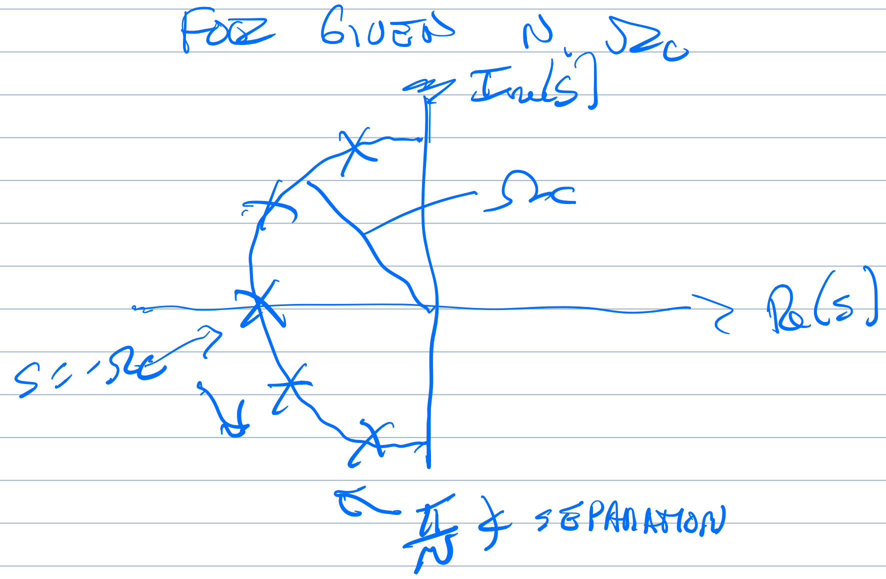


N, Ω_c

\# POLES ORDER

CUTOFF FREQ

FOR GIVEN N, Ω_c



SPECIFICATIONS:

$\omega_p = .15\pi$

$\omega_s = .35\pi$

PASSBAND RIPPLE $-3\,dB \le |H(e^{j\omega})| \le 0\,dB$
$|\omega| \le \omega_p$

$|H(e^{j\omega})| \le -20\,dB,$
$\omega_s \le |\omega| \le \pi$

$$dB = 20\log_{10}\left(\frac{X}{X_{REF}}\right)$$, AMPLITUDE, VOLTAGE, CURRENT, PRESSURE, VELOCITY, ETC.

$$dB = 10\log_{10}\left(\frac{X}{X_{REF}}\right)$$, INTENSITY, ENERGY POWER

$$dB = 20\log_{10}\left(\frac{X}{1}\right)$$

$$\frac{dB}{20} = \log_{10}(X)$$

$$10^{dB/20} = X = 1-\delta_p$$

$$1-\delta_p = 10^{-3/20} = .7079$$

$$\delta_s = 10^{-20/20} = .1$$

DEFINE $k_1 \triangleq \frac{1}{(1-\delta_p)^2} - 1 = .9953$

$$k_2 \triangleq \frac{1}{\delta_s^2} - 1 = 99$$

$$\boxed{\text{DESIGN USING IMPULSE INVARIANCE}}$$

$$\omega = \Omega T \; ; \; \Omega = \frac{\omega}{T}$$

$$\Omega_p = \frac{\omega_p}{T} = \frac{.47124}{T}$$

$$\Omega_s = \frac{\omega_s}{T} = \frac{1.0996}{T}$$

AT PASSBAND EDGE,

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}} = (1-\delta_p)^2$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{1}{(1-\delta_p)^2} - 1 = k_1$$

AT STOPBAND EDGE,

$$\frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}} = \delta_s^2$$

$$\left(\frac{\Omega_s}{\Omega_p}\right)^{2N} = \frac{1}{\delta_s^2} - 1 = k_2$$

$$\frac{\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}}{\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}} = \frac{k_1}{k_2}$$

$$N = \frac{1}{2}\,\frac{\log(k_1/k_2)}{\log(\Omega_s/\Omega_p)} = 2.7144$$

$$\Rightarrow 3$$

MATCHING AT PASSBAND

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = k_1 \; ; \; \Omega_c = \frac{\Omega_p}{k_1^{1/2N}} = \frac{.4716}{6}$$

MATCHING AT STOP BAND

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = k_2 \Rightarrow \Omega_c = \frac{\Omega_s}{k_2^{1/2N}} = \frac{.5112}{T}$$

$$\boxed{\text{DESIGN USING BILINEAR TRANSFORMATION}}$$

SPECIFICATIONS:

$\omega_p = .15\pi$

$\omega_s = .35\pi$

PASSBAND RIPPLE $\quad -3\,dB \le |H(e^{j\omega})| \le 0\,dB$

$|\omega| \le \omega_p$

$$|H(e^{j\omega})| \le -20\,dB,$$

$\omega_s \le |\omega| \le \pi$

DEFINE $\quad k_1 \triangleq \dfrac{1}{(1-\delta_p)^2} - 1 = .9953$

$$k_2 \triangleq \frac{1}{\delta_s^2} - 1 = 99$$

WITH IMPULSE INVARIANCE $\quad \Omega = \dfrac{\omega}{T}$

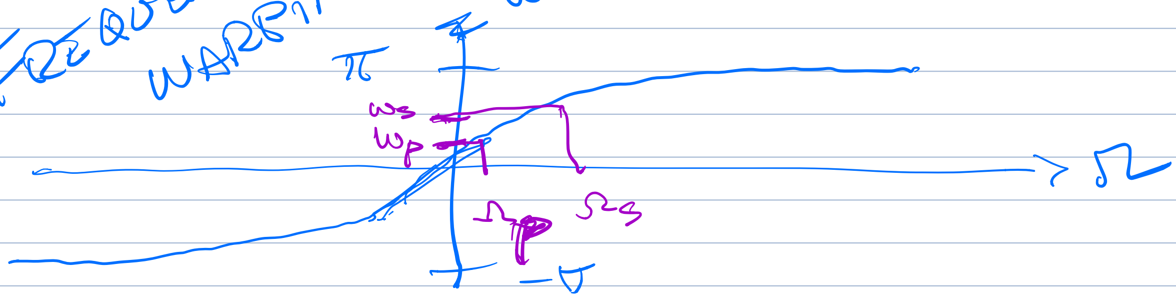
WITH BILINEAR TRANSFORMATION

$$\Omega = \frac{2}{T} \tan\left(\frac{\omega}{2}\right)$$

$$\Omega_p = \frac{2}{T} \tan\left(\frac{\omega_p}{2}\right) = \frac{.4802}{T}$$

$$\Omega_s = \frac{2}{T} \tan\left(\frac{\omega_s}{2}\right) = \frac{1.226}{T}$$

FREQUENCY WARPING



$$N = \frac{1}{2}\,\frac{\log(k_2/k_1)}{\log(\Omega_s/\Omega_p)} = 2.4546 \Rightarrow 3$$

$$\Omega_c = \frac{.4806}{T}$$ MATCHING @ PASSBAND

$$\Omega_c = \frac{.5698}{T}$$ MATCHING @ STOPBAND