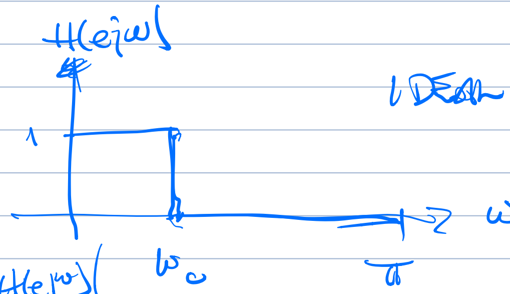
3/29/21

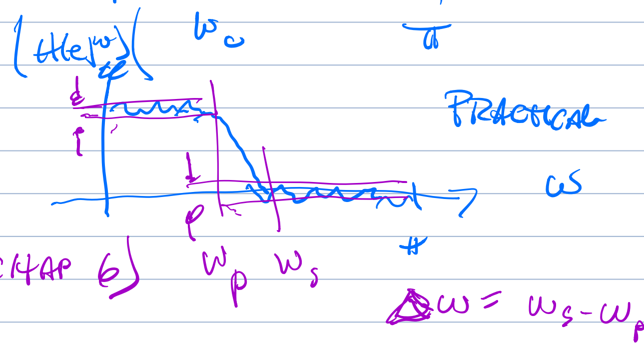
Digital Filter Implementation

(OS48 6.0 - 6.5)

Building a Filter ...

1. Establish Specs
2. Develop Equations that meet specs
   (Filter Design, OS48 Chap 7)
3. Implement Eqs. in Hardware or Software
   (Filter Implementation, OS48 Chap 6)





Two Filter Types

IIR - $h[n]$ Right-sided, Left-sided, Both-sided

Poles + Zeros

FIR - $h[n]$ Finite Duration

Zeros only except for $z=0$

Advantages of IIR Filter

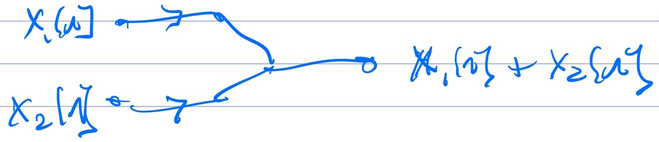
* Most efficient design to realize specs
* Design methods to transform good CT filter designs into good DT filter designs

Advantages of FIR Filters

* Always stable
* Always can be linear phase
* Can implement convolution using FFTs

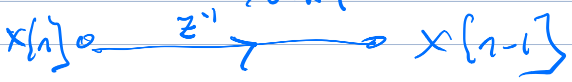
SIGNAL FLOW NOTATION

ADDITION



MULT BY CONST

UNIT DELAY

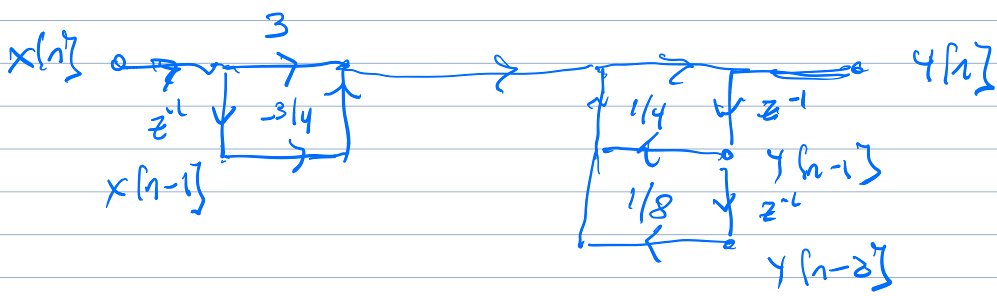


$$y[n] - \frac{1}{4}y[n-1] - \frac{1}{8}y[n-2] = 3x[n] - \frac{3}{4}x[n-1]$$

IIR IMPLEMENTATION

$$Y(z)\left(1 - \frac{1}{4}z^{-1} - \frac{1}{8}z^{-2}\right) = X(z)\left(3 - \frac{3}{4}z^{-1}\right)$$

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{8}y[n-2] + 3x[n] - \frac{3}{4}x[n-1]$$



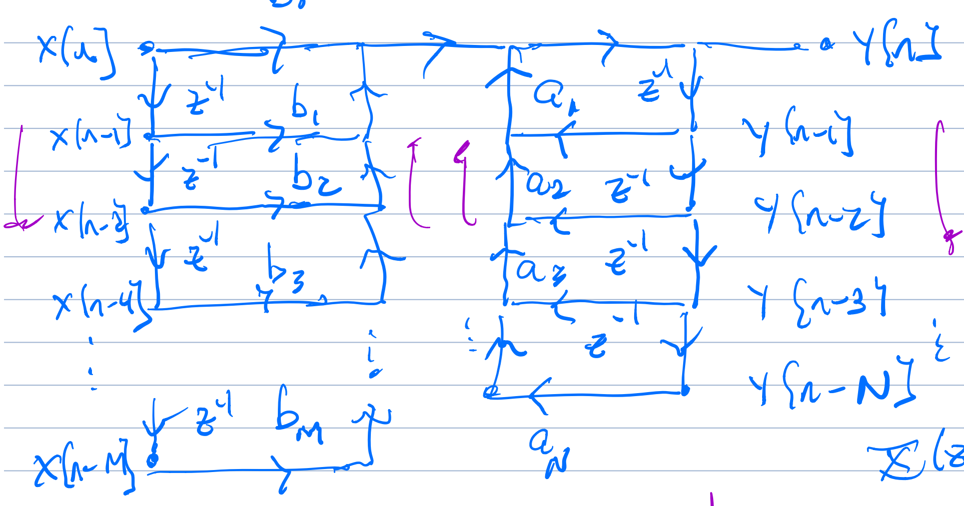
IN GENERAL,

$$H(z) = \frac{\sum_{l=0}^{M} b_l z^{-l}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{Y(z)}{X(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2} + \ldots + b_M z^{-M}}{1 - a_1 z^{-1} - a_2 z^{-2} - \ldots - a_N z^{-N}} = \frac{B(z)}{A(z)}$$

[OR $1 + a_1 z^{-1} + a_2 z^{-2} + \ldots + a_N z^{-N}$]

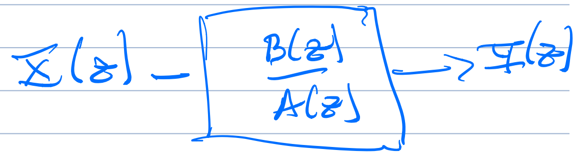
$$y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{l=0}^{M} b_l x[n-l]$$

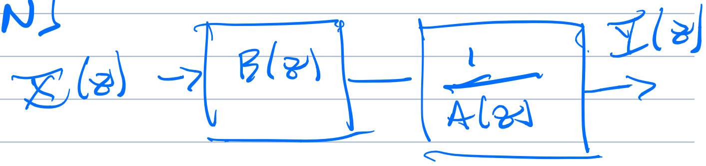
DIRECT FORM I

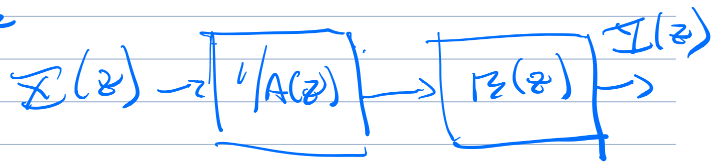


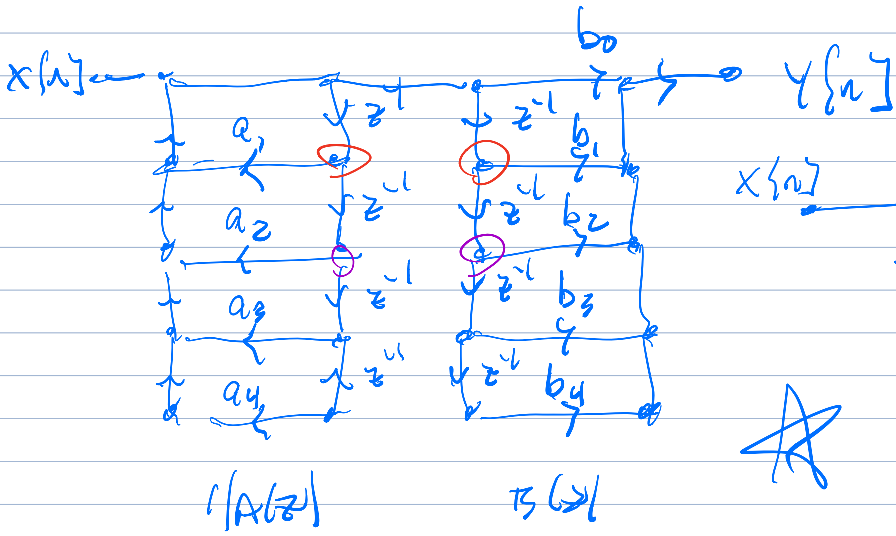
FEED FORWARD $B(z)$

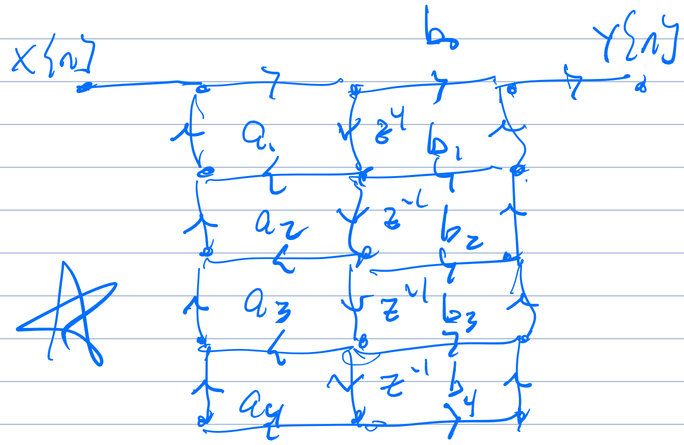
$1/A(z)$ FEEDBACK











DIRECT FORM II

THE CANONICAL FORM

ALTERNATE FORMS

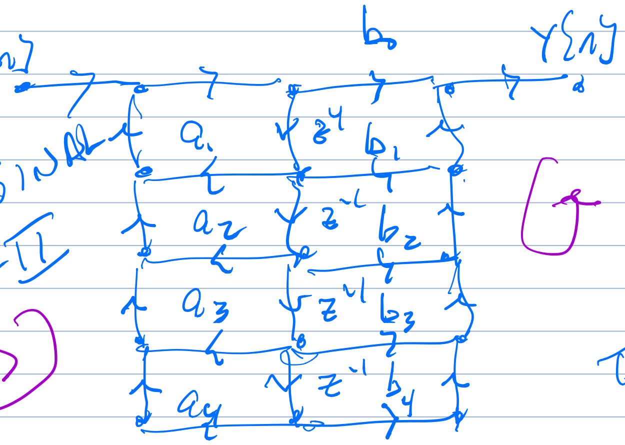
TRANSPOSED FORMS

NETWORK TRANSPOSITION LEMMA (S. J. MASON)

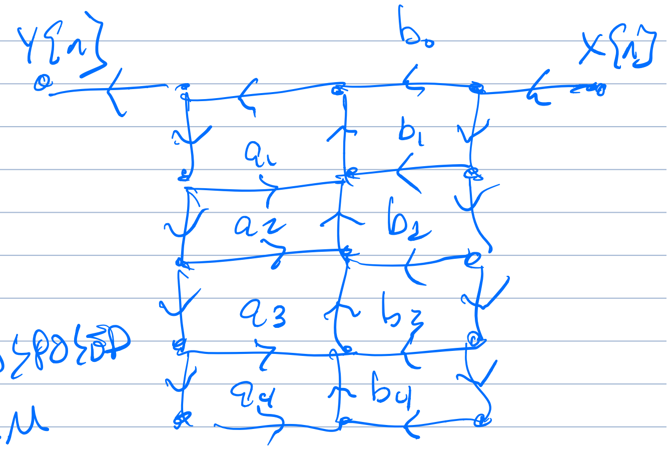
IF

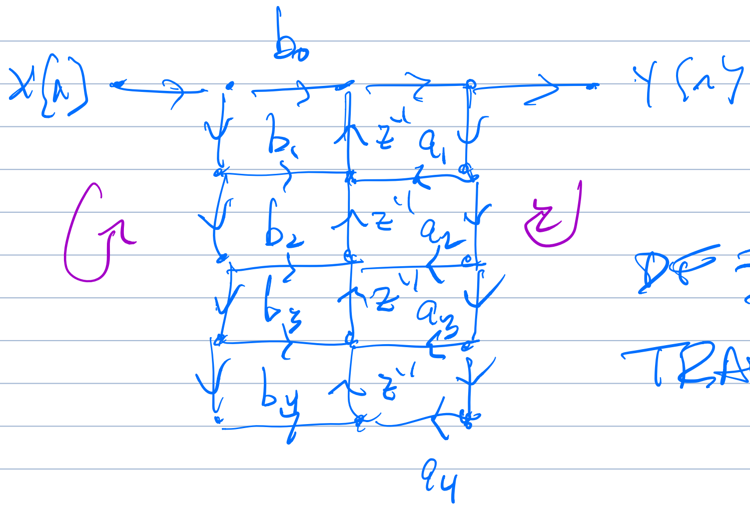
1. REVERSE ARROWS
2. INTERCHANGE INPUT + OUTPUT $\Rightarrow$ SAME TRANSFER FN.

ORIGINAL DF-II



TRANSPOSED FORM

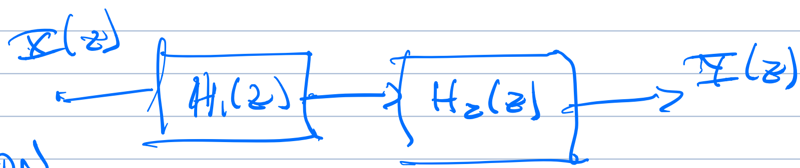




DF-II TRANSPOSED

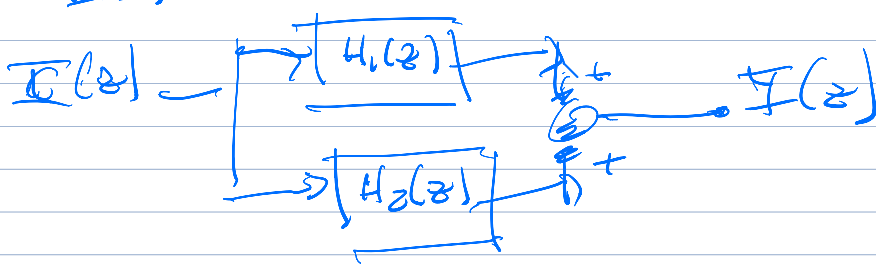
BREAKING $H(z)$ INTO SUBNETWORKS

CASCADE COMBINATION



$$\frac{Y(z)}{X(z)} = H_1(z) \cdot H_2(z)$$

PARALLEL COMBINATION



$$\frac{Y(z)}{X(z)} = H_1(z) + H_2(z)$$

CASCADE FILTER FORM

$$H(z) = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 \prod_{\ell=1}^{M} (1 - c_\ell z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$$

$$= b_0 \prod_{k=1}^{\max(M,N)/2} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

CONSIDER 2 POLES @ d_k d_k^*

$$(1 - d_k z^{-1})(1 - d_k^* z^{-1})$$
$$= 1 - z^{-1}(d_k + d_k^*) + z^{-2} d_k \cdot d_k^*$$
$$= 1 - z^{-1}\, 2\mathrm{Re}(d_k) + z^{-2} |d_k|^2$$

$$a_{1k} = 2\mathrm{Re}(d_k)$$
$$a_{2k} = -|d_k|^2$$