Filter Realization Process

Digital Filter Implementation (DSYP 6.0-6.5)

* Establish Specs
* Develop Equations that satisfy specs (design) DSYP chap 7
* Implement EQs in software or hardware DSYP chap 6

TYPES OF FILTERS.

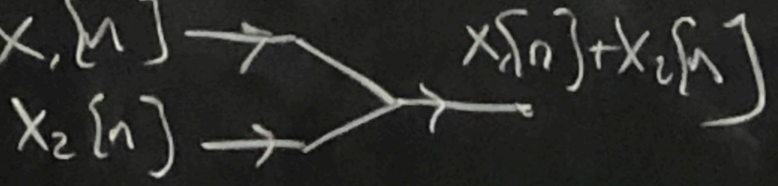
IIR

FIR

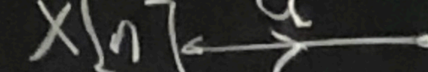
$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + 3x[n] - \frac{3}{4}x[n-1]$$

BUILDING BLOCKS

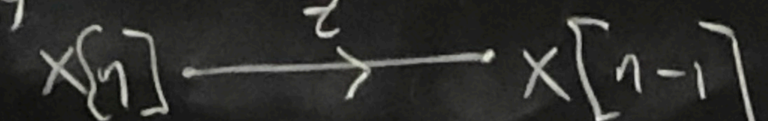
ADDER



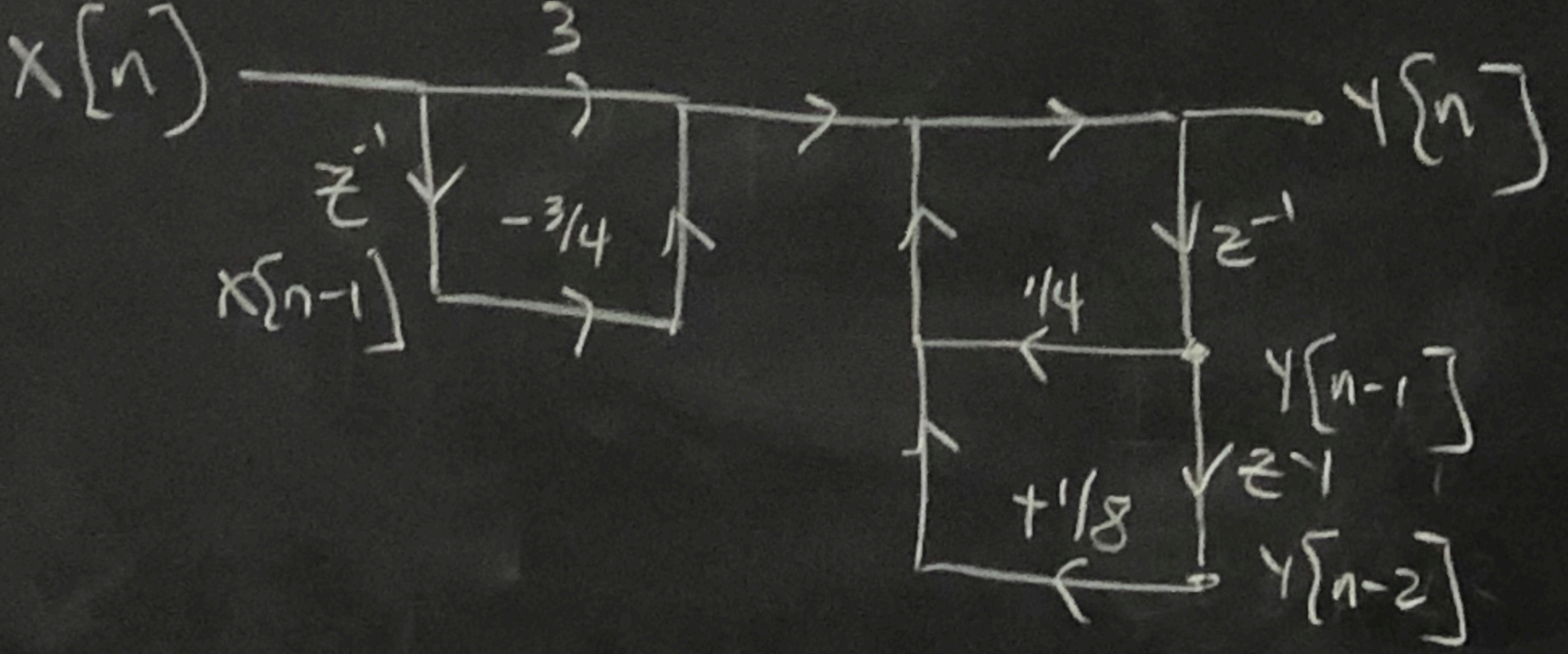
MULT BY CONST



SINGLE DELAY



$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + 3x[n] - \frac{3}{4}x[n-1]$$

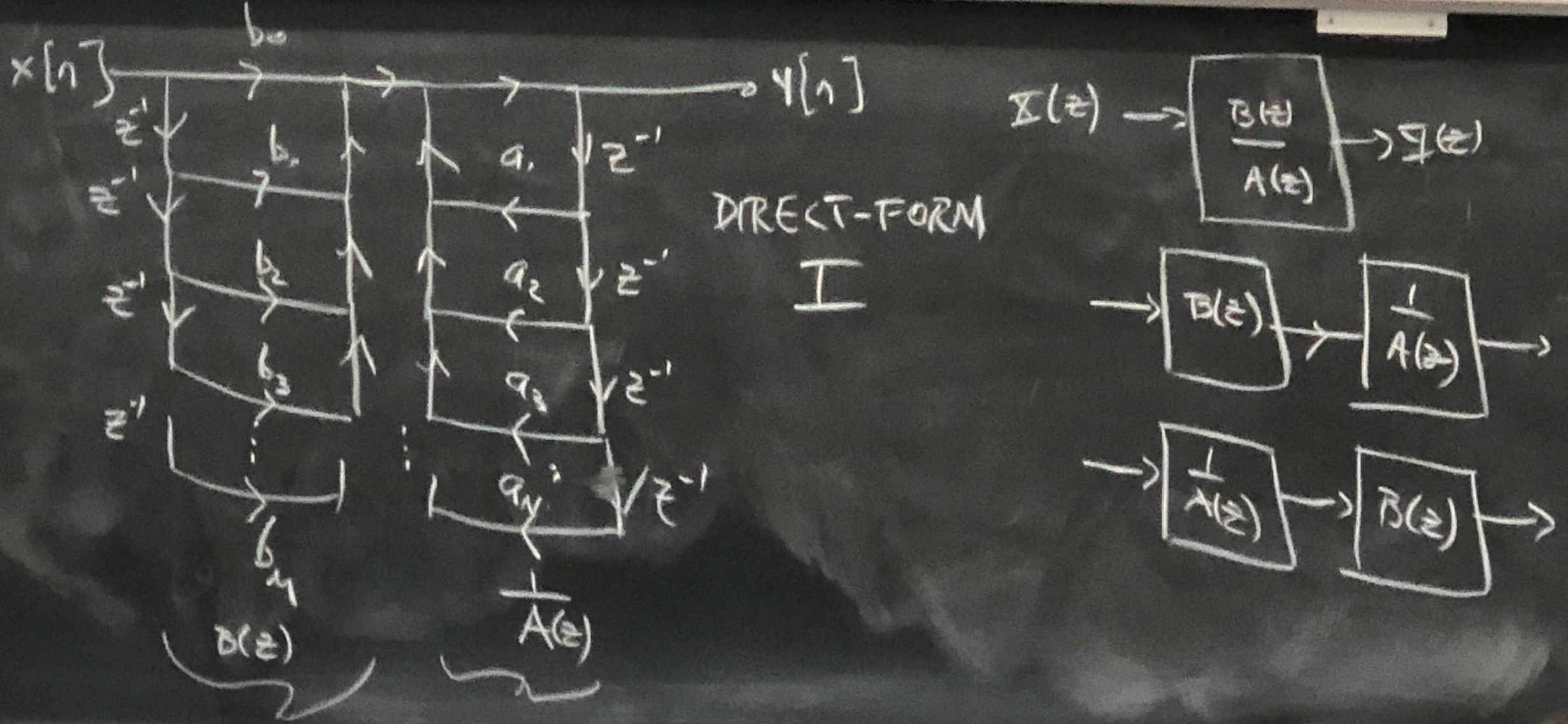


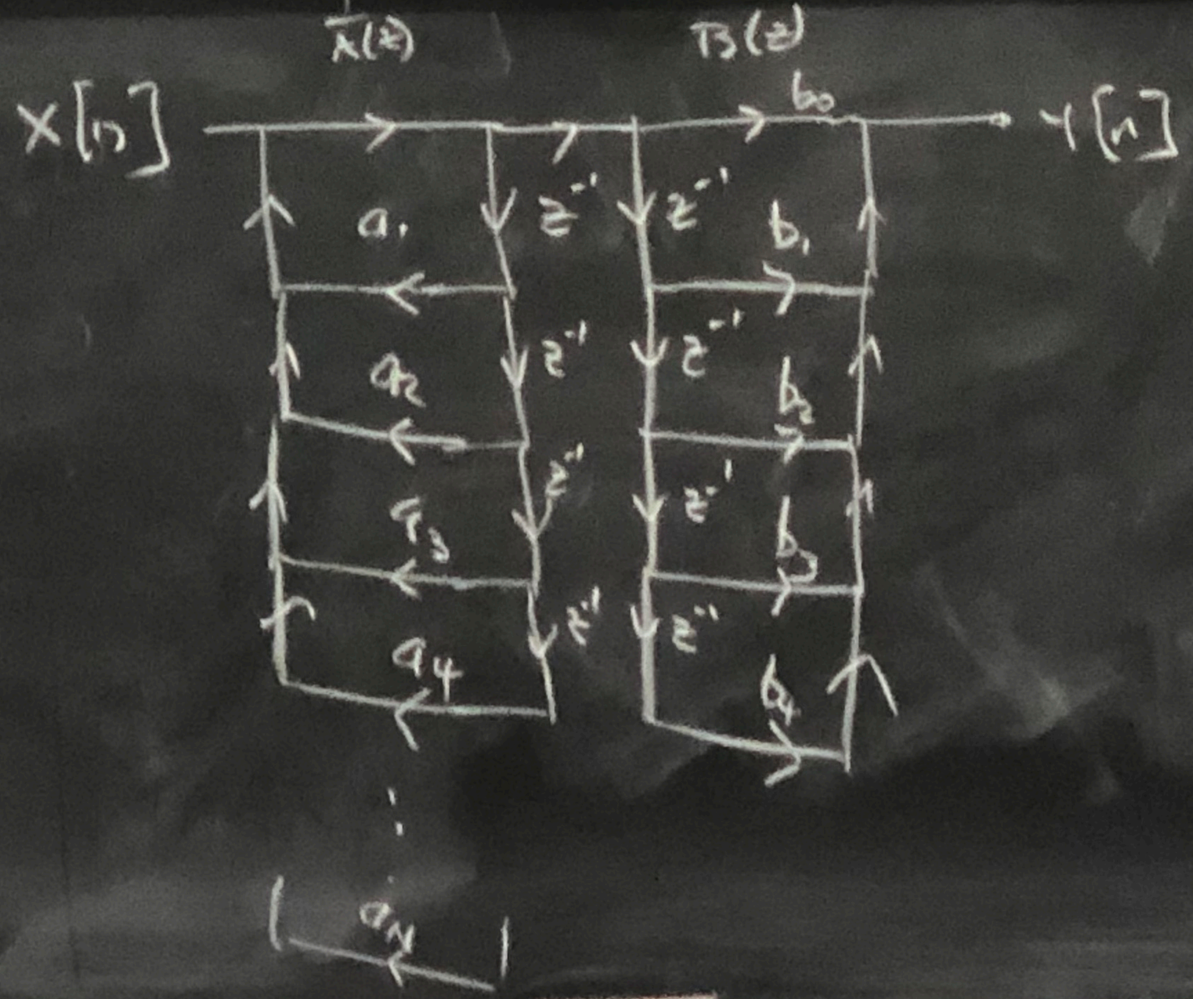
IN GENERAL,

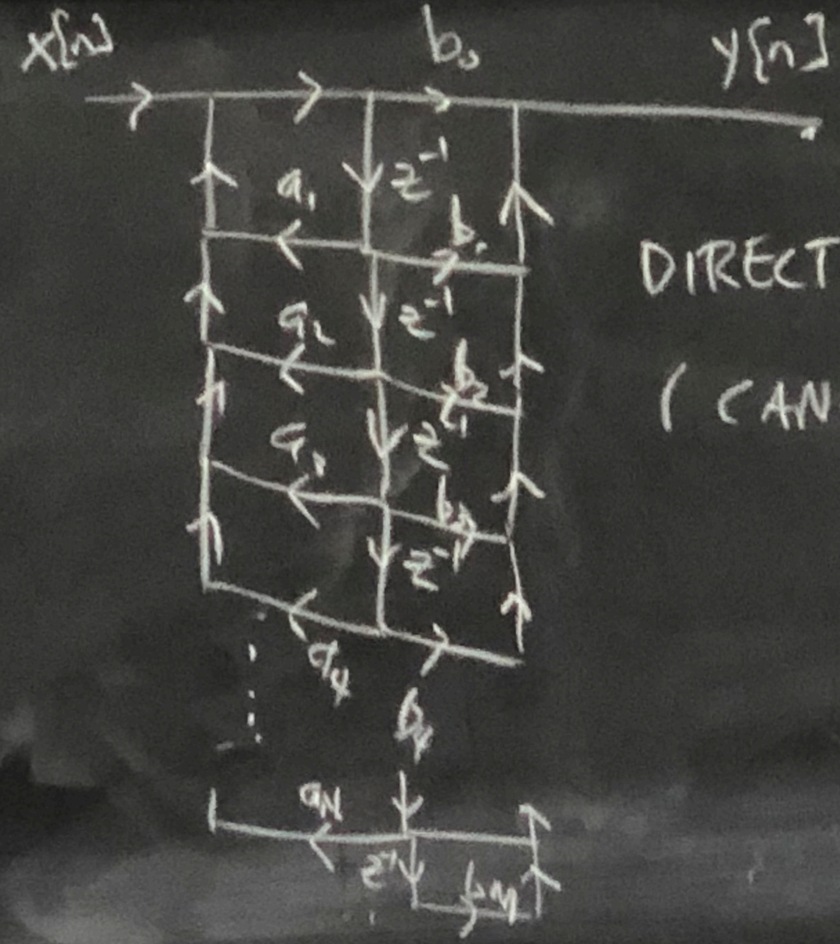
$$H(z) = \frac{\sum_{m=0}^{M} b_m z^{-m}}{1 - \sum_{n=1}^{N} a_n z^{-n}} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \dots + b_M z^{-M}}{1 - a_1 z^{-1} - a_2 z^{-2} - \dots - a_N z^{-N}} = \frac{Y(z)}{X(z)}$$

$$Y[n] - \sum_{k=1}^{N} a_k Y[n-k] = \sum_{m=0}^{M} b_m x[n-m]$$

$$Y[n] = \sum_{k=1}^{N} a_k Y[n-k] + \sum_{m=0}^{M} b_m x[n-m]$$


x[n]
b0
y[n]
z⁻¹
b1
b2
b3
bM
a1
a2
a3
aN
B(z)
1/A(z)
DIRECT-FORM I
X(z)
B(z)/A(z)
Y(z)
B(z)
1/A(z)
1/A(z)
B(z)





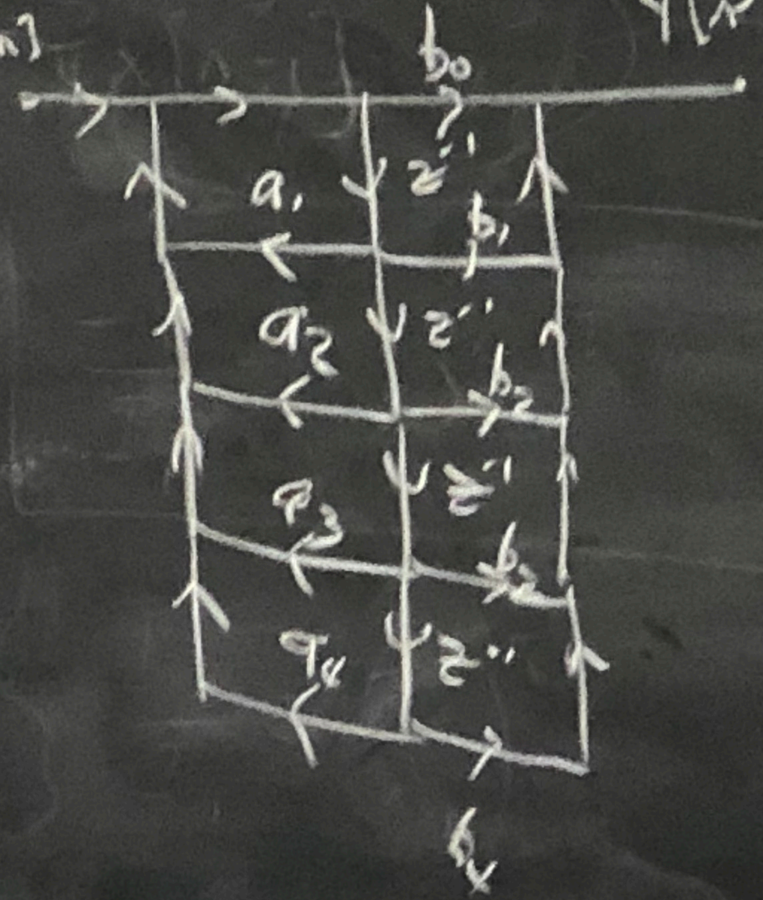
DIRECT-FORM II

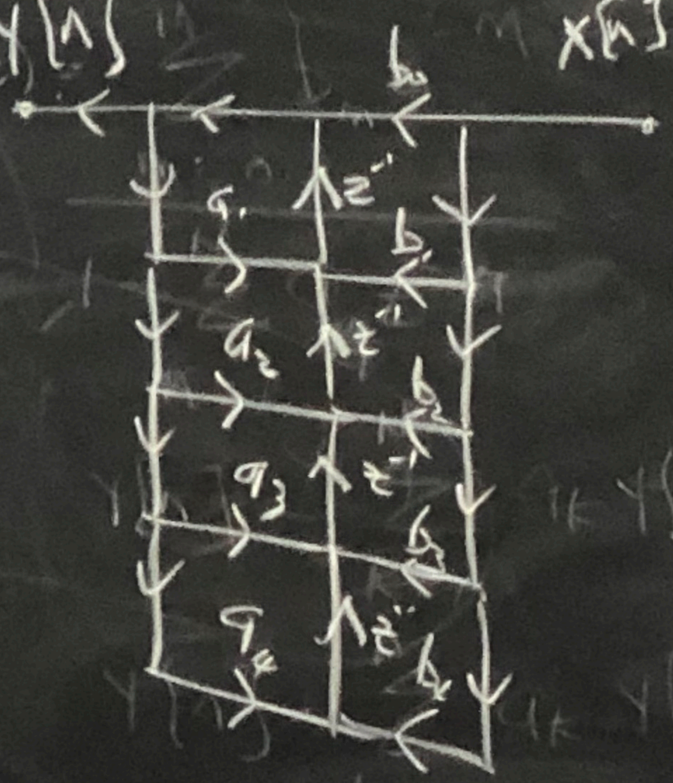
(CANONICAL FORM)

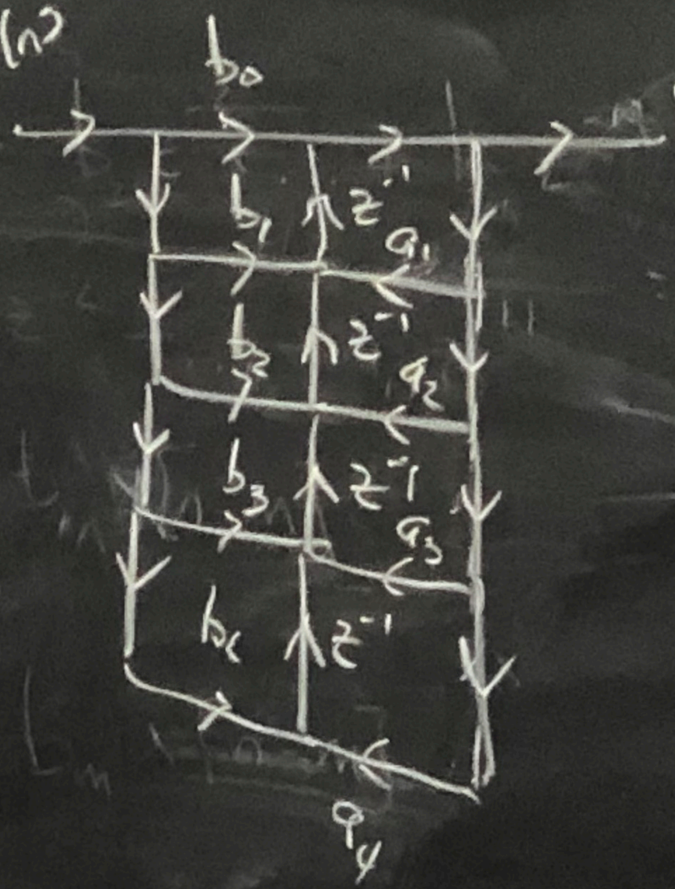
NETWORK TRANSPOSITION THM.
(S. J. MASON)

1. REVERSE ARROWS
2. INTERCHANGE INPUTS + OUTPUTS

⇒ SAME TRANSFER FN!







DF - II TRANSPOSED

BREAKING INTO SMALLER STRUCTURES

$$H(z) = \frac{\sum_{m=0}^{M} b_m z^{-m}}{1 - \sum_{n=1}^{N} a_n z^{-n}} = \frac{b_0 \prod_{\ell=1}^{M} \left(1 - c_\ell z^{-1}\right)}{\prod_{k=1}^{N} \left(1 - d_k z^{-1}\right)}$$

$$= b_0 \prod_{k=1}^{\frac{M}{2}} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

SECOND-ORDER SYSTEM:

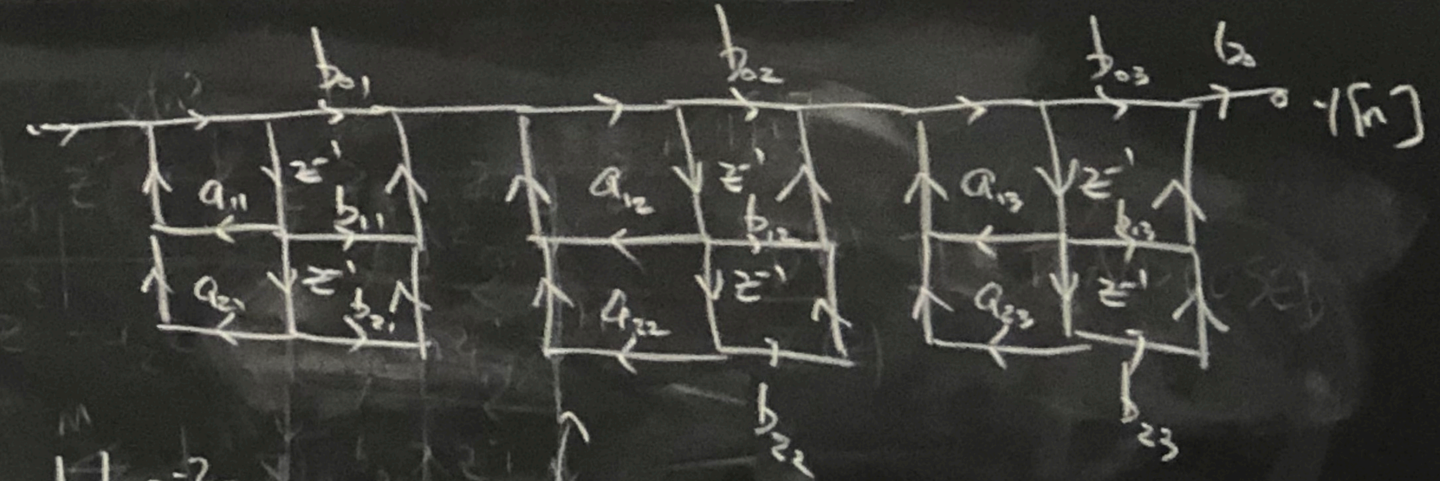
(a) TWO REAL POLES, d_1, d_2

$$(1-d_1 z^{-1})(1-d_2 z^{-1}) = 1-(d_1+d_2)z^{-1}+d_1 d_2 z^{-2}$$

(b) COMPLEX CONJ POLE PAIR, $d_1 = re^{j\theta}$, $d_2 = re^{-j\theta}$

$$(1-d_1 z^{-1})(1-d_2 z^{-2}) = 1-2r\cos\theta\, z^{-1}+r^2 z^{-2}$$

$$1-a_1 z^{-1}-a_2 z^{-2}$$



CASCADE FORM

PARALLEL FORM

$$H(z) = \sum_{k=1}^{N} \frac{A_k}{1 - d_k z^{-1}} + \sum_{l=0}^{M-N} C_l z^{-l}$$

$$= \sum_{l=0}^{M-N} c_l z^{-l} + \sum_{k=1}^{\frac{N}{2}} \frac{e_{0k} + e_{1k} z^{-1}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

TAKING TWO

$$\frac{A_1}{1 - d_1 z^{-1}} + \cdots$$

$$= \frac{e_0 + e_1}{1 - a_1 z^{-1}}$$

TAKING SUMS -
TWO AT A TIME:

$$\frac{A_1}{1-d_1 z^{-1}} + \frac{A_2}{1-d_2 z^{-1}} = \frac{(A_1+A_2) - (A_2 d_1 + A_1 d_2) z^{-1}}{1-(d_1+d_2)z^{-1} + d_1 d_2 z^{-2}}$$

$$= \frac{e_0 + e_1 z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$

TAKING SUMS TWO AT A TIME:

$$\frac{A_1}{1-d_1 z^{-1}} + \frac{A_2}{1-d_2 z^{-1}} = \frac{e_0 + e_1 z^{-1}}{1 - a_1 z^{-1} - a_2 z^{-2}}$$