FIR FILTER STRUCTURES
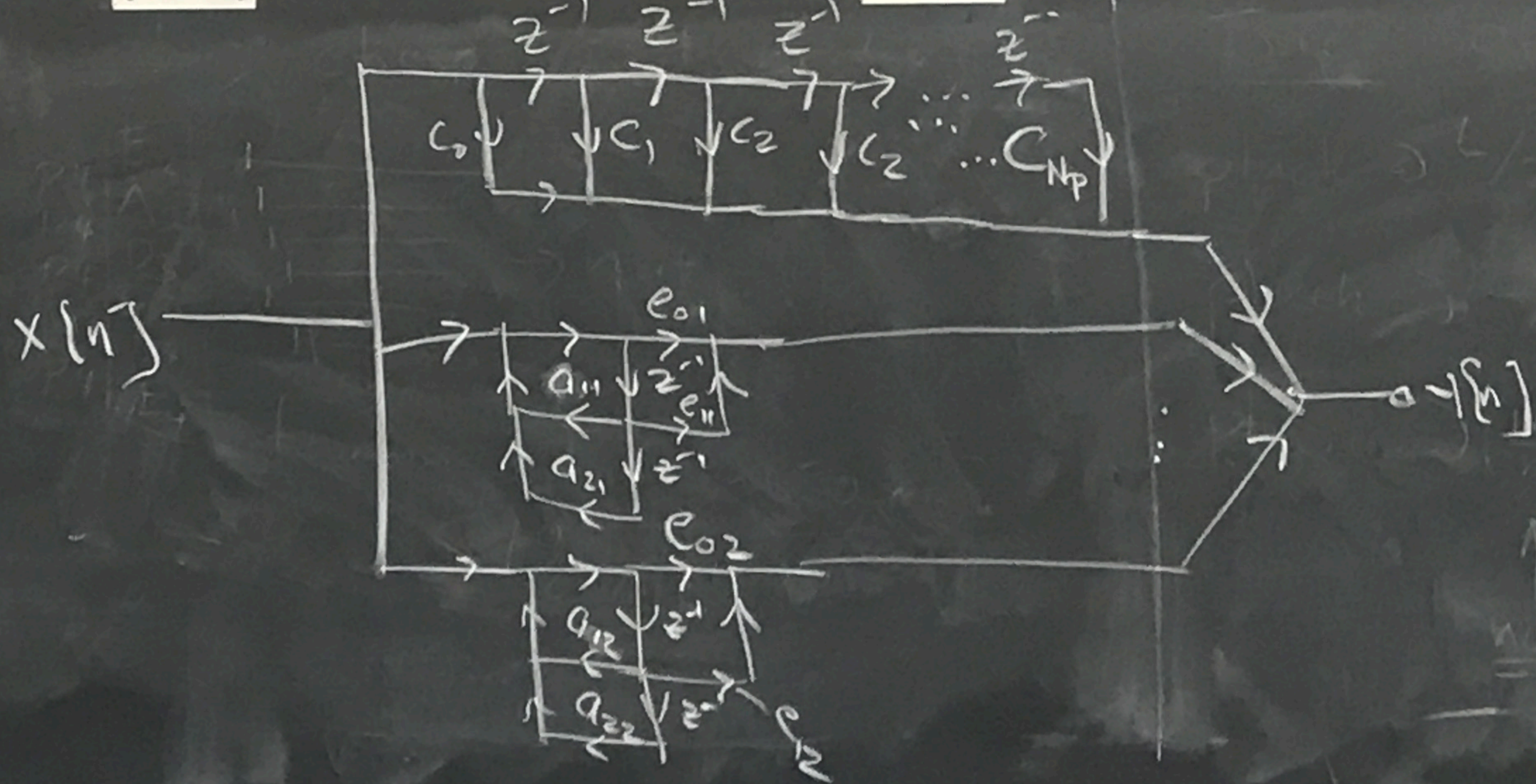
(OSYP 6.4, 6.5)

PARALLEL FORM

$$H(z) = \sum_{k=0}^{N_P} c_k z^{-k} + \sum_{k=1}^{N} \frac{A_k}{1 - d_k z^{-k}}$$

$$= \sum_{k=0}^{N_P} c_k z^{-k} + \sum_{k=1}^{N'} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

$$N' = \text{CEIL}\left(\frac{N}{2}\right)$$
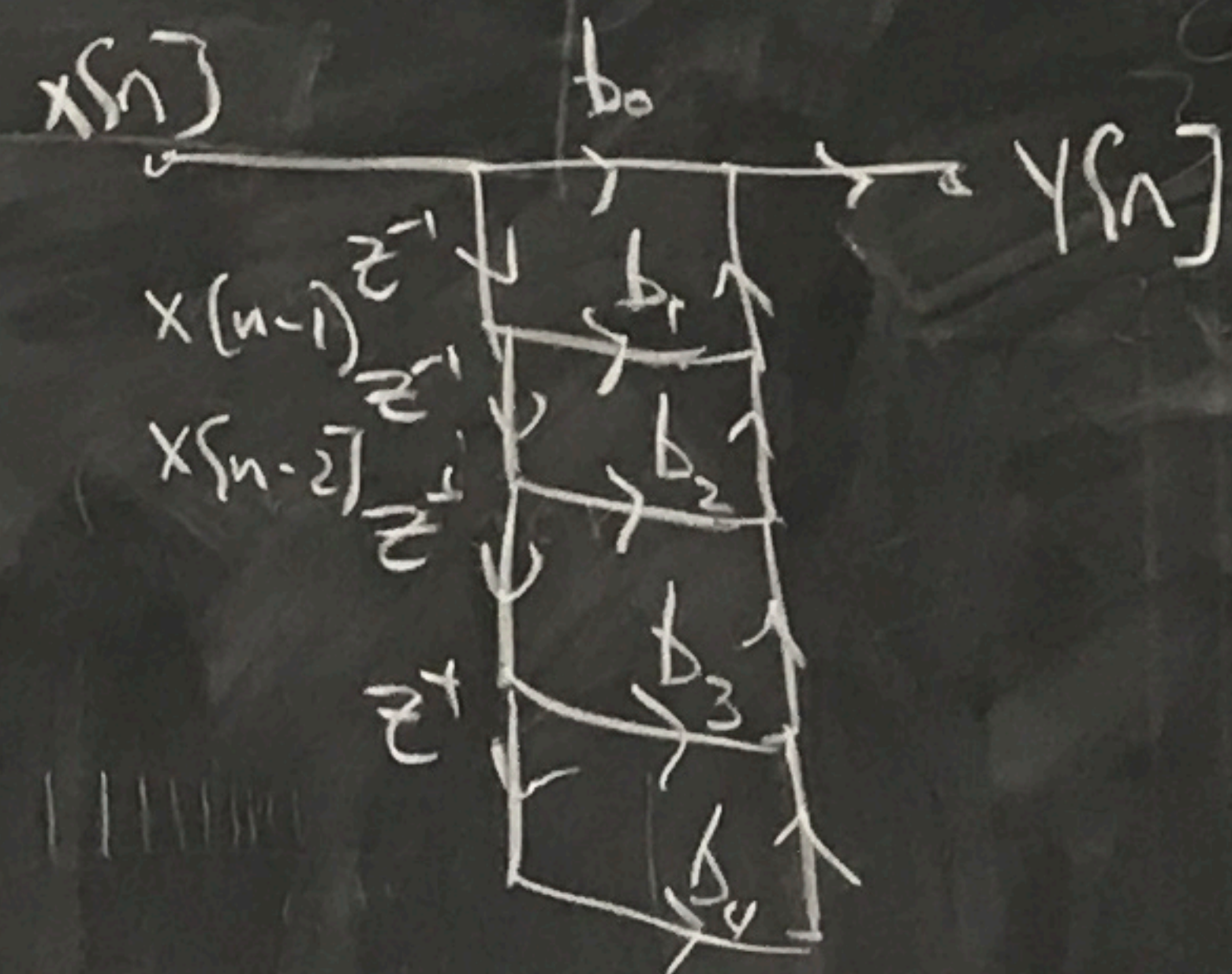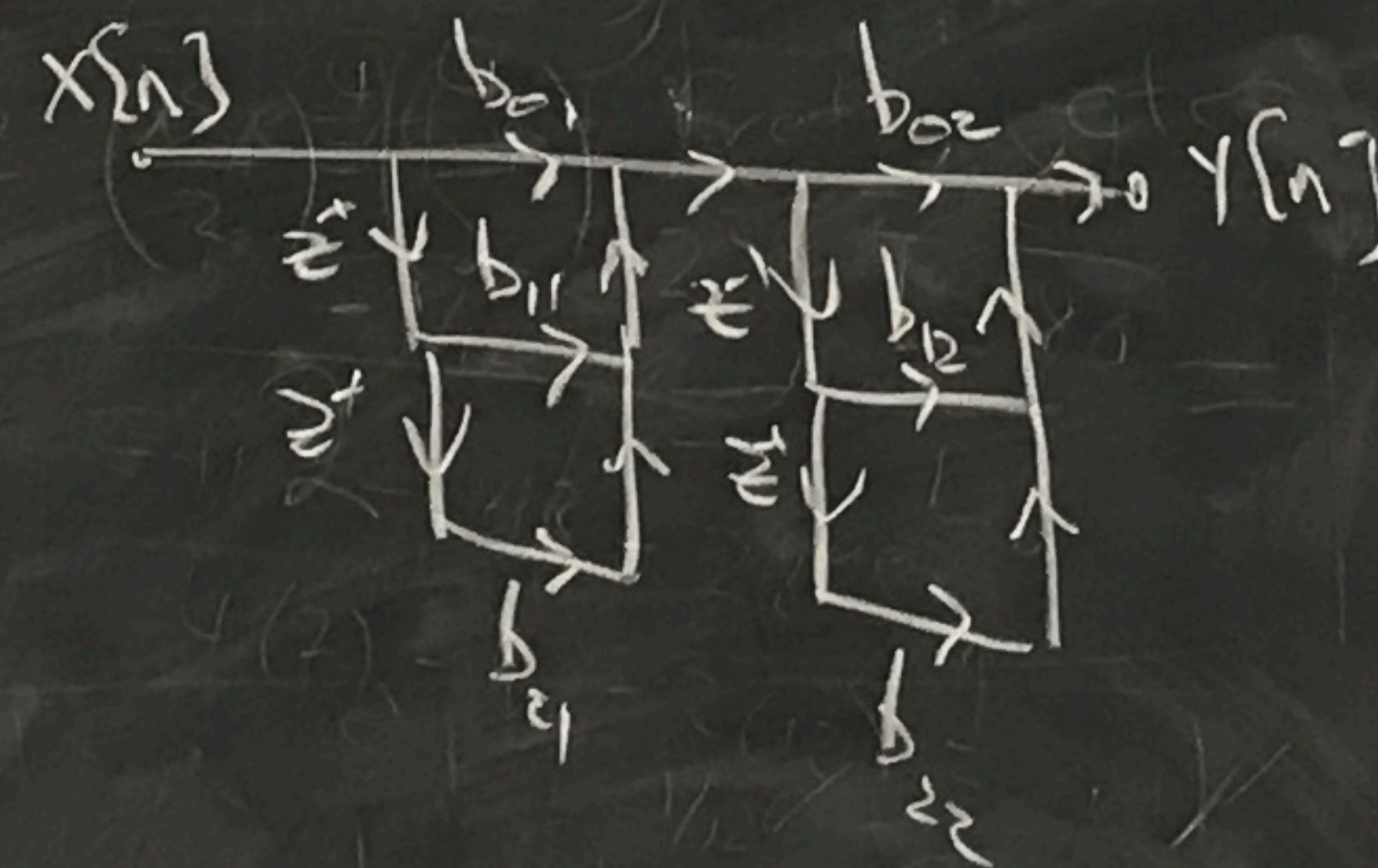
FIR FILTER STRUCTURES

DIRECT FORM

CASCADE FORM

$$H(z) = B(z) = \sum_{\ell=0}^{M} b_\ell z^{-\ell}$$

$$h[n] = \sum_{\ell=0}^{M} b_\ell \delta[n-\ell]$$

$$\frac{\partial d_\ell}{\partial a_k} = \left. \frac{\partial A(z)}{\partial a_k} \right|_{z=d_\ell} = \frac{-z^{-k}}{-z^{-1} \displaystyle\prod_{\substack{\ell=1 \\ \ell \ne i}}^{N} (1-d_\ell z^{-1})} \Bigg|_{z=d_i}$$

$$= \left. \frac{-z^{N-k}}{-\displaystyle\prod_{\substack{\ell=1 \\ \ell \ne i}}^{N} (z-d_\ell)} \right|_{z=d_i}$$

FIR LINEAR PHASE FORM

ex
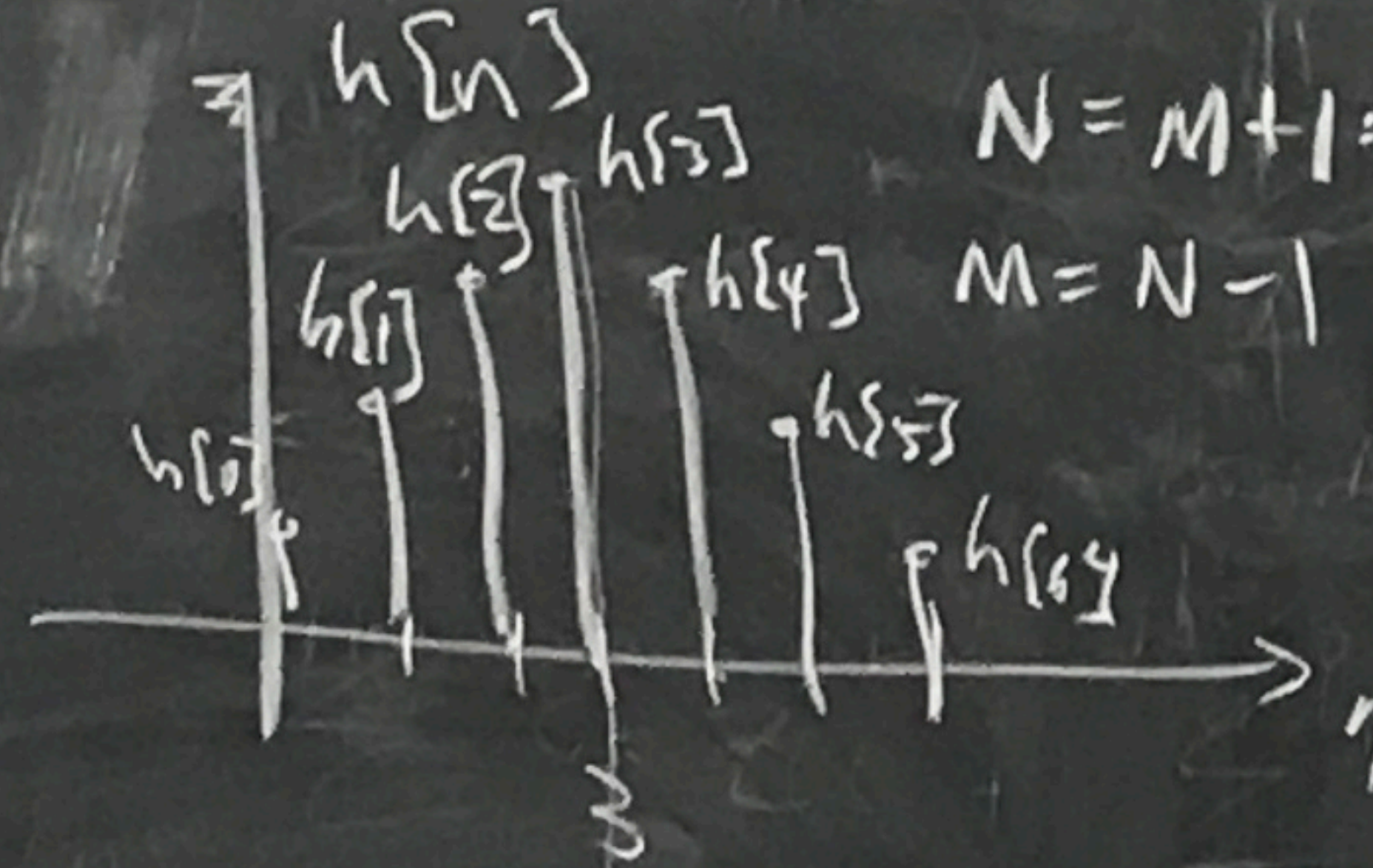
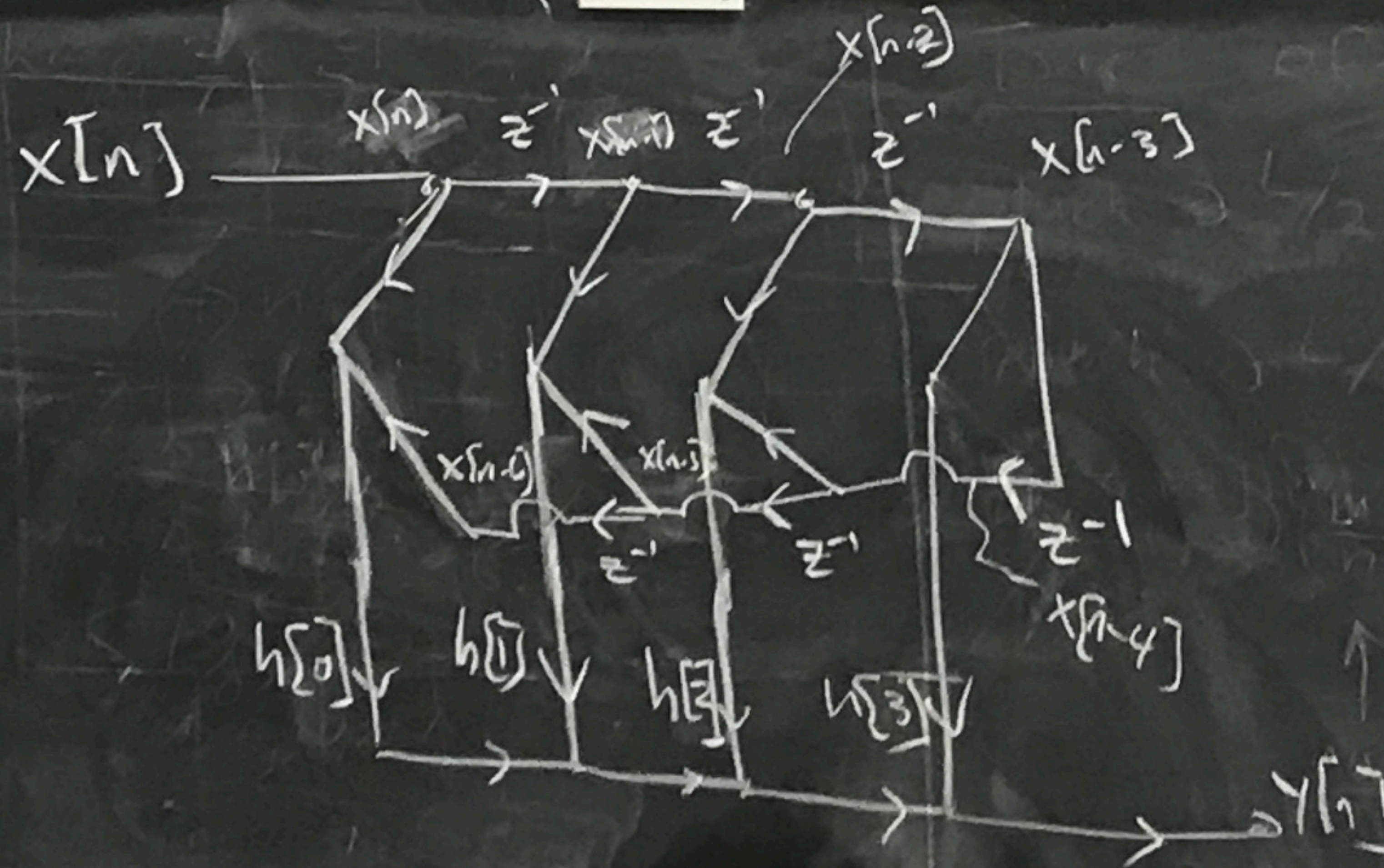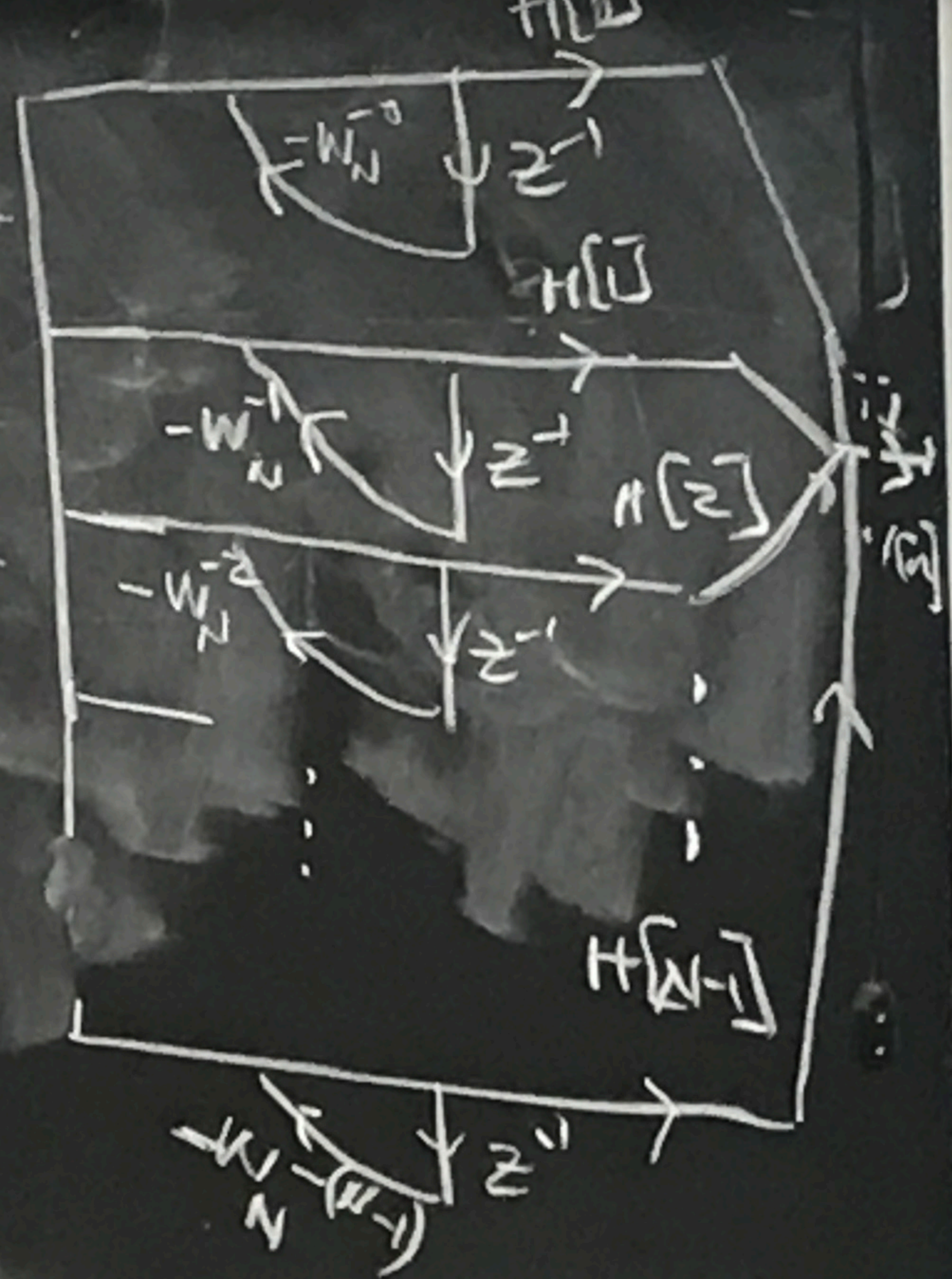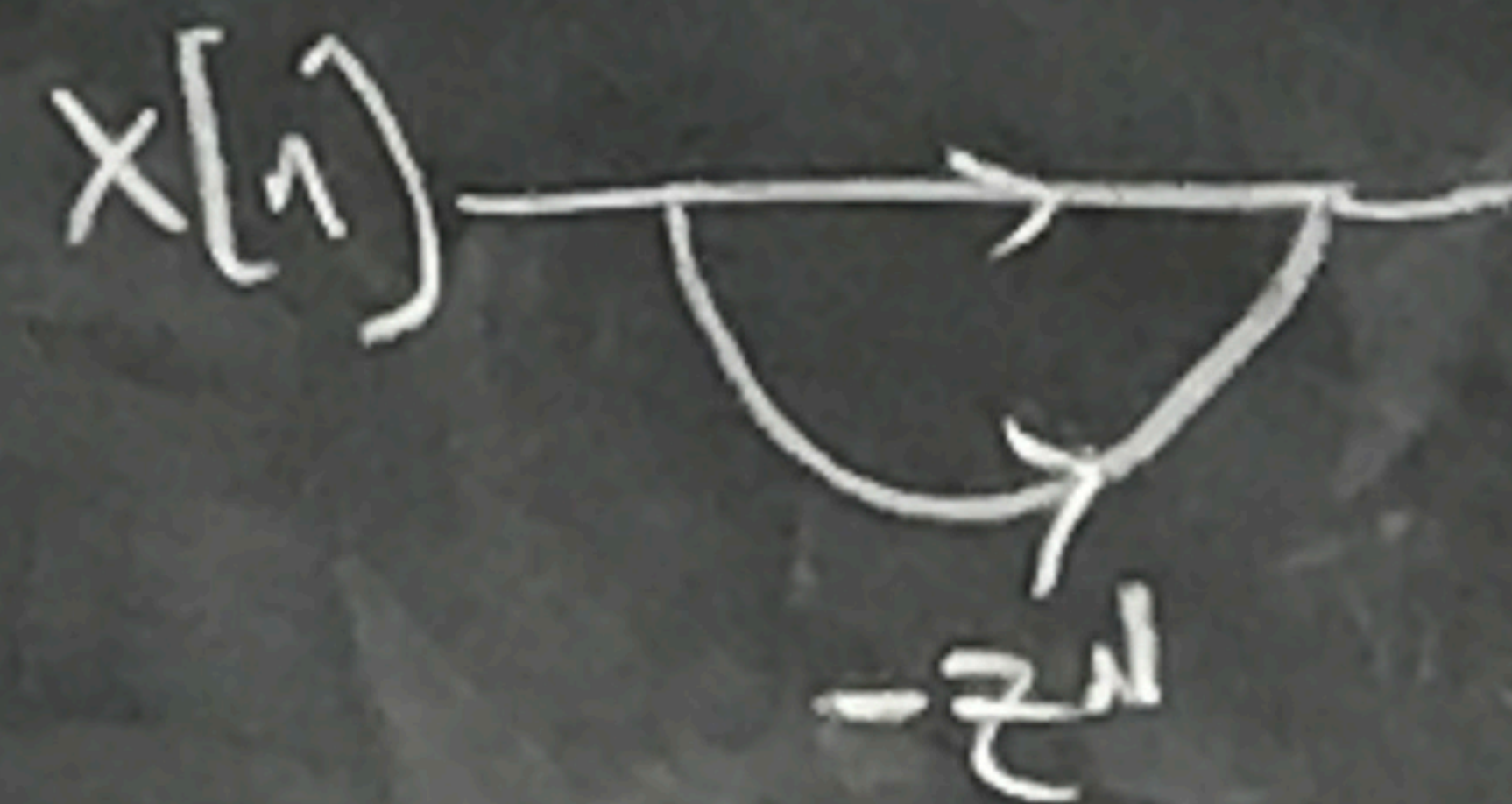$N = M+1 = 7$

$M = N-1$

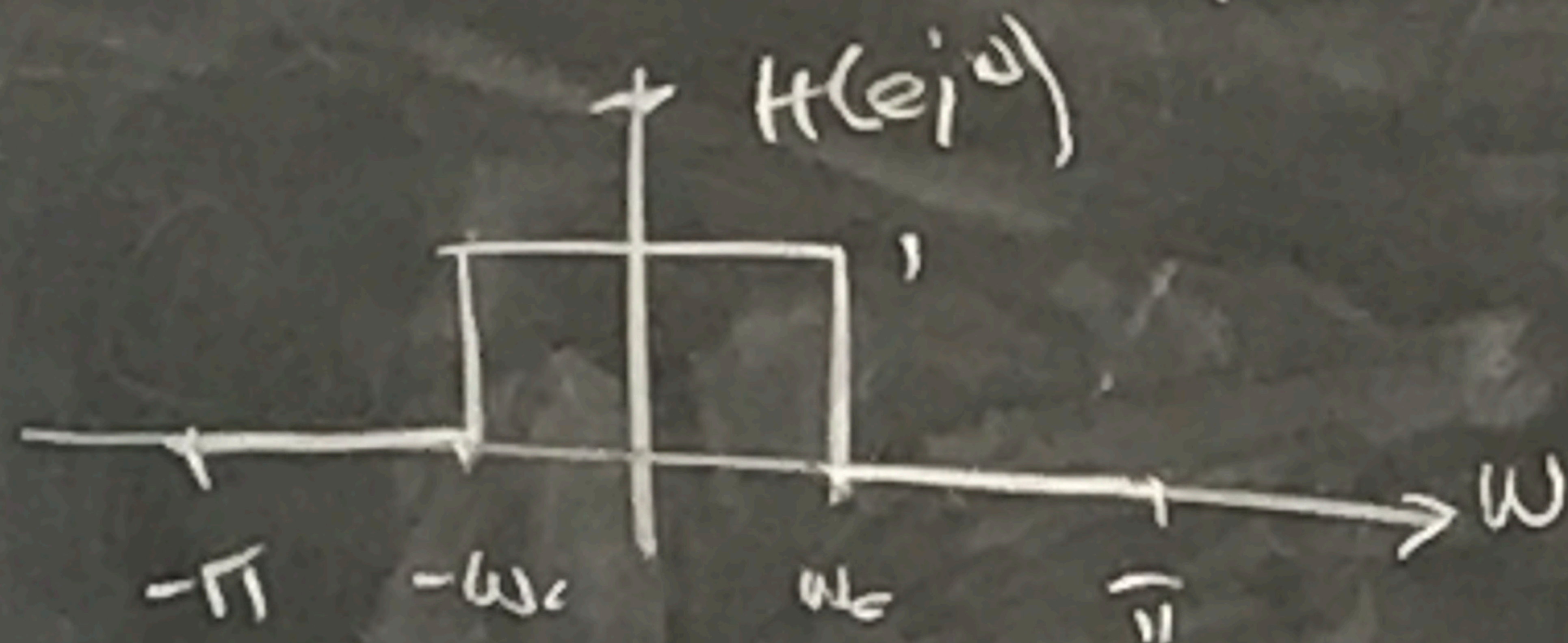$$h[n] = h[N-1-n]$$
$$= h[M-n]$$

$$Y[n] = \sum_{k=0}^{N-1} h[k] x[n-k]$$

$$H(z) = \frac{1 - z^{-N}}{N} \cdot \sum_{k=0}^{N-1} H[k] \frac{1}{1 - W_N^{-k} z^{-1}}$$
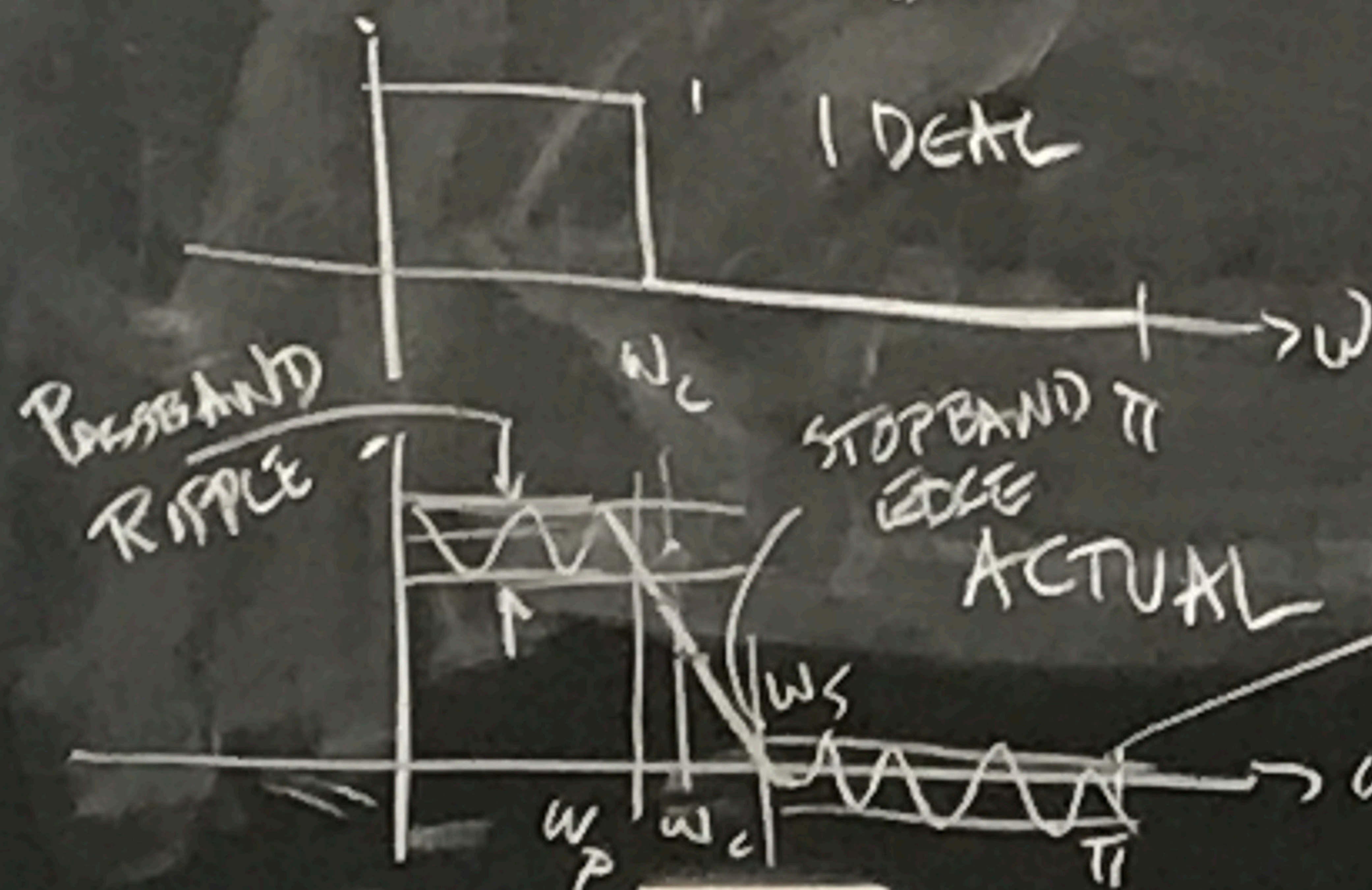
Frequency-sampling form.
See handout for derivation.

Ex. IDEAL LPF

$H(e^{j\omega})$

$-\pi$ $-\omega_c$ ω_c π ω

$h[n] =$

$$\frac{\sin(\omega_c n)}{\pi n}$$

FILTER DESIGN
ISSUES

IDEAL

ω_c ω

PASSBAND
RIPPLE

STOPBAND
EDGE
ACTUAL

π

TRANSITION BAND

$$\Delta\omega = \omega_s - \omega_p$$

STOPBAND
RIPPLE

ω_p ω_c ω_s π ω