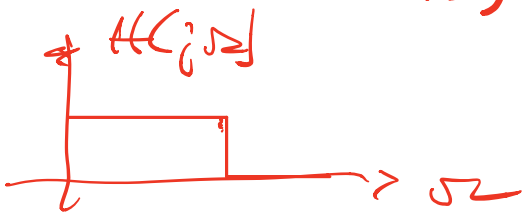
DIGITAL FILTER IMPLEMENTATION

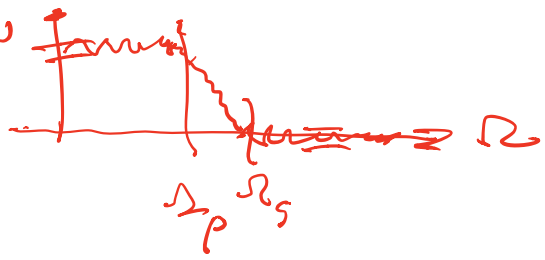
(COSC 4P 6.0 - 6.5)

BUILDING A FILTER:

1. ESTABLISH SPECS
2. DEVELOP EQUATIONS THAT SATISFY SPECS. (DESIGN, COSC 4P 7)



3. IMPLEMENT EQS. IN HARDWARE OR SOFTWARE (IMPLEMENTATION COSC 4P 6)



TODAY: IIR IMPLEMENTATION

IIR $h[n]$ RIGHT-SIDED, LEFT-SIDED, BOTH-SIDED

POLES + ZEROS

FIR $h[n]$ FINITE DURATION

ZEROS ONLY (EXCEPT $z=0$)

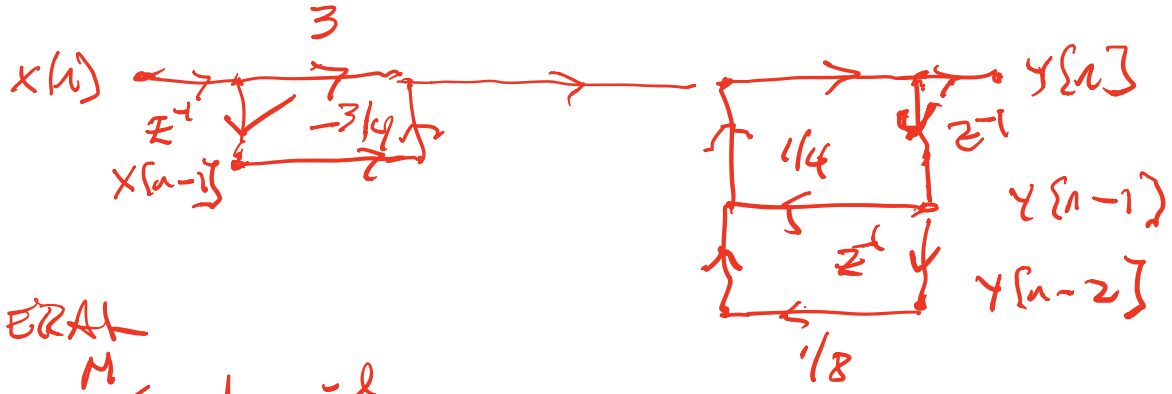
SIGNAL FLOW NOTATION

ADDITION

MULT BY CONSTANT

UNIT DELAY

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + 3x[n] - \frac{3}{4}x[n-1]$$



IN GENERAL

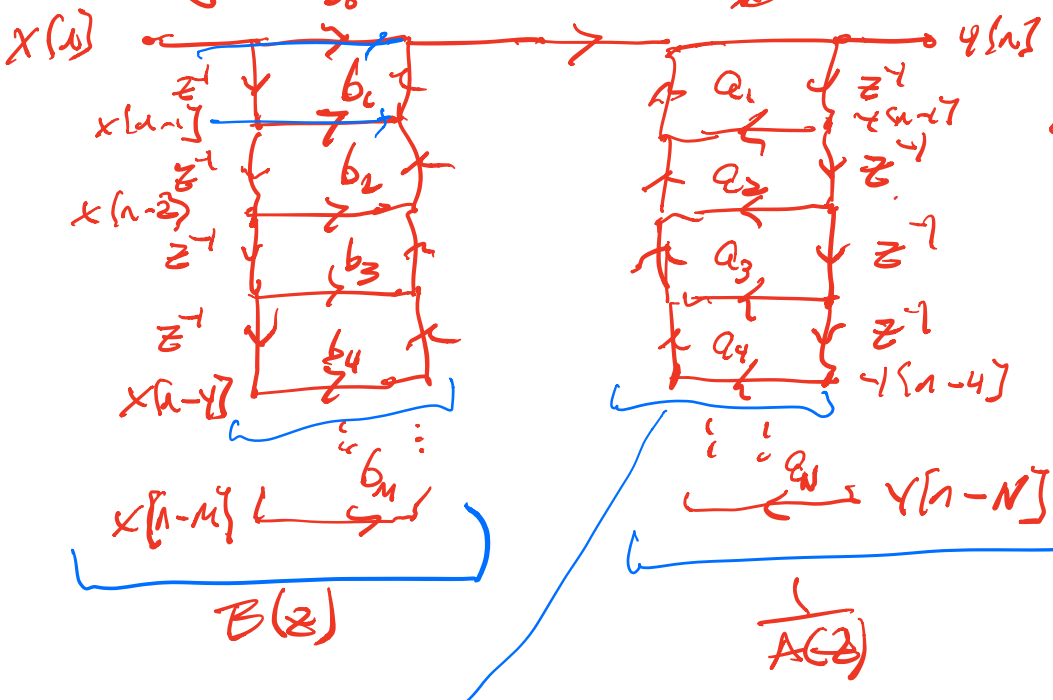
$$H(z) = \frac{\sum_{l=0}^{M} b_l z^{-l}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 - a_1 z^{-1} - a_2 z^{-2} + \dots + -a_N z^{-N}} = \frac{B(z)}{A(z)}$$

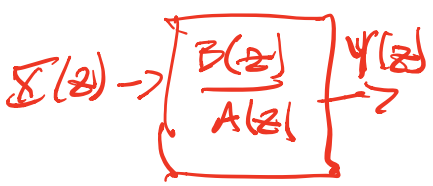
$$= \frac{Y(z)}{X(z)}$$

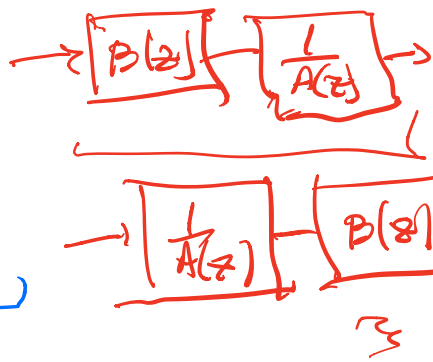
$$y[n] - \sum_{k=1}^{N} a_k y[n-k] = \sum_{l=0}^{M} b_l x[n-l]$$

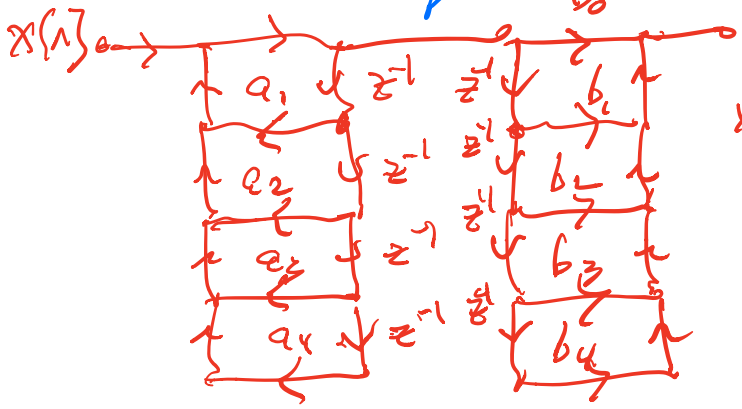
DIRECT FORM I

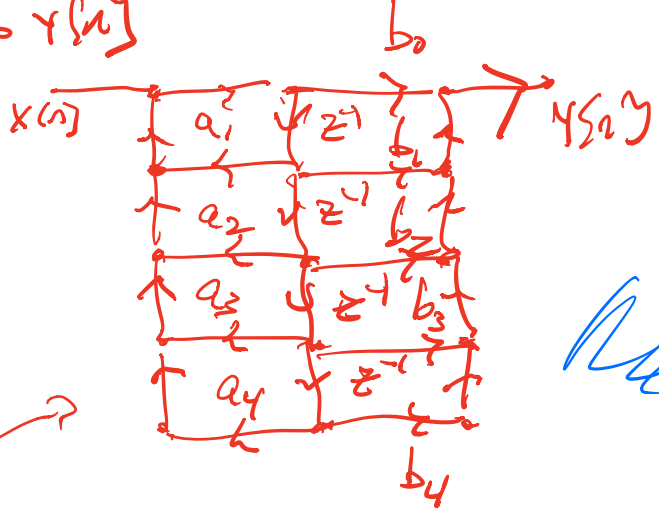
$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{l=0}^{M} b_l x[n-l]$$







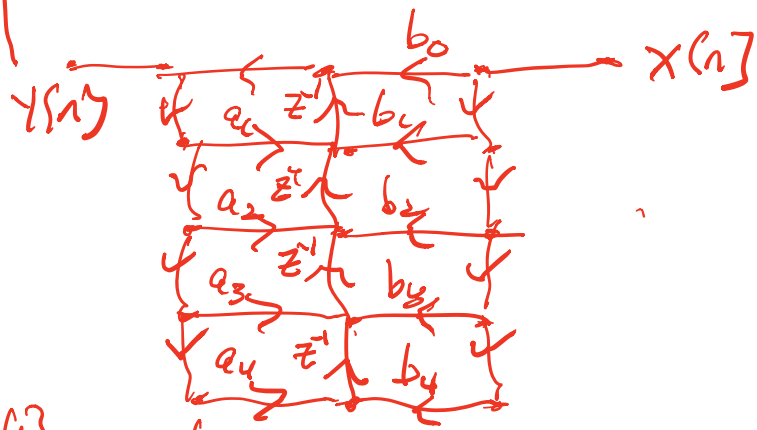




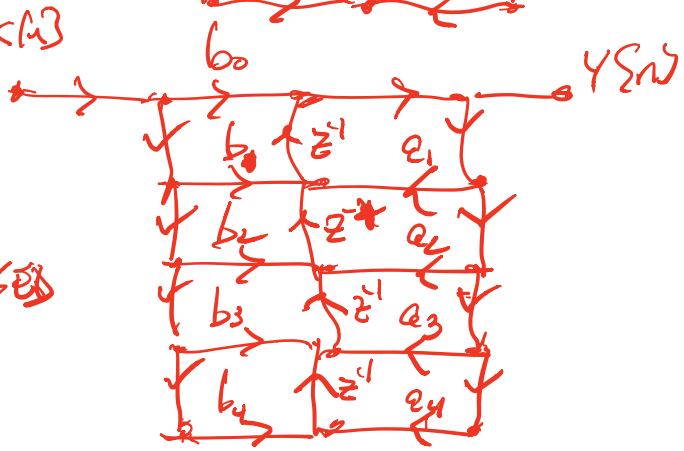
DIRECT-FORM II
CANONICAL FORM

TRANSPOSED FORMS

THEN: 1. REVERSE ARROWS
2. INTERCHANGE INPUT AND OUTPUT
$\Rightarrow$ SAME TRANSFER FN.



DF-II TRANSPOSED



BREAKING INTO SMALLER STRUCTURES

$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 \prod_{\ell=1}^{M} (1 - c_\ell z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$$

$$H(z) = b_0 \prod_{k=1}^{\max(M/2,\, N/2)} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} + a_{2k} z^{-2}}$$

2 REAL ZEROS

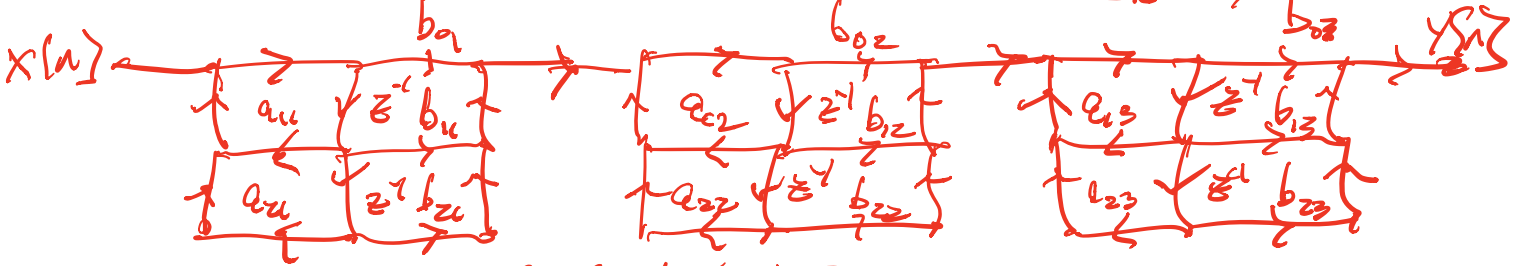
c_1, c_2 $\quad (1 - c_1 z^{-1})(1 - c_2 z^{-1}) = 1 - (c_1 + c_2) z^{-1} + c_1 c_2 z^{-2}$

2 COMPLEX ZEROS

c_1, c_1^* $\quad (1 - c_1 z^{-1})(1 - c_1^* z^{-1}) = 1 - (c_1 + c_1^*) z^{-1} + c_1 c_1^* z^{-2}$

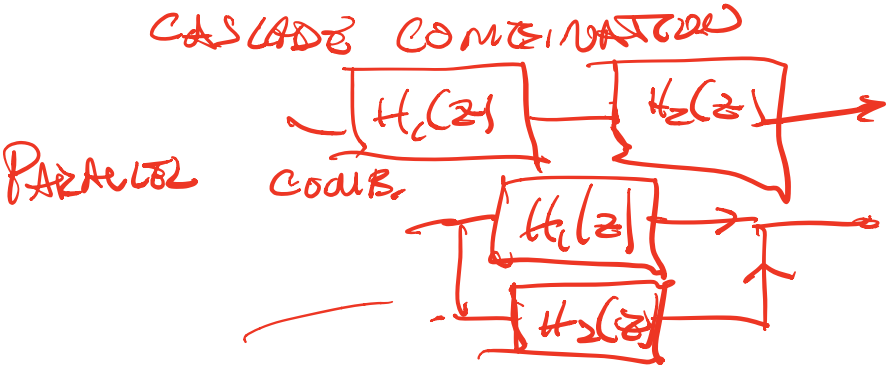
$$= 1 - 2\,\mathrm{Re}[c_1]\, z^{-1} + |c_1|^2 z^{-2}$$

$$H(z) = b_0 \prod_{k=1}^{\max(M/2,\, N/2)} \left(\frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} + a_{2k} z^{-2}} \right) = \frac{Y(z)}{X(z)}$$



CASCADE FORM

CASCADE COMBINATION



$$H(z) = H_1(z)\, H_2(z)$$

$$H(z) = H_1(z) + H_2(z)$$

PARALLEL FORM USING PARTIAL FRACTIONS

$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \sum_{k=1}^{N} \frac{A_k}{1 - d_k z^{-1}} + \sum_{k=0}^{M-N} f_k z^{-k}$$

FOR $N \geq M$

$$A_k = \left. H(z)(1 - d_k z^{-1}) \right|_{z = d_k}$$

CONSIDER BOTH d_1, d_2

$$\frac{A_1}{1 - d_1 z^{-1}} + \frac{A_2}{1 - d_2 z^{-1}} = \frac{(A_1 + A_2) - (A_1 d_2 + A_2 d_1) z^{-1}}{1 - (d_1 + d_2) z^{-1} + d_1 d_2 z^{-2}}$$

IF $d_2 = d_1^*$

$A_2 = A_1^*$

PARALLEL FORM