

10/27/25

DISCRETE-TIME FILTER IMPLEMENTATION

COS4P 6.0-6.5

SCHEDULE CHANGES THIS WEEK

★ WEDNESDAY CLASS OVER ZOOM

MTG ID 991 7278 4564

PASSCODE 49 33 11

★ REPLACEMENT OFF:

- XANA 2-3

- BEN 3:30-4:30

★ FRIDAY NOON - 2 EXERCISES ONLY

DESIGN + IMPLEMENTATION of Digital Filters

Process of Design:

1. ESTABLISH SPECIFICATIONS

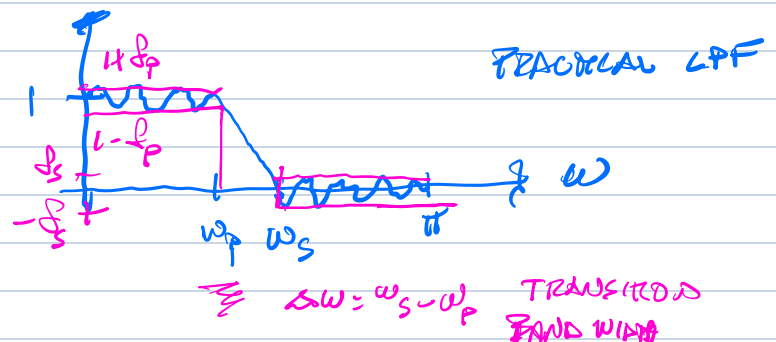
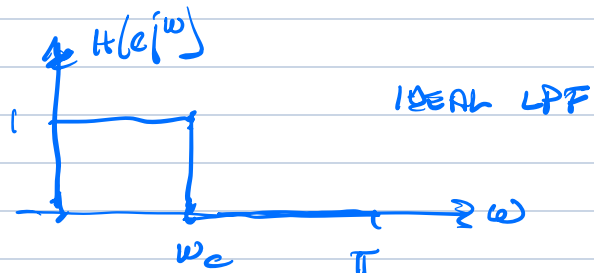
$f_p, f_s, \omega_p, \omega_s$

2. DESIGN FILTER - DEVELOP
EQU TO REALIZE SPECS

COS4P CHAP 7

3. IMPLEMENT FILTER IN
HARDWARE OR SOFTWARE

COS4P CHAP 6



TWO FILTER TYPES

IIR $h(n)$ RIGHT-SIDED, LEFT-SIDED, BOTH SIDES
BOTH POLES + ZEROS

FIR $h(n)$ FINITE-DURATION
ZEROS ONLY @ $z=0$

ADVANTAGES OF IIR FILTERS

* MOST EFFICIENT DESIGN GIVEN SPECS

* COULD CONVERT OPTIMAL CT SOLUTIONS TO DT STRUCTURES

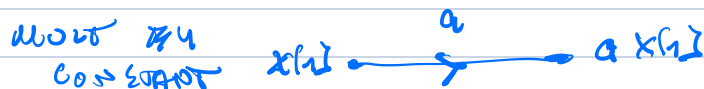
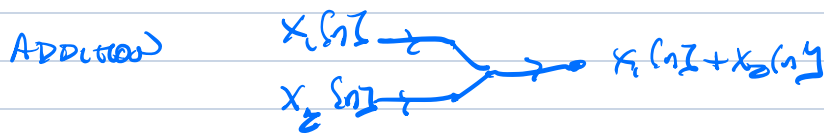
ADVANTAGES OF FIR FILTERS

* ALWAYS STABLE!

* CAN ^{ALWAYS} BE LINEAR PHASE

* CAN IMPLEMENT FILTERING WITH FFTs

SIGNAL FLOWGRAPH NOTATION



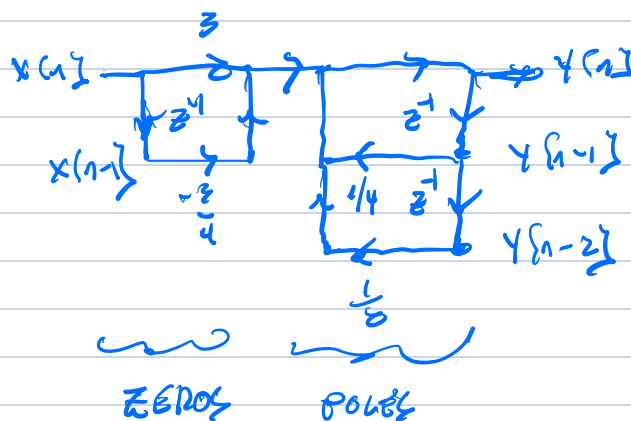
LIR FILTER IMPLEMENTATION

Consider $y[n] - \frac{1}{4}y[n-1] - \frac{1}{2}y[n-2] = 3x[n] - \frac{3}{4}x[n-1]$

$$y[n] = \frac{1}{4}y[n-1] + \frac{1}{2}y[n-2] + 3x[n] - \frac{3}{4}x[n-1]$$

$$Y(z)(1 - \frac{1}{4}z^{-1} - \frac{1}{2}z^{-2}) = X(z)(3 - \frac{3}{4}z^{-1})$$

$$H(z) = \frac{Y(z)}{X(z)} = \frac{3 - \frac{3}{4}z^{-1}}{1 - \frac{1}{4}z^{-1} - \frac{1}{2}z^{-2}}$$

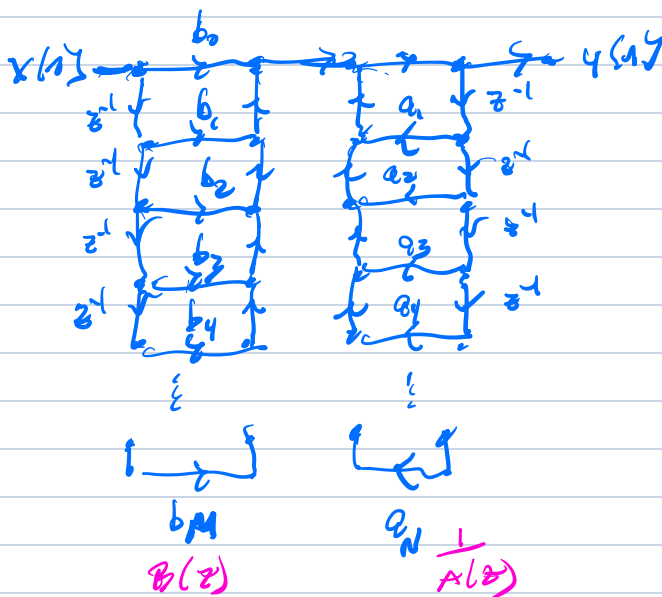


IN GENERAL

$$H(z) = \frac{Y(z)}{X(z)} = \frac{\sum_{l=0}^M b_l z^{-l}}{1 - \sum_{k=1}^N a_k z^{-k}} = \frac{B(z)}{A(z)} \quad y[n] = \sum_{k=1}^N a_k y[n-k] + \sum_{l=0}^M b_l x[n-l]$$

(+ FOR MATLAB, ELEMENTS)

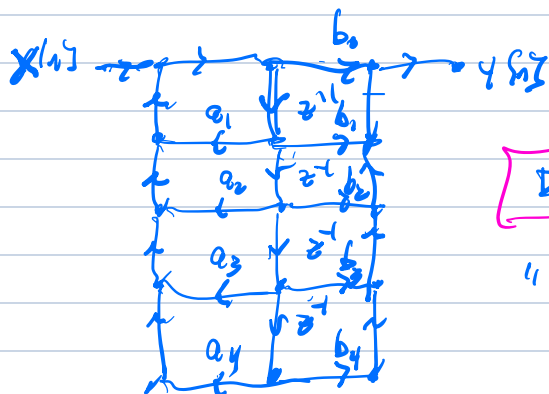
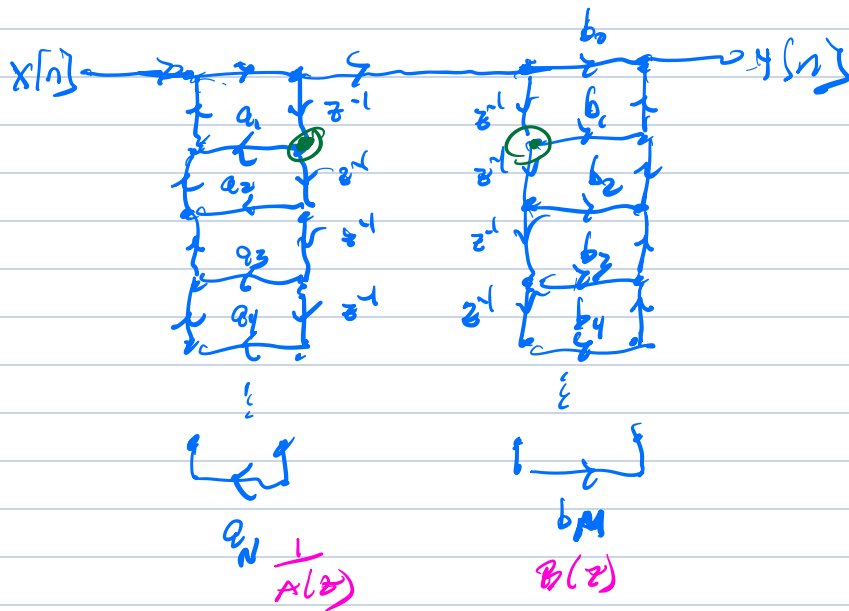
DIRECT FORM I



$$x[n] \rightarrow [A(z)] \rightarrow y[n]$$

$$x[n] \rightarrow [B(z)] \rightarrow \left[\frac{1}{A(z)} \right] \rightarrow y[n]$$

$$x[n] \rightarrow \left[\frac{B(z)}{A(z)} \right] \rightarrow y[n]$$



DIRECT FORM II

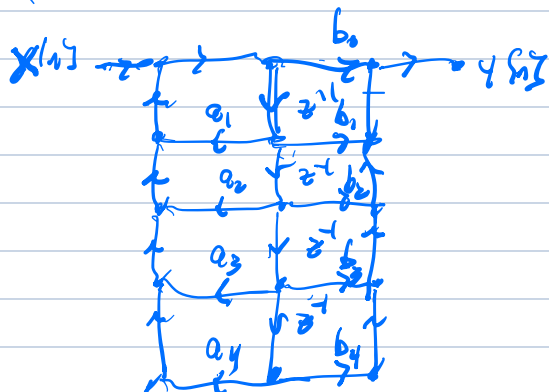
"THE CANONICAL FORM"

ALTERNATE FORMS

& TRANSPOSITION

& FACTORIZATION / DECOMPOSITION

TRANSPOSITION

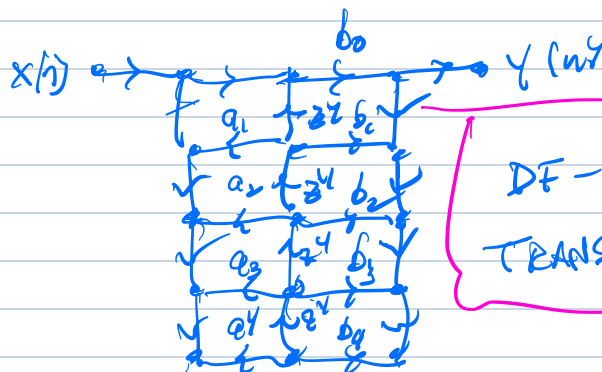
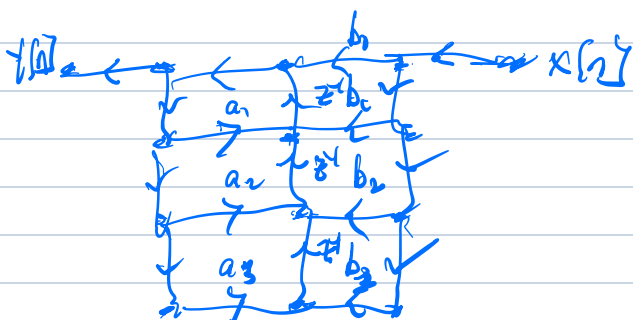


MASON NETWORK TRANSPOSITION RULE

1. REVERSE ARROWS

2. INTERCHANGE INPUT & OUTPUT

⇒ SAME RESPONSE

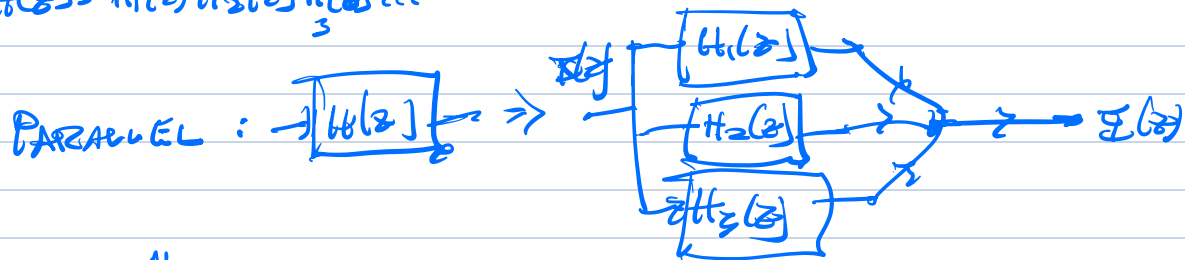


DF-II
TRANSPOSED

DECOMPOSITION $H(z)$: CASCADE + PARALLEL FORM



$$H(z) = H_1(z) H_2(z) H_3(z) \dots$$



$$H(z) = \frac{\sum_{l=0}^M b_l z^{-l}}{1 - \sum_{k=1}^N a_k z^{-k}} = \frac{b_0 \prod_{l=1}^M (1 - c_l z^{-1})}{\prod_{k=1}^N (1 - d_k z^{-1})} \quad \begin{matrix} \sum c_l \in \mathbb{R} \\ \sum d_k \in \mathbb{R} \end{matrix}$$

ZEROS POLES

SECOND ORDER DECOMPOSITION

$$H(z) = b_0 \prod_{k=1}^{\max(\frac{M}{2}, \frac{N}{2})} \frac{b_{0k} + b_{1k} z^{-1} + b_{2k} z^{-2}}{1 - a_{1k} z^{-1} - a_{2k} z^{-2}}$$

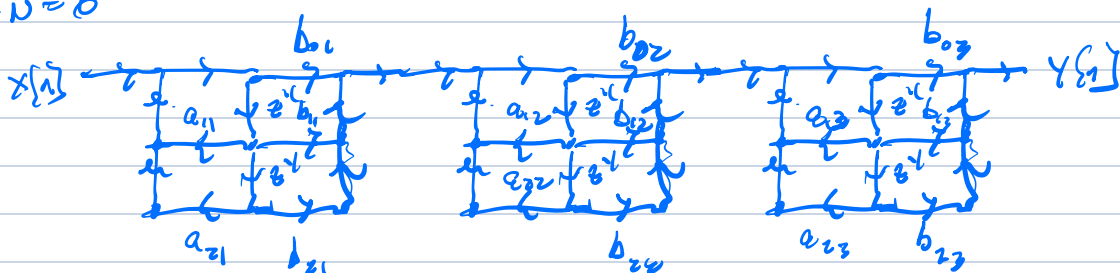
TWO REAL POLES: d_1, d_2

$$(1 - d_{1k} z^{-1})(1 - d_{2k} z^{-1}) = 1 - (d_{1k} + d_{2k}) z^{-1} + d_{1k} d_{2k} z^{-2}$$

COMPLEX CONJ. POLE PAIR

$$(1 - d_k z^{-1})(1 - d_k^* z^{-1}) = 1 - 2\operatorname{Re}\{d_k\} z^{-1} + |d_k|^2 z^{-2}$$

$$M = N = 6$$



PARALLEL FORM

PARTIAL FRACTION DECOMPOSITION

$$H(z) = \frac{\prod_{l=1}^M (1 - c_l z^{-1})}{\prod_{k=1}^N (1 - d_k z^{-1})} = \sum_{k=1}^N \frac{A_k}{1 - d_k z^{-1}} + \sum_{l=0}^{M-N} g_l z^{-l}$$

$g_0 + g_1 z^{-1} + g_2 z^{-2} + \dots$

TWO REAL POLES

$$\frac{A_1}{1 - d_1 z^{-1}} + \frac{A_2}{1 - d_2 z^{-2}} = \frac{(A_1 + A_2) - z^{-1}(d_1 + d_2)}{1 - (d_1 + d_2)z^{-1} + d_1 d_2 z^{-2}}$$

TWO COMPLEX CONJ. POLES

$$\frac{z \operatorname{Re}(A) - z^{-1} z \operatorname{Re}(d)}{1 - z \operatorname{Re}(d) z^{-1} + |d|^2 z^{-2}}$$

