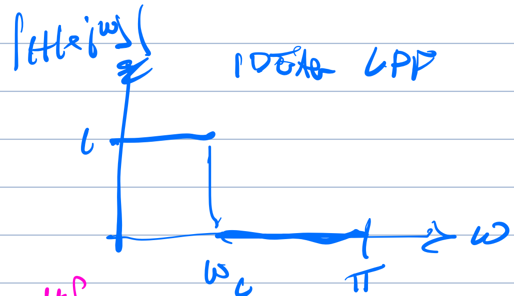
3/18/24

DISCRETE-TIME FILTER IMPLEMENTATION

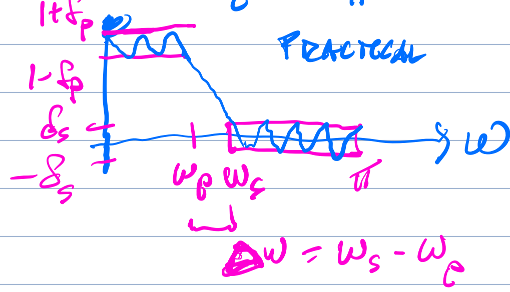
[OS4E 6.0–6.5]

DESIGN & IMPLEMENTATION OF DIGITAL FILTERS

1. ESTABLISH SPECIFICATIONS



2. DESIGN FILTER ... DERIVE / DEVELOP EQUATIONS TO REALIZE DESIGN PARAMETERS
   OS4E CHAP 7



3. IMPLEMENT FILTER IN HARDWARE OR SOFTWARE
   OS4E CHAP 6

TWO FILTER TYPES

IIR h[n] RIGHT-SIDED, LEFT-SIDED, BOTH-SIDED
$\mathcal{Z}$ BOTH POLES & ZEROS

FIR h[n] FINITE DURATION
$\mathcal{Z}$ ZEROS ONLY, EXCEPT $z=0$

ADVANTAGES OF IIR FILTERS

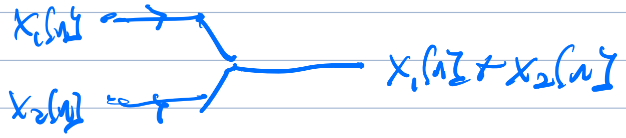
* MOST EFFICIENT DESIGN FOR SPECS
* CAN CONVERT OPTIMAL CT SOLUTIONS TO DT SOLUTIONS

ADVANTAGES OF FIR FILTERS

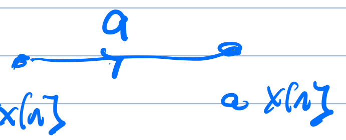
* ALWAYS STABLE!
* CAN OBTAIN LINEAR PHASE (ALWAYS!)
* CAN IMPLEMENT CONVOLUTION USING FFTS + OLA/OLS

SIGNAL FLOWGRAPH NOTATION....

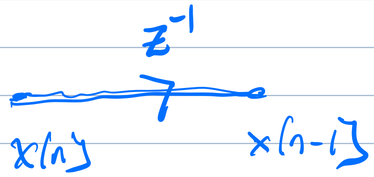
ADDITION



MULT BY CONST.



UNIT DELAY

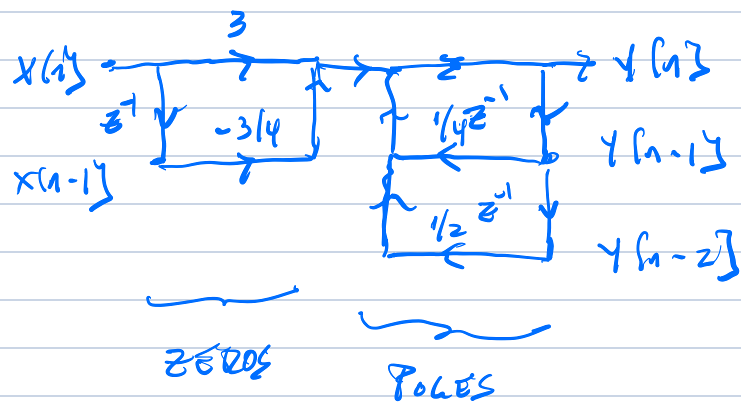


CONSIDER $y[n] - \frac{1}{4}y[n-1] - \frac{1}{2}y[n-2] = 3x[n] - \frac{3}{4}x[n-1]$

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{2}y[n-2] + 3x[n] - \frac{3}{4}x[n-1]$$

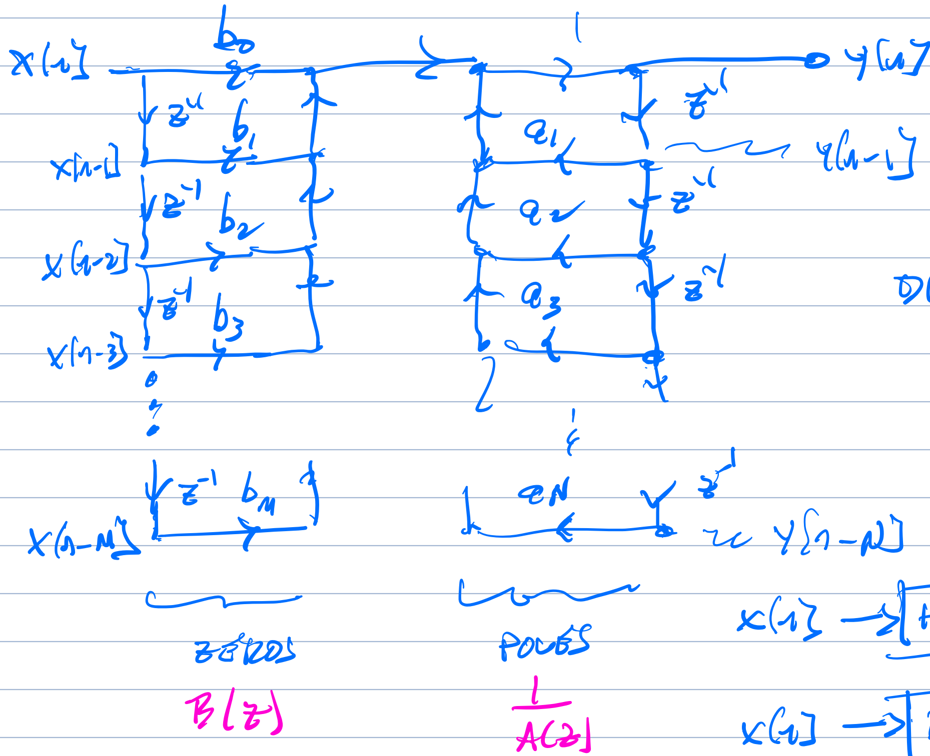
$$Y(z)\left(1 - \frac{1}{4}z^{-1} - \frac{1}{2}z^{-2}\right) = X(z)\left(3 - \frac{3}{4}z^{-1}\right)$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 - \frac{3}{4}z^{-1}}{1 - \frac{1}{4}z^{-1} - \frac{1}{2}z^{-2}} = \frac{3z\left(z - \frac{3}{4}\right)}{z^2 - \frac{1}{4}z - \frac{1}{2}}$$

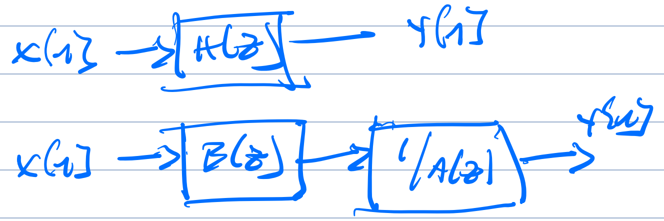


IN GENERAL,

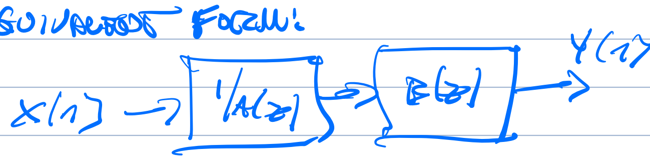
$$H(z) = \frac{Y(z)}{X(z)} = \frac{B(z)}{A(z)} = \frac{\sum_{\ell=0}^{M} b_\ell z^{-\ell}}{1 - \sum_{k=1}^{N} a_k z^{-k}} \; ; \quad y[n] = \sum_{k=1}^{N} a_k y[n-k] + \sum_{\ell=0}^{M} b_\ell x[n-\ell]$$

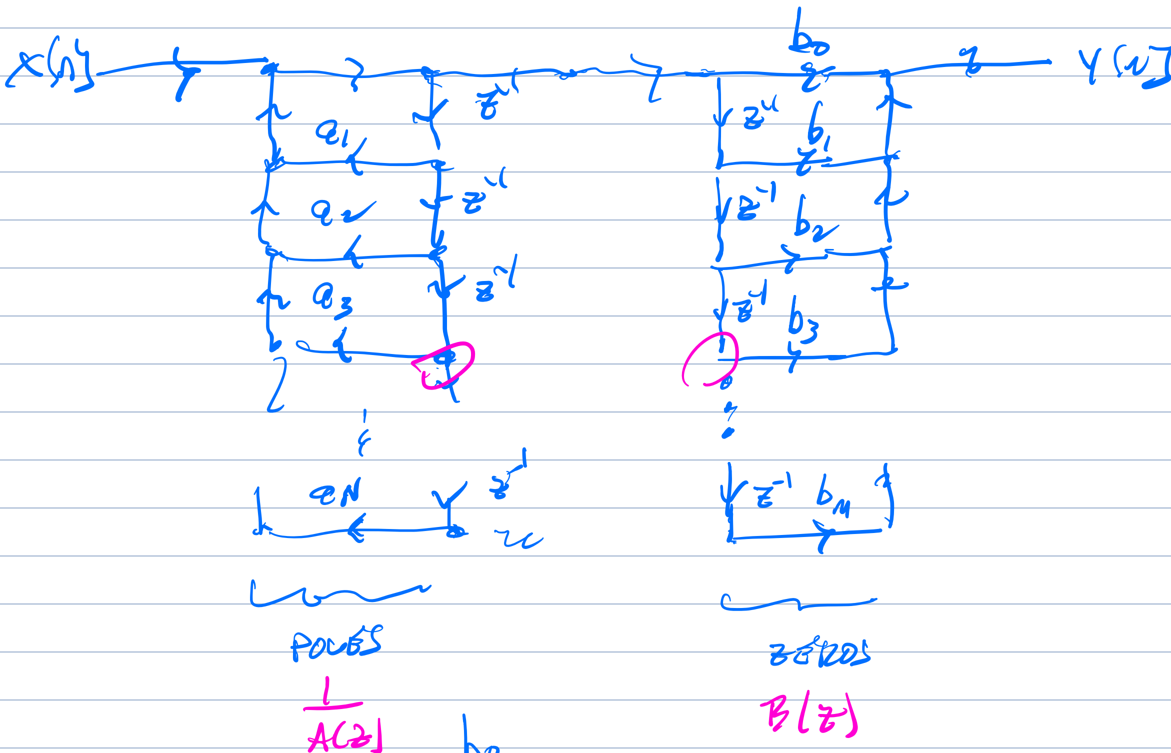


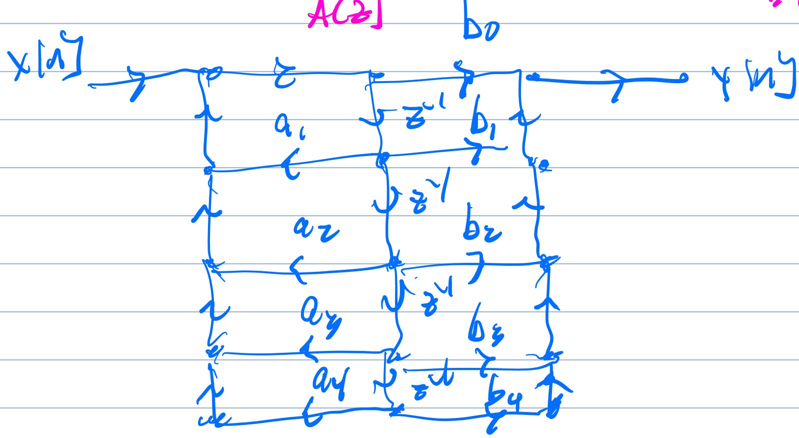
DIRECT-FORM I



EQUIVALENT FORM:





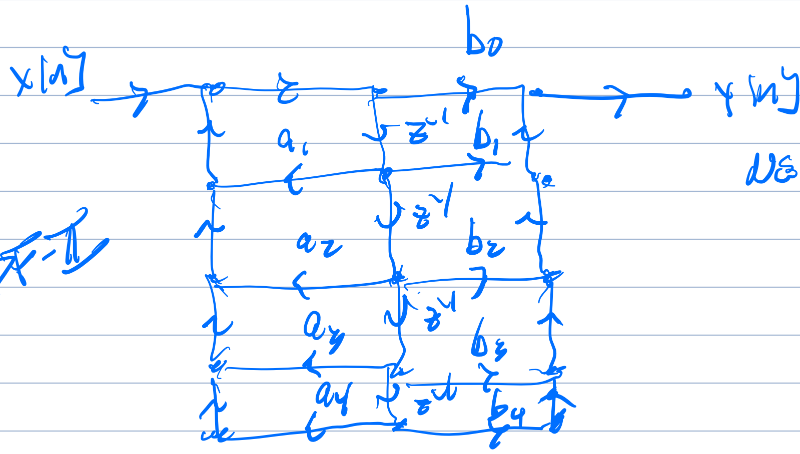


DIRECT-FORM II

"THE CANONICAL FORM"

ALTERNATE FORMS

- TRANSPOSITION
- FACTORIZATION / DECOMPOSITION



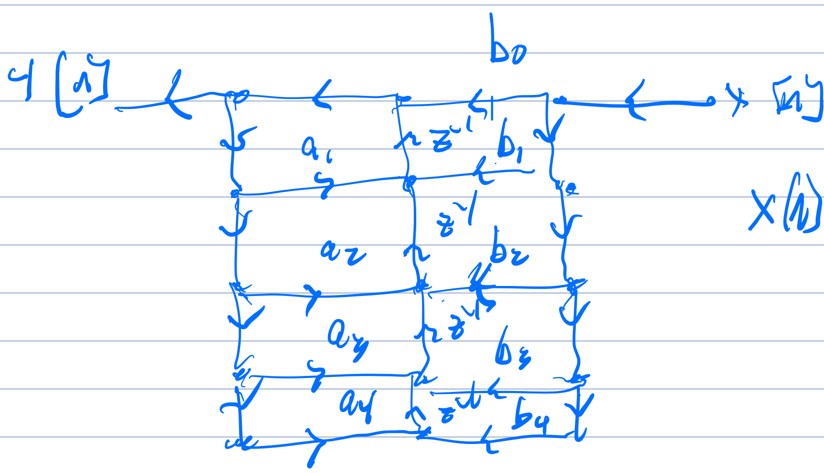
DF-II

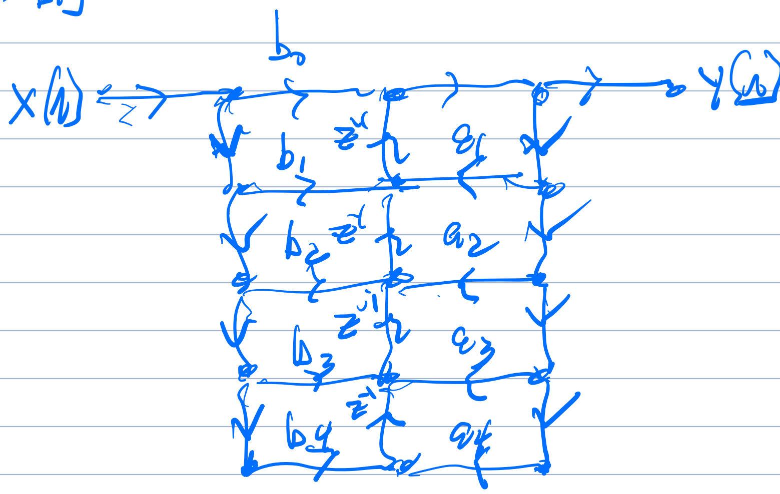
DF-II

NETWORK TRANSPOSITION THM:

1. REVERSE ARROWS
2. INTERCHANGE INPUT & OUTPUT

⇒ SAME TRANSFER FN.



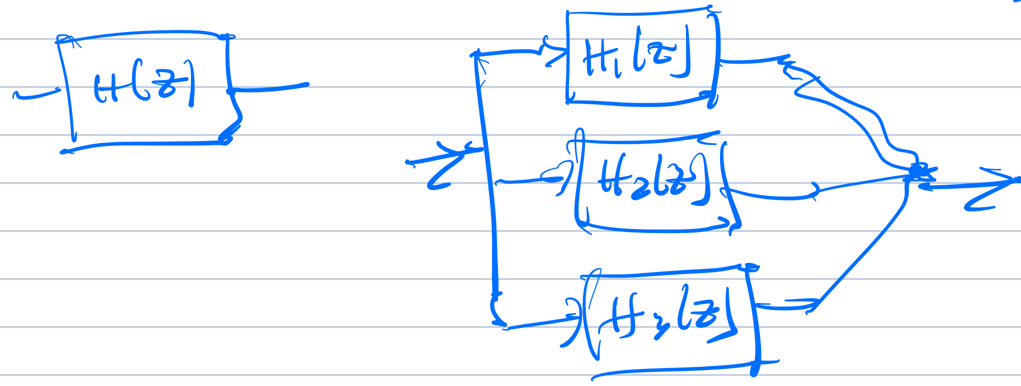


DF-II TRANSPOSE

ALTERNATE FORMS: CASCADE & PARALLEL

GOAL: DECOMPOSE "LADDERS" INTO MULTIPLE SECOND-ORDER LADDER

CASCADE: 

PARALLEL: 

CASCADE DECOMPOSITION:

$$H(z) = \frac{\sum_{l=0}^{M} b_l z^{-l}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{b_0 \prod_{l=1}^{M} (1 - c_l z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})}$$

$\{c_l\}$ ZEROS

$\{d_k\}$ POLES

SECOND-ORDER DECOMPOSITION:

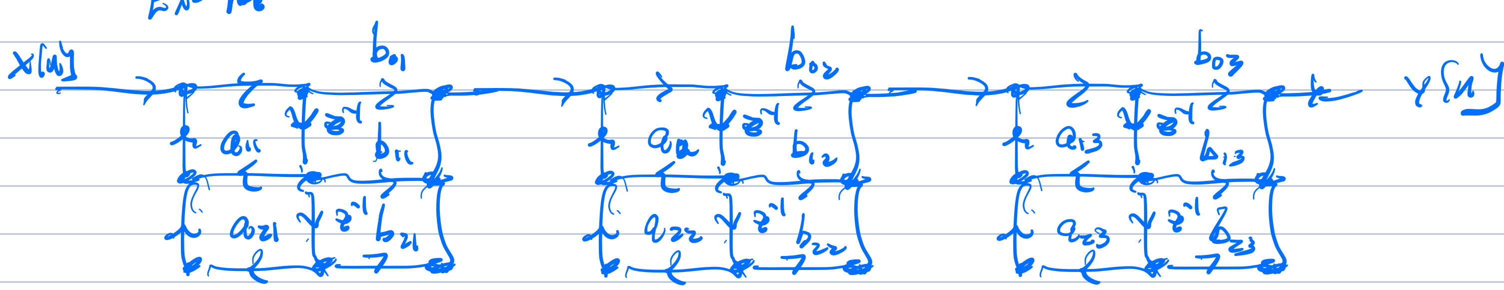
$$H(z) = b_0 \prod_{k=1}^{\mathrm{MAX}\left(\frac{M}{2}, \frac{N}{2}\right)} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

TWO REAL POLES: $(1 - d_1 z^{-1})(1 - d_2 z^{-2}) = 1 - (d_1 + d_2) z^{-1} + d_1 d_2 z^{-2}$

COMPLEX POLE PAIR: $(1 - d z^{-1})(1 - d^* z^{-1}) = 1 - z^{-1}(d + d^*) + d d^* z^{-2}$

$= 1 - 2\,\mathrm{Re}(d) z^{-1} + |d|^2 z^{-2}$

EX: $M = N = 6$



PARALLEL FORM

PARTIAL FRACTIONS:

FOR $N \geq M$

$$H(z) = \frac{b_0 \prod_{\ell=1}^{M} (1 - c_\ell z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})} = \sum_{k=1}^{N} \frac{A_k}{1 - d_k z^{-1}}$$

FOR TWO REAL POLES, d_1, d_2; A_1, A_2

$$\frac{A_1}{1 - d_1 z^{-1}} + \frac{A_2}{1 - d_2 z^{-1}} = \frac{(A_1 + A_2) - z^{-1}(A_1 d_2 + A_2 d_1)}{1 - (d_1 + d_2) z^{-1} + d_1 d_2 z^{-2}}$$

FOR TWO COMPLEX CONJUGATE POLES,

d, d^*; A, A^*

$$\frac{A}{1 - d z^{-1}} + \frac{A^*}{1 - d^* z^{-1}} = \frac{(A + A^*) - (A d^* + A^* d) z^{-1}}{1 - (d + d^*) z^{-1} + d d^* z^{-2}}$$

$$= \frac{2 \operatorname{Re}\{A\} - 2 \operatorname{Re}[A d^*] z^{-1}}{1 - 2 \operatorname{Re}\{d\} z^{-1} + |d|^2 z^{-2}}$$