Carnegie Mellon | DSP | Electrical & Computer ENGINEERING

Fundamentals of Signal Processing (18-491)
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THE FREQUENCY-SAMPLING FIR FILTER IMPLEMENTATION

In this handout we derive and discuss the frequency-sampled implementation of FIR filters.

The frequency-sampled form is easy to derive. We begin with the definition of the z-transform of an FIR filter:

$$H(z) = \sum_{n=0}^{N-1} h[n] z^{-n}$$

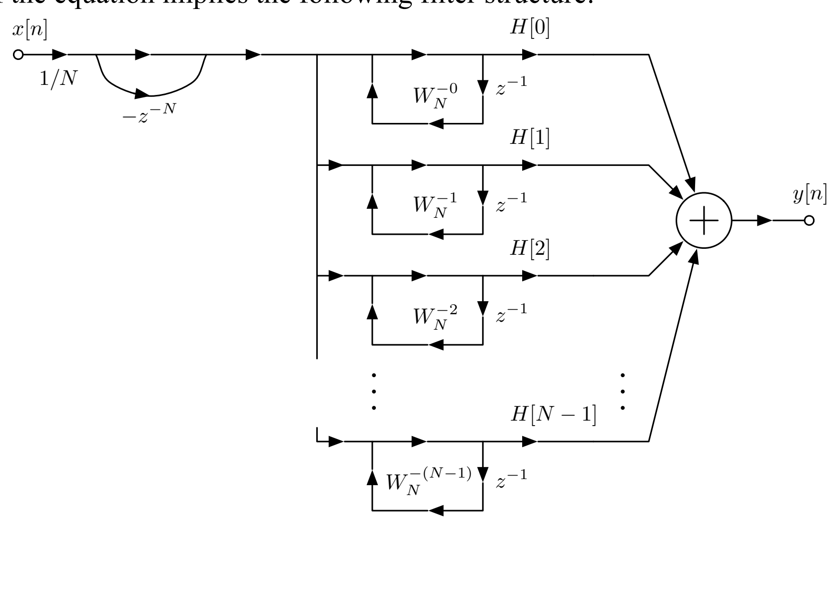
Expressing $h[n]$ as in inverse DFT, we obtain

$$H(z) = \sum_{n=0}^{N-1} \frac{1}{N} \sum_{k=0}^{N-1} H[k] W_N^{-nk} z^{-n} = \sum_{k=0}^{N-1} H[k] \frac{1}{N} \sum_{n=0}^{N-1} (W_N^{-k} z^{-1})^n$$

Using the finite sum of exponentials formula to realize the inner sum in the last term, we obtain

$$H(z) = \sum_{k=0}^{N-1} H[k] \frac{1}{N} \left(\frac{1 - W_N^{-Nk} z^{-N}}{1 - W_N^{-k} z^{-1}} \right) = \frac{1}{N}(1 - z^{-N}) \sum_{k=0}^{N-1} \frac{H[k]}{1 - W_N^{-k} z^{-1}}$$

The latter form of the equation implies the following filter structure:



Some comments:

- The variable coefficients in this form are the DFT coefficients $H[k]$. This is convenient if one wishes to manipulate the response of a filter directly from the frequency-domain representation. In contrast, the variable coefficients of the direct-form FIR implementation are the coefficients of the unit sample response.
- In many practical cases, a large number of the DFT coefficients may be zero, so the frequency-sampled form has the potential to be very computationally efficient.
- While the coefficients of the filter representing the twiddle factors W_N^{-k} and $H[k]$ are complex, $H[k]$ and $H[N-1-k]$ will be complex conjugates of one another, as will be W_N^{-k} and $W_N^{-(N-1-k)}$, if $h[n]$ is real. Because of this, a frequency-sampled FIR implementation with real coefficients in second-order sections can be obtained by combining the responses for channels k and $N-1-k$. This topic is explored in Problem 7.4.
- The frequency-sampled FIR implementation has a N feedback loops in parallel, each with a pole right on the unit circle, which makes the filter potentially unstable. This problem can be resolved by multiplying the coefficients W_N^{-k} and the factor of $-z^{-N}$ at the input by a constant r that is close to but less than 1.