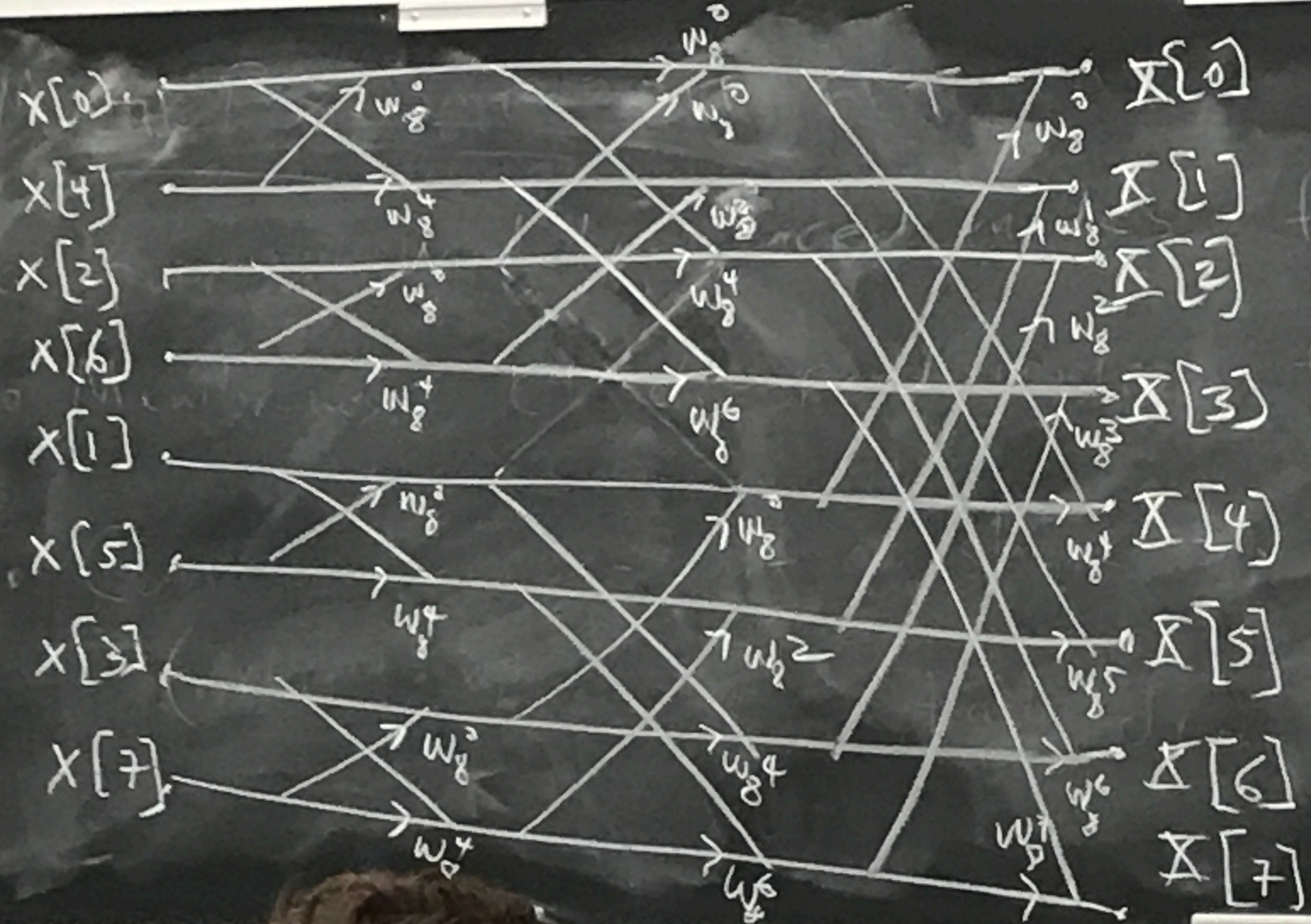
$$X[k] = \sum_{n=0}^{N-1} x(n)\, W_N^{nk}$$

$$x(n) = \frac{1}{N} \sum_{k=0}^{N-1} X[k]\, W_N^{-nk}$$

ALTERNATE STRUCTURES, IMPLEMENTATIONS OF THE FFT



THE 8-PT DIT FFT

0	000
4	100
2	010
6	110
1	001
5	101
3	011
7	111

BIT-REVERSED ORDER

OBTAINING THE IDFT.

I. NEGATE SIGNS OF W_N^k, MULT BY $\frac{1}{N}$ AT END

II. $$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-nk}$$

1. COMPL. CONJ INPUT $X[k]$
2. COMPUTE "NORMAL" DFT
3. COMPLEX CONJ. RESULTS, DIVIDE BY N

$$N x^*[n] = \sum_{k=0}^{N-1} X^*[k] W_N^{nk}$$

$$\left(W_N^{-nk}\right)^* = \left(e^{+j\frac{2\pi nk}{N}}\right)^* = e^{-j\frac{2\pi nk}{N}} = W_N^{nk}$$

DECIMATION IN FREQUENCY

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x[n] W_N^{nk} + \sum_{n=\frac{N}{2}}^{N-1} x[n] W_N^{nk}$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] W_N^{nk} + (-1)^k x[n+\tfrac{N}{2}] W_N^{nk} \right)$$

$$= \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + (-1)^k x[n+\tfrac{N}{2}] \right) W_N^{nk}$$

FOR k EVEN LET $k = 2r$

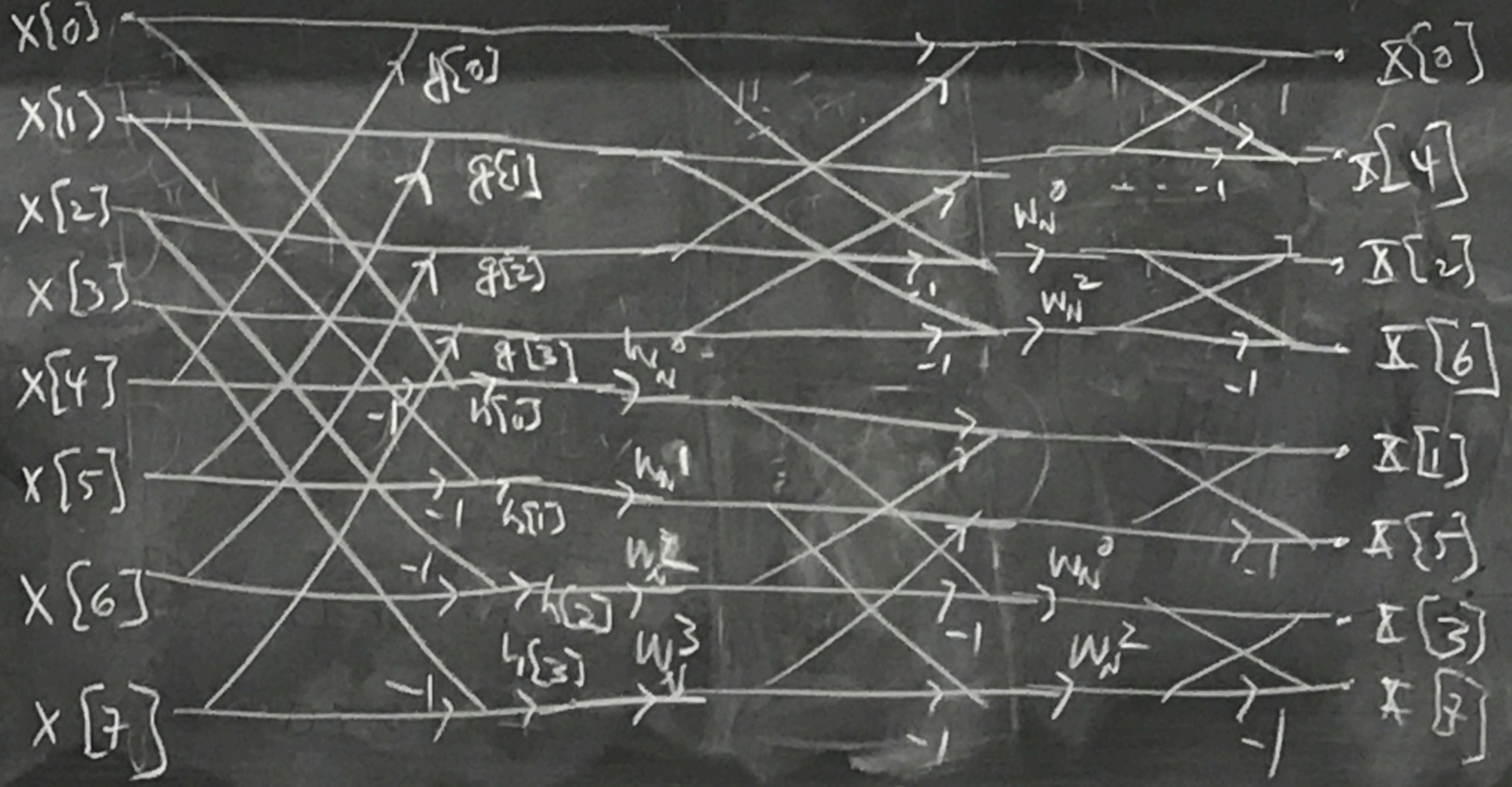
EVEN k

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} \Big[\underbrace{x[n] + (-1)^{2r} x[n+\tfrac{N}{2}]}_{x[n] + x[n+\frac{N}{2}]} \Big] W_N^{2rn} = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + (-1)^{2r} x[n+\tfrac{N}{2}] \right) W_{\frac{N}{2}}^{rn}$$

ODD k

$k = 2r+1$

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} \Big[\underbrace{x[n] - (-1)^{2r} x[n+\tfrac{N}{2}]}_{(x[n] - x[n+\frac{N}{2}])\, W_N^n} \Big] W_N^{(2r+1)n} = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] - (-1)^{2r} x[n+\tfrac{N}{2}] \right) W_N^n \cdot W_{\frac{N}{2}}^{rn}$$

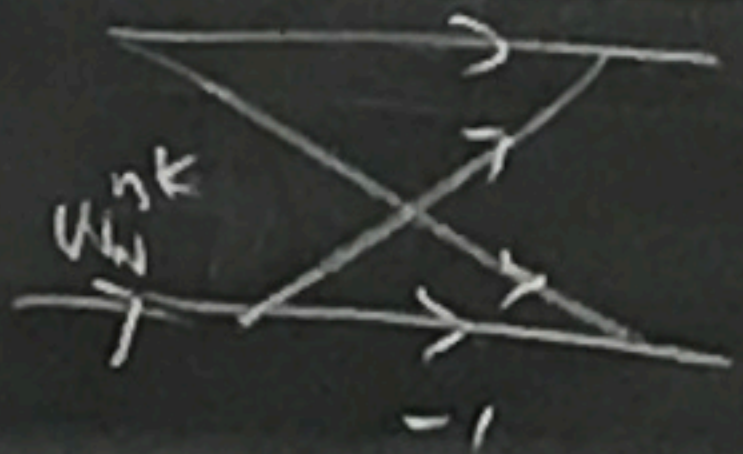


x[0]
x[1]
x[2]
x[3]
x[4]
x[5]
x[6]
x[7]
g[0]
g[1]
g[2]
g[3]
h[0]
h[1]
h[2]
h[3]
W_N^0
W_N^1
W_N^2
W_N^3
W_N^0
W_N^2
W_N^0
W_N^2
-1
X[0]
X[4]
X[2]
X[6]
X[1]
X[5]
X[3]
X[7]

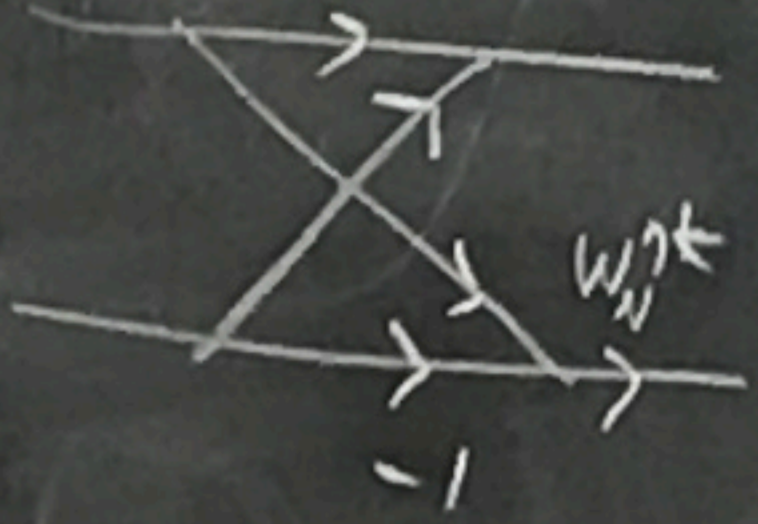
TRANSPOSE:

1. INTERCHANGE INPUTS + OUTPUTS
2. REVERSE ARROWS

DIT



DIF



FAST DFTs FOR $N \neq 2^{\nu}$

$$N \neq 2^{\nu} = p_1 p_2 p_3 \cdots p_{\nu}$$

LET $q_1 = \frac{N}{p_1}$, $q_2 = \frac{N}{p_1 p_2}$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} = \sum_{r=0}^{q_1-1} x[p_1 r] W_N^{p_1 r k} + \sum_{r=0}^{q_1-1} x[p_1 r+1] W_N^{(p_1 r+1)k} + \cdots + \sum_{r=0}^{q_1-1} x[p_1 r + (p_1-1)] W_N^{(p_1 r+(p_1-1))k}$$

$$= \sum_{r=0}^{q_1-1} \sum_{\ell=0}^{p_1-1} x[p_1 r+\ell] W_N^{(p_1 r+\ell)k} = \sum_{r=0}^{q_1-1} \left(\sum_{\ell=0}^{p_1-1} x[p_1 r+\ell] W_{N/p_1=q_1}^{rk} \right) W_N^{\ell k}$$