3/20/24

FIR FILTER STRUCTURES

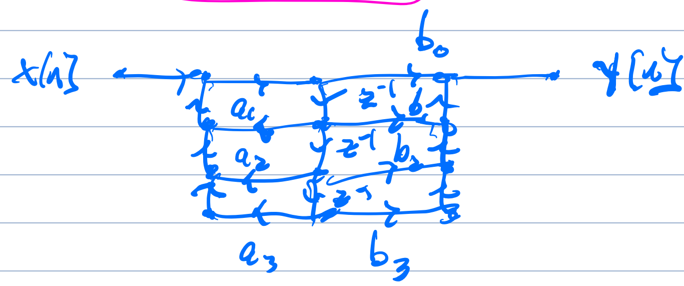
(DS&P 6.4-6.5 + NOTES ON THE FREQUENCY-SAMPLING FORM)

$\boxed{\text{QUIZ 2 WEDNESDAY 4/3}}$

IIR REVIEW

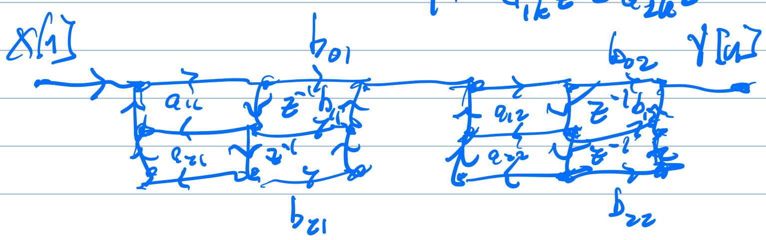
$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

IIR DF-II

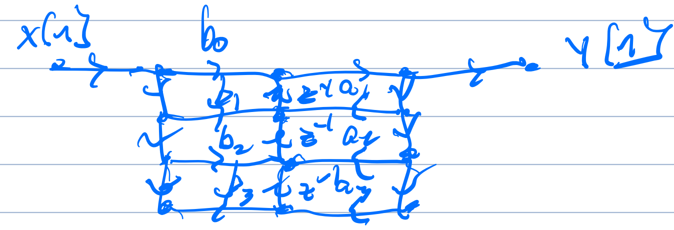


IIR CASCADE FORM

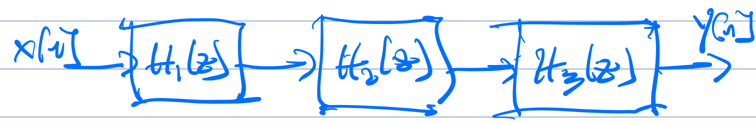
$$H(z) = b_0 \prod_{k=1}^{\max\left(\frac{N}{2}, \frac{M}{2}\right)} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$



IIR TRANSPOSED FORM BASED ON DF-II



CASCADE FORM



IIR PARALLEL FORM

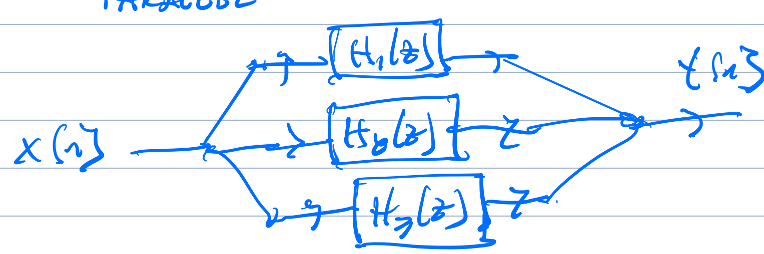
PARTIAL FRACTION DECOMPOSITION

$$H(z) = \frac{\sum_{\ell=0}^{N} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

IF # POLES UNIQUE

$$= \sum_{k=1}^{N} \frac{A_k}{1 - d_k z^{-1}} + \sum_{\ell=0}^{M-N} f_\ell z^{-\ell}$$

PARALLEL FORM

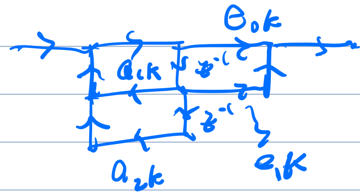


TWO REAL POLES:

$$\frac{A_1}{1-d_1z^{-1}}+\frac{A_2}{1-d_2z^{-1}} = \frac{(A_1+A_2) - z^{-1}(A_1d_2+A_2d_1)}{1-(d_1+d_2)z^{-1}+d_1d_2z^{-2}}$$

COMPLEX POLE PAIR

$$\frac{A}{1-dz^{-1}}+\frac{A^*}{1-d^*z^{-1}} = \frac{2\mathrm{Re}(A) - z^{-1}\,2\mathrm{Re}(Ad^*)}{1-2\mathrm{Re}(d)z^{-1}+|d|^2z^{-2}}$$

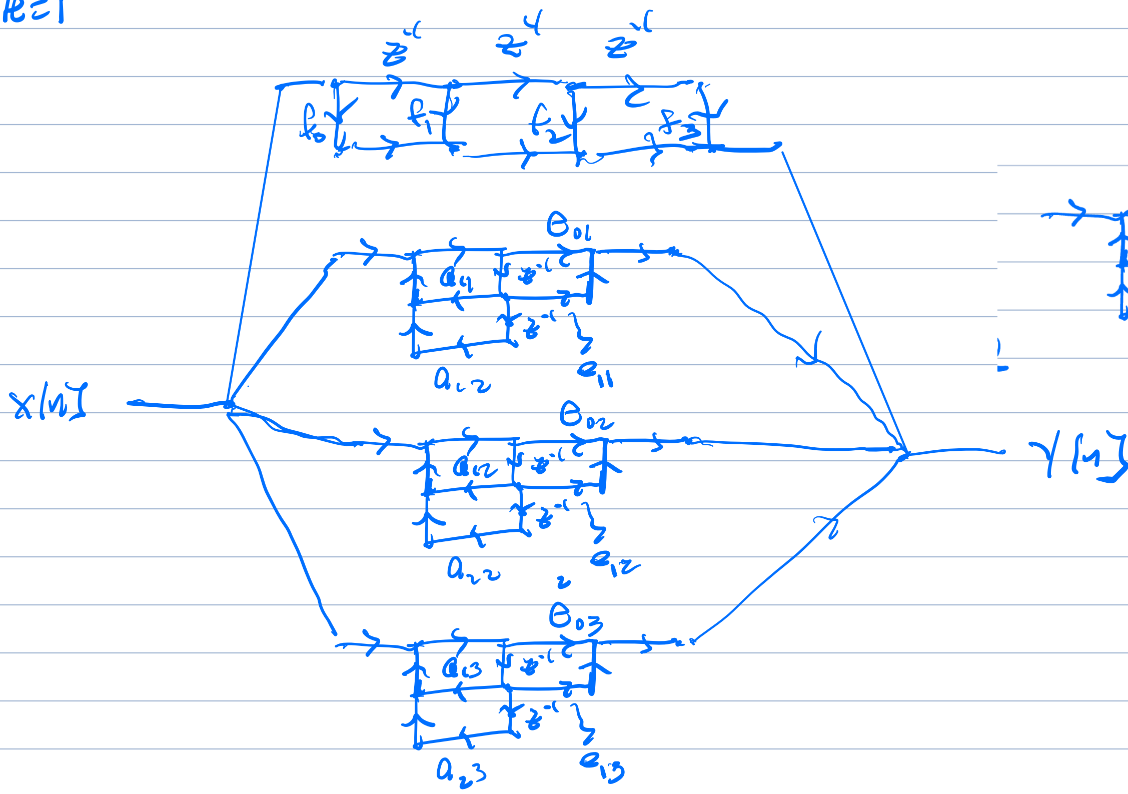


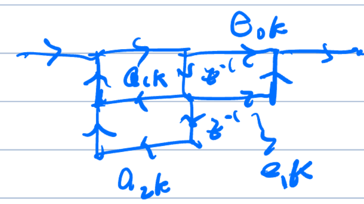
$$\frac{A_1}{1-d_1z^{-1}}+\frac{A_2}{1-d_2z^{-1}}$$

$$= \frac{(A_1+A_2) - z^{-1}(d_1+d_2)}{1-(d_1+d_2)z^{-1}+d_1d_2z^{-2}}$$

COMPLETE PARALLEL FORM

$$H(z) = \sum_{k=1}^{N}\frac{A_k}{1-d_kz^{-1}} + \sum_{\ell=0}^{M-N} f_\ell z^{-\ell}$$





EFFECTS OF COEFFICIENT QUANTIZATION

$$H(z) = \frac{\sum_{l=0}^{M} b_l z^{-l}}{1 - \sum_{k=0}^{N} a_k z^{-k}} = \frac{B(z)}{A(z)} = \frac{b_0 \prod_{l=1}^{M} (1 - c_l z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$$

WANT TO DETERMINE EFFECT OF $\{a_k\}$ ON $\{d_k\}$

$$A(z) = 1 - \sum_{k=0}^{N} a_k z^{-k} = \prod_{r=1}^{N} (1 - d_r z^{-1})$$

LET'S EVALUATE $\dfrac{\partial d_l}{\partial a_k}$

$$\left.\frac{\partial A(z)}{\partial a_k}\right|_{z=d_l} = \left.\frac{\partial A(z)}{\partial d_l}\right|_{z=d_l} \cdot \frac{\partial d_l}{\partial a_k}$$

$$\frac{\partial d_l}{\partial a_k} = \frac{\left.\frac{\partial A(z)}{\partial a_k}\right|_{z=d_l}}{\left.\frac{\partial A(z)}{\partial d_l}\right|_{z=d_l}} = \frac{\left.-z^{-k}\right|_{z=d_l}}{\left.-z^{-1} \prod_{\substack{r=1 \\ r \neq l}}^{N} (1 - d_r z^{-1})\right|_{z=d_l}} \cdot \frac{z^N}{z^N}$$

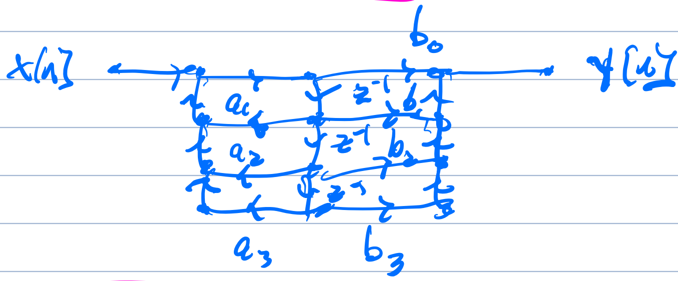
$$\frac{\partial d_l}{\partial a_k} = \frac{\left.-z^{N-k}\right|_{z=d_l}}{\left.-\cancel{z^{-1} z} \prod_{\substack{r=1 \\ r \neq l}}^{N} (z - d_r)\right|_{z=d_l}} = \frac{-d_l^{N-k}}{-\prod_{\substack{r=1 \\ r \neq l}}^{N} (d_l - d_r)}$$

FIR FILTER IMPLEMENTATION BASED ON IIR DESIGNS

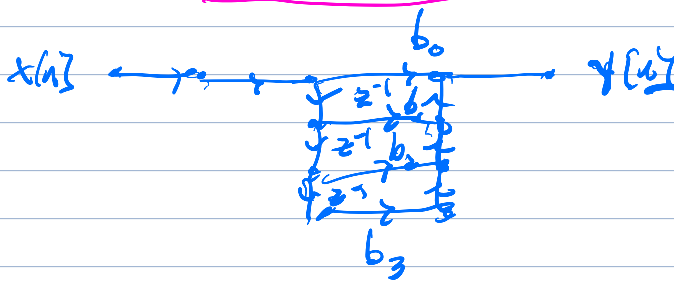
$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}}$$

FIR $\Rightarrow \{a_k\} = 0$

IIR DF-II

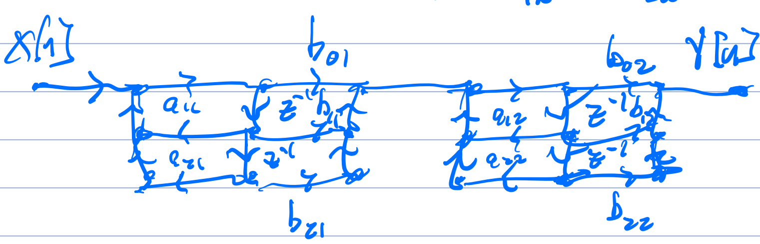


FIR DF-II

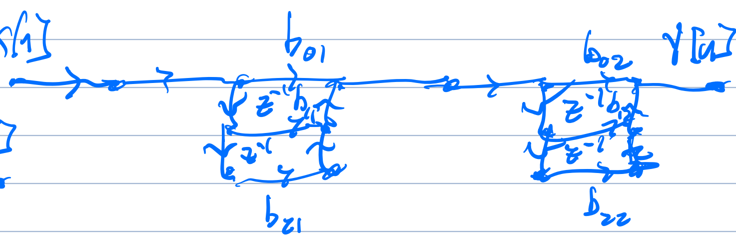


IIR CASCADE FORM

$$H(z) = b_0 \prod_{k=1}^{\max(\frac{N}{2}, \frac{M}{2})} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$



FIR CASCADE FORM

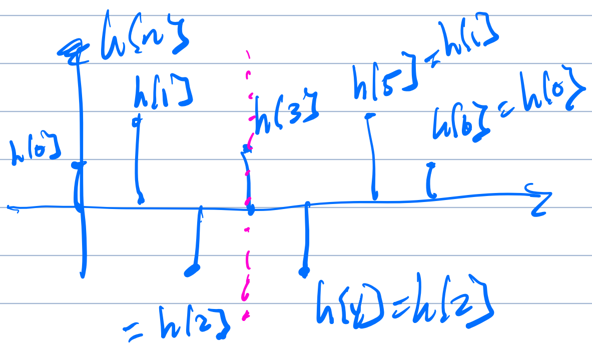


FIR PARALLEL FORM

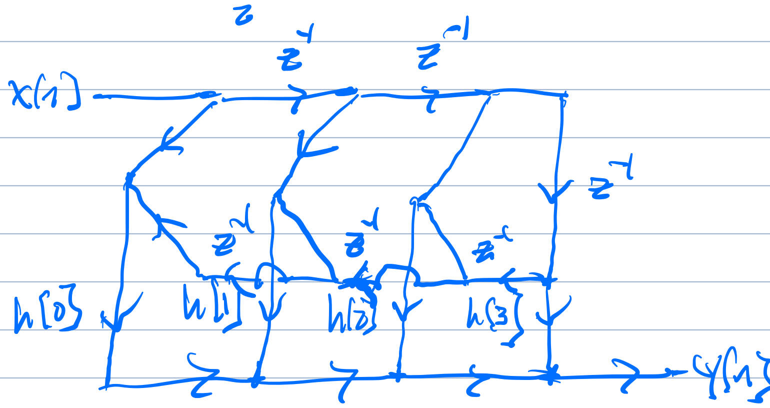
[DOES NOT EXIST BECAUSE WE NEED POLES FOR PARTIAL FRACTIONS]

FIR LINEAR PHASE FORM

IF $H(e^{j\omega})$ IS LINEAR PHASE, $h[n]$ SYMMETRIC ABOUT $n = \frac{N-1}{2}$



$N = 7 \quad \frac{N-1}{2} = 3$



FIR FREQUENCY-SAMPLING FORM

FOR FIR FILTERS

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n} \quad = \cancel{\sum_{n=0}^{M} b_n z^{-n}}$$

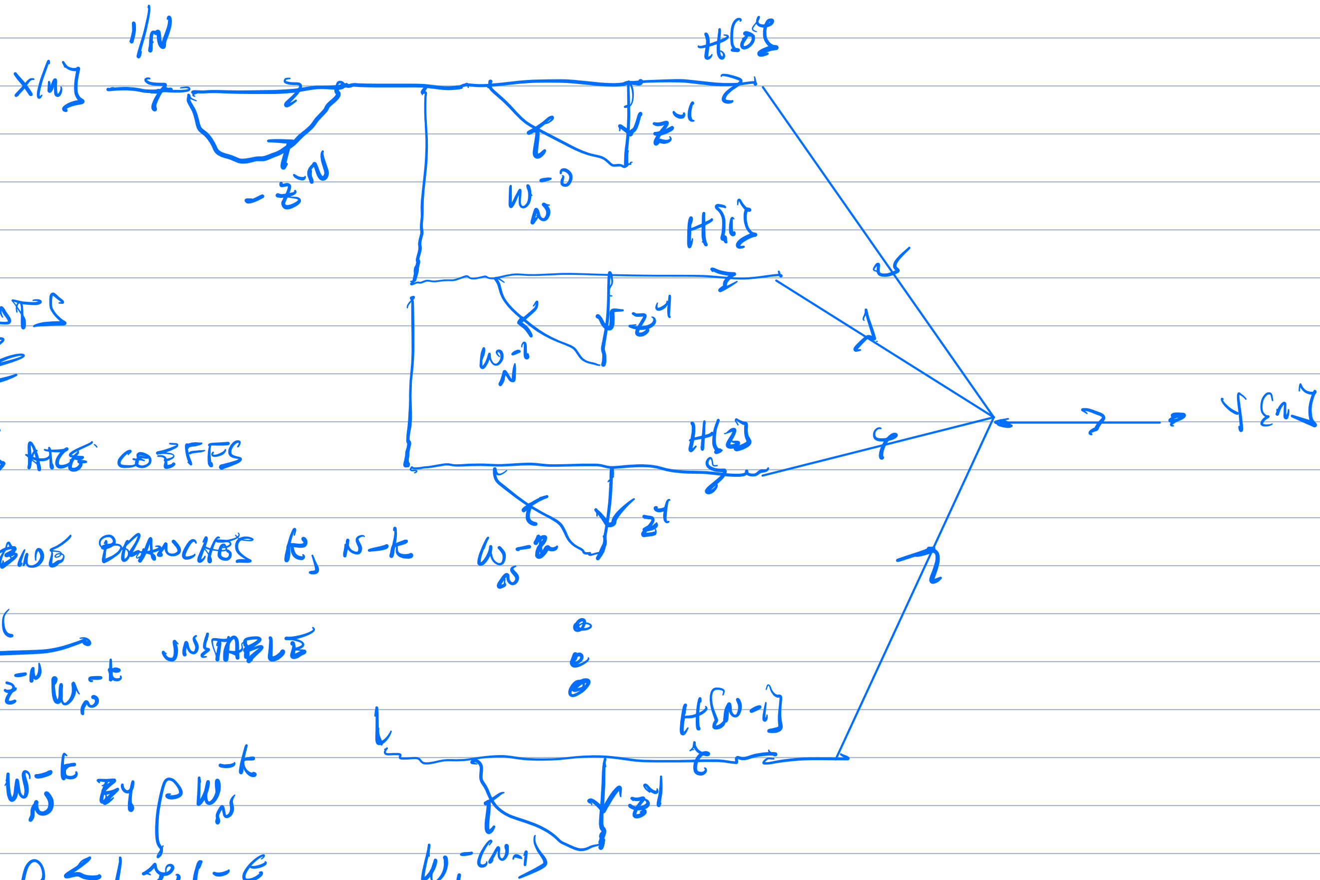
$$= \sum_{n=0}^{N-1} \underbrace{\frac{1}{N}\sum_{k=0}^{N-1} H[k] W_N^{-nk}}_{h[n]} z^{-n}$$

$$= \sum_{k=0}^{N-1} \frac{1}{N} H[k] \sum_{n=0}^{N-1} \left(W_N^{-k} z^{-1}\right)^n$$

$$W_N^{-Nk} = e^{+j\frac{2\pi}{N}Nk} = 1$$

$$H(z) = \sum_{k=0}^{N-1} \frac{1}{N} H[k] \frac{1 - \cancel{W_N^{-Nk}} z^{-N}}{1 - W_N^{-k} z^{-1}}$$

$$H(z) = \frac{1}{N}\left(1 - z^{-N}\right) \sum_{k=0}^{N-1} \frac{H[k]}{1 - W_N^{-k} z^{-1}}$$



COMMENTS

1. $H[k]$ ARE COEFFS
2. COMBINE BRANCHES k, $N-k$
3. $\frac{1}{1 - z^{-N} W_N^{-k}}$ UNSTABLE

REPLACE W_N^{-k} BY ρW_N^{-k}

$\rho < 1$ $\approx 1 - \epsilon$