3/22/24 IIR DESIGN EXAMPLE

* IMPULSE INVARIANCE

Let $h[n] = T\, h_c(t)$

* BILINEAR TRANSFORMATION

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}}$$

$$\Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$

CT FILTER DESIGNS

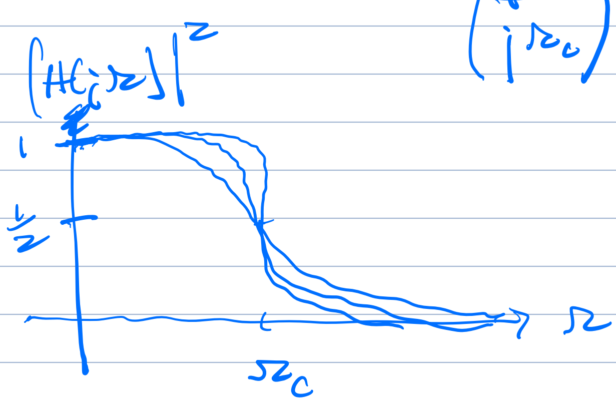
* BUTTERWORTH
* CHEBYSCHEV
* ELLIPTICAL

BUTTERWORTH FILTER

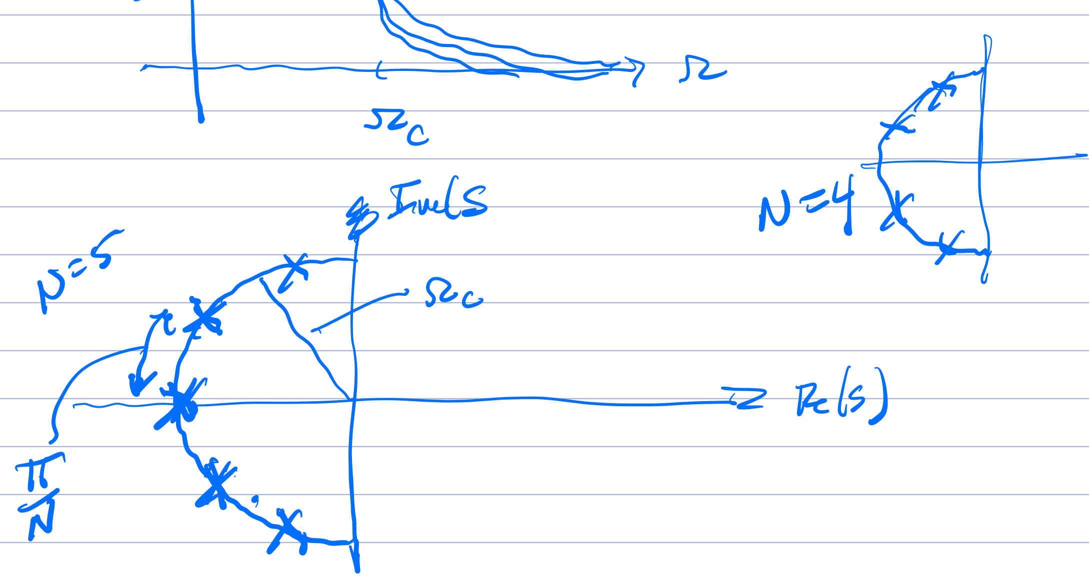
$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{j\Omega}{j\Omega_c}\right)^{2N}}$$

N = # POLES

Ω_c = CUTOFF FREQ



FOR A GIVEN Ω_c N



CONSIDER $\omega_p = .15\pi$

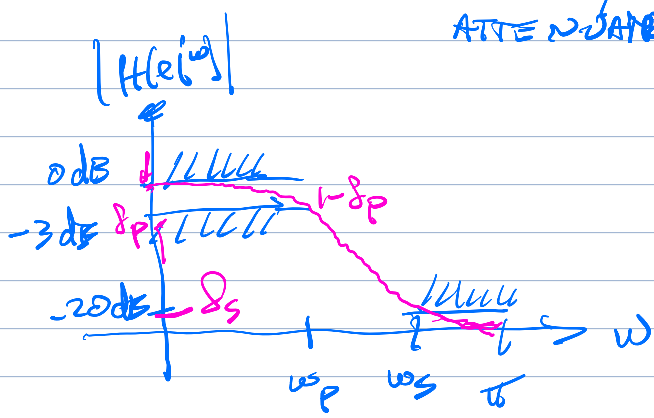
$\omega_s = .35\pi$

PASSBAND RIPPLE $\quad -3dB \le |H(e^{j\omega})| \le 0dB$

$|\omega| < \omega_p$

STOPBAND RIPPLE / ATTENUATION $\quad |H(e^{j\omega})| \le -20dB$

$\omega \le |\omega| \le \pi$



DECIBEL CALCULATION $\quad dB$

$$dB = 20 \log_{10}\left(\frac{x}{x_{REF}}\right)$$ AMPLITUDE, VOLTAGE, CURRENT, VELOCITY, FORCE, PRESSURE

$$dB = 10 \log_{10}\left(\frac{x}{x_{REF}}\right)$$ INTENSITY, ENERGY, POWER

$\boxed{\text{DESIGN USING IMPULSE INVARIANCE}}$

$$\omega = \Omega T \; ; \; \Omega = \frac{\omega}{T}$$

$$\Omega_p = \frac{\omega_p}{T} = \frac{.47124}{T}$$

$$\Omega_s = \frac{\omega_s}{T} = \frac{1.0996}{T}$$

$$dB = 20 \log_{10}(x)$$

$$\frac{dB}{20} = \log_{10}(x)$$

$$x = 10^{dB/20}$$

$$1-\delta_p = 10^{-3/20} = .7079$$

$$\delta_s = 10^{-20/20} = .1$$

DEFINE $\quad K_1 \triangleq \dfrac{1}{(1-\delta_p)^2} - 1 = .9953$

$$K_2 = (1/\delta_s)^2 - 1 = 99$$

AT PASSBAND EDGE

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_p}{\Omega_c}\right)^{2N}} = (1-\delta_p)^2$$

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = \frac{1}{(1-\delta_p)^2} - 1 = k_1 = .9953$$

AT STOPBAND EDGE

$$|H(j\Omega)|^2 = \frac{1}{1+\left(\frac{\Omega_s}{\Omega_c}\right)^{2N}} = \delta_s^2$$

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = \frac{1}{\delta_s^2} - 1 = k_2 = 99$$

$$\frac{\left(\frac{\Omega_p}{\cancel{\Omega_c}}\right)^{2N}}{\left(\frac{\Omega_s}{\cancel{\Omega_c}}\right)^{2N}} = \frac{k_1}{k_2}$$

$$\left(\frac{\Omega_p}{\Omega_s}\right)^{2N} = \frac{k_1}{k_2}$$

$$2N \log\left(\frac{\Omega_p}{\Omega_s}\right) = \log\left(\frac{k_1}{k_2}\right)$$

$$N = \frac{\log(k_1/k_2)}{2\log(\Omega_p/\Omega_s)} = \frac{\log(k_2/k_1)}{2\log(\Omega_s/\Omega_p)}$$

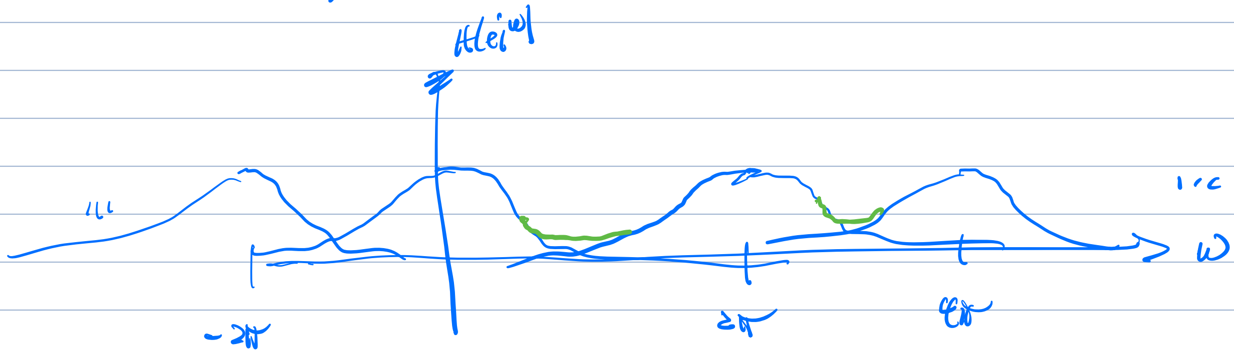
$$N = \frac{1}{2}\frac{\log(k_2/k_1)}{\log(\Omega_s/\Omega_p)} = 2.7144 \Rightarrow 3$$

MATCHING $H(j\Omega)$ ~~AT~~ PASS BAND EDGE

$$\left(\frac{\Omega_p}{\Omega_c}\right)^{2N} = k_1 \;; \quad \Omega_c = \frac{\Omega_p}{k_1^{1/2N}} = \frac{.4716}{T}$$

MATCHING EXACTLY AT STOP BAND

$$\left(\frac{\Omega_s}{\Omega_c}\right)^{2N} = k_2 \;; \quad \Omega_c = \frac{\Omega_s}{k_2^{1/2N}} = \frac{.5112}{T}$$



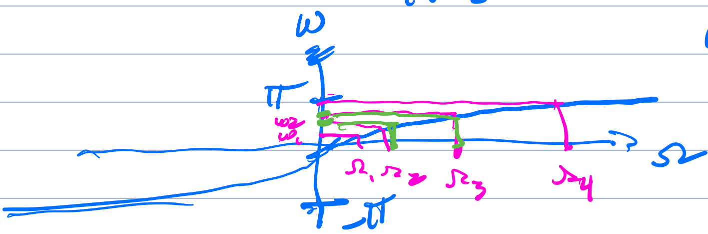
LPF: 1. MATCH SPECS @ PASS BAND

2. CHECK RESPONSE @ $\omega = \omega_s$

DESIGN USING BILINEAR TRANSFORM

$$s = \frac{2}{T}\,\frac{1-z^{-1}}{1+z^{-1}} \qquad \Omega = \frac{2}{T}\tan\left(\frac{\omega}{2}\right)$$

$$\omega = 2\tan^{-1}\left(\frac{\Omega T}{2}\right)$$



$k_1 = .8853$

$k_2 = 99$

$$\Omega_p = \frac{2}{T}\tan\left(\frac{\omega_p}{2}\right) = \frac{.4802}{T}$$

$$\Omega_s = \frac{2}{T}\tan\left(\frac{\omega_s}{2}\right) = \frac{1.226}{T}$$

$$N = \frac{1}{2}\,\frac{\log(k_2/k_1)}{\log(\Omega_s/\Omega_p)} = 2.4546 \Rightarrow 3$$

$\Omega_c = \dfrac{.4806}{T}$ MATCHING @ PASSBAND

$\Omega_c = \dfrac{.5698}{T}$ MATCHING @ STOPBAND

PROTOTYPE FILTER

$$H(s) = \frac{\Omega_c^3}{(s - \Omega_c e^{j\pi})(s - \Omega_c e^{j(\pi - \pi/3)})(s - \Omega_c e^{j(\pi + \frac{\pi}{3})})}$$

$$H(s) = \frac{.1850 / T^3}{s^3 + \frac{1.397}{T} s^2 + \frac{.6494}{T^2} s + \frac{.1850}{T^3}}$$

$$H(z) = \frac{.1850 / T^3}{\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^3 + \frac{1.397}{T}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right)^2 + \frac{.6494}{T^2}\left(\frac{2}{T}\frac{1-z^{-1}}{1+z^{-1}}\right) + \frac{.1850}{T^3}}$$

$$H(z) = \frac{.0132\left(1 + 3z^{-1} + 3z^{-2} + z^{-3}\right)}{1 - 1.907z^{-1} + 1.3315z^{-2} - .3244z^{-3}}$$