INTRO TO THE

FFT

(OSYP 9.0-9.2)

DFT

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi nk}{N}}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi nk}{N}}$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-nk}$$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$$

NUMBER OF MULTIPLYS
REQUIRED FOR COMPUTATIONS

DIRECT CONVOLUTION

$$y[n] = \sum_{k=-\infty}^{\infty} h[k]\,x[n-k]$$

IF $x[n]$ IS OF LENGTH N
$h[n]$ IS OF LENGTH M

$\#$ MULTS $\approx NM$

FILTERING w DFTs

N-PT. DFT

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{+nk}$$

$O(N^2)$

N^2 COMPLEX MULTS

FILTERING $\Rightarrow$ $3(N+M-1)^2 + (N+M-1)$ COMPLEX MULTS

$O(N^2)$

COOLEY-TUKEY DECIMATION IN TIME

LET $N = 2^{\nu}$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk} = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}} ; \quad W_N = e^{-j\frac{2\pi}{N}}$$

$N = 8$

$$= \sum_{n \text{ EVEN}} x[n] W_N^{nk} + \sum_{n \text{ ODD}} x[n] W_N^{nk}$$

$n = 2r$ $\quad$ $n = 2r+1$

$$W_N^{2rk} = e^{-j\frac{2\pi}{N}(2rk)} = e^{-j\frac{2\pi}{N/2}(rk)} = W_{\frac{N}{2}}^{rk}$$

$$= \sum_{r=0}^{\frac{N}{2}-1} x[2r] W_N^{2rk} + \sum_{r=0}^{\frac{N}{2}-1} x[2r+1] W_N^{(2r+1)k}$$

$$W_N^{(2r+1)k} = W_N^{2rk} \cdot W_N^{k} = W_N^{k} \cdot W_{\frac{N}{2}}^{rk}$$

$$X[k] = \sum_{r=0}^{\frac{N}{2}-1} x[2r]\, W_{\frac{N}{2}}^{rk} + W_N^{k} \sum_{r=0}^{\frac{N}{2}-1} x[2r+1]\, W_{\frac{N}{2}}^{rk}$$

$N = 8$

ORIGINAL DFT $\approx$ 64 MULTS

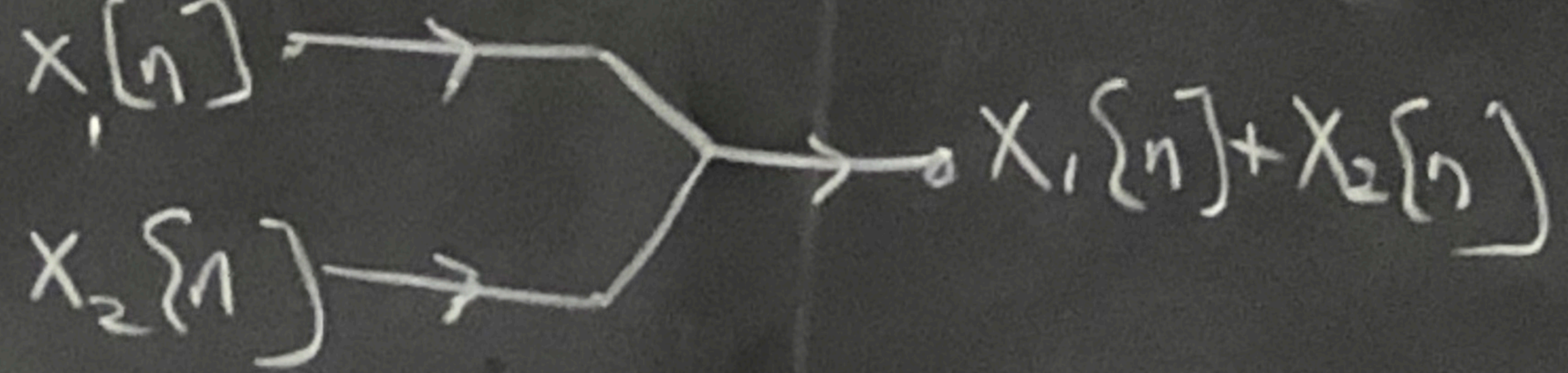
EVEN | ODD DFT

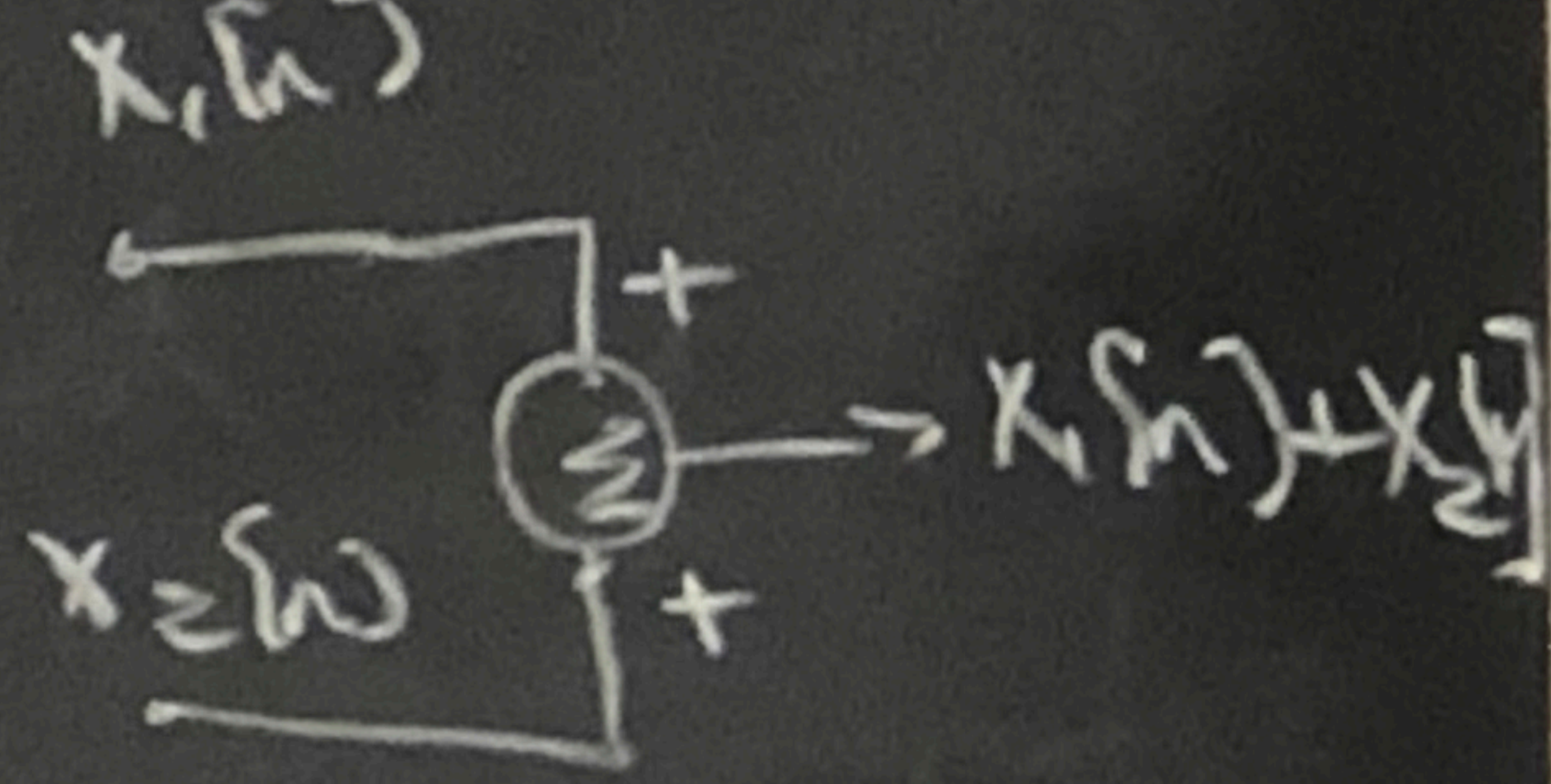
$2(16) + 8 = 40$ MULTS

$2\left(\frac{N}{2}\right)^2 + N$

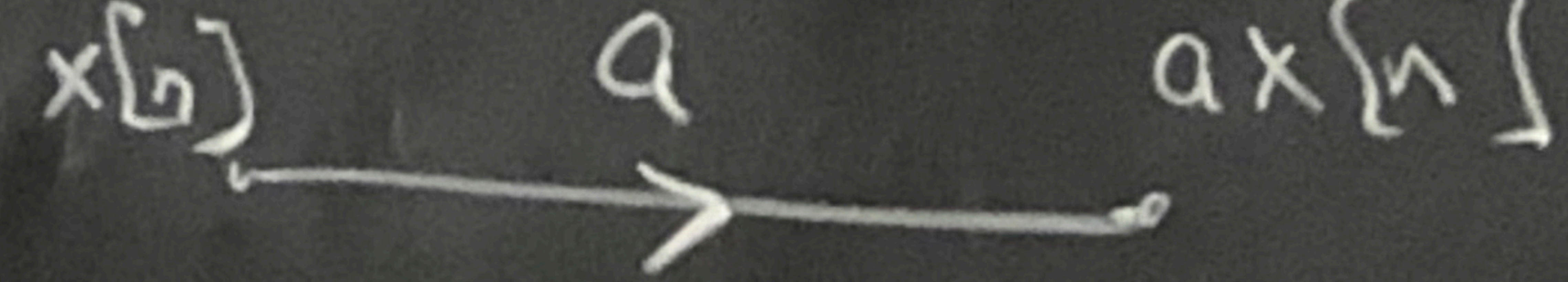
SIGNAL FLOWGRAPH NOTATION

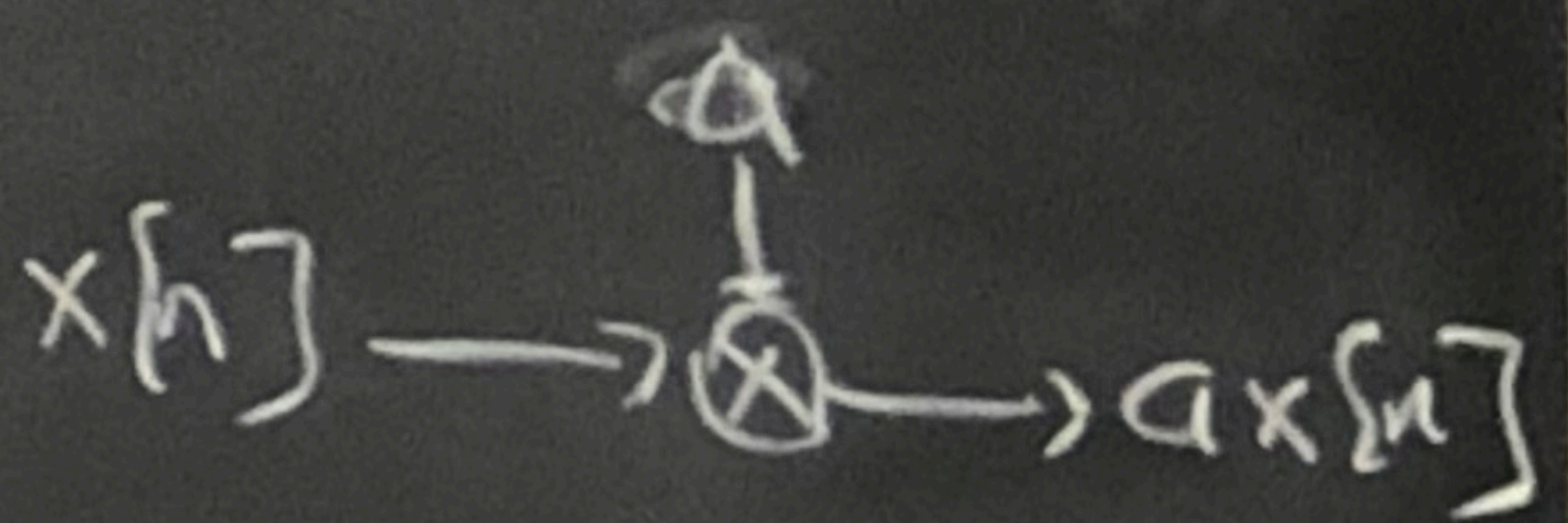
ADDITION



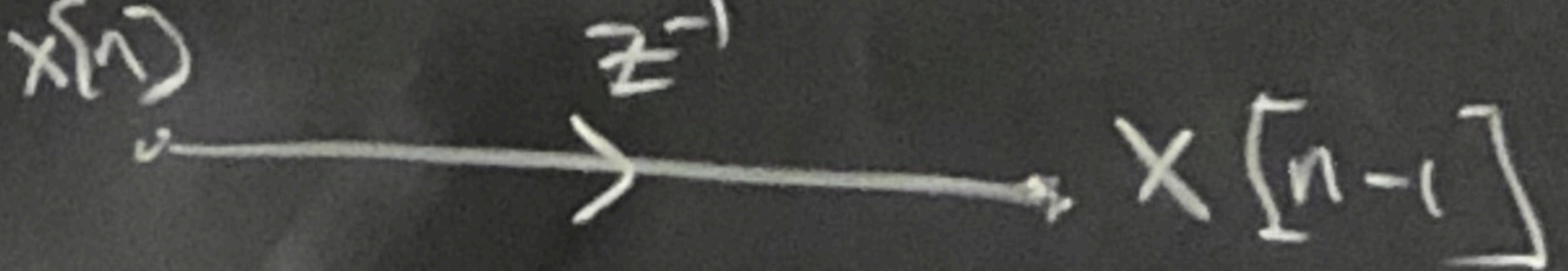


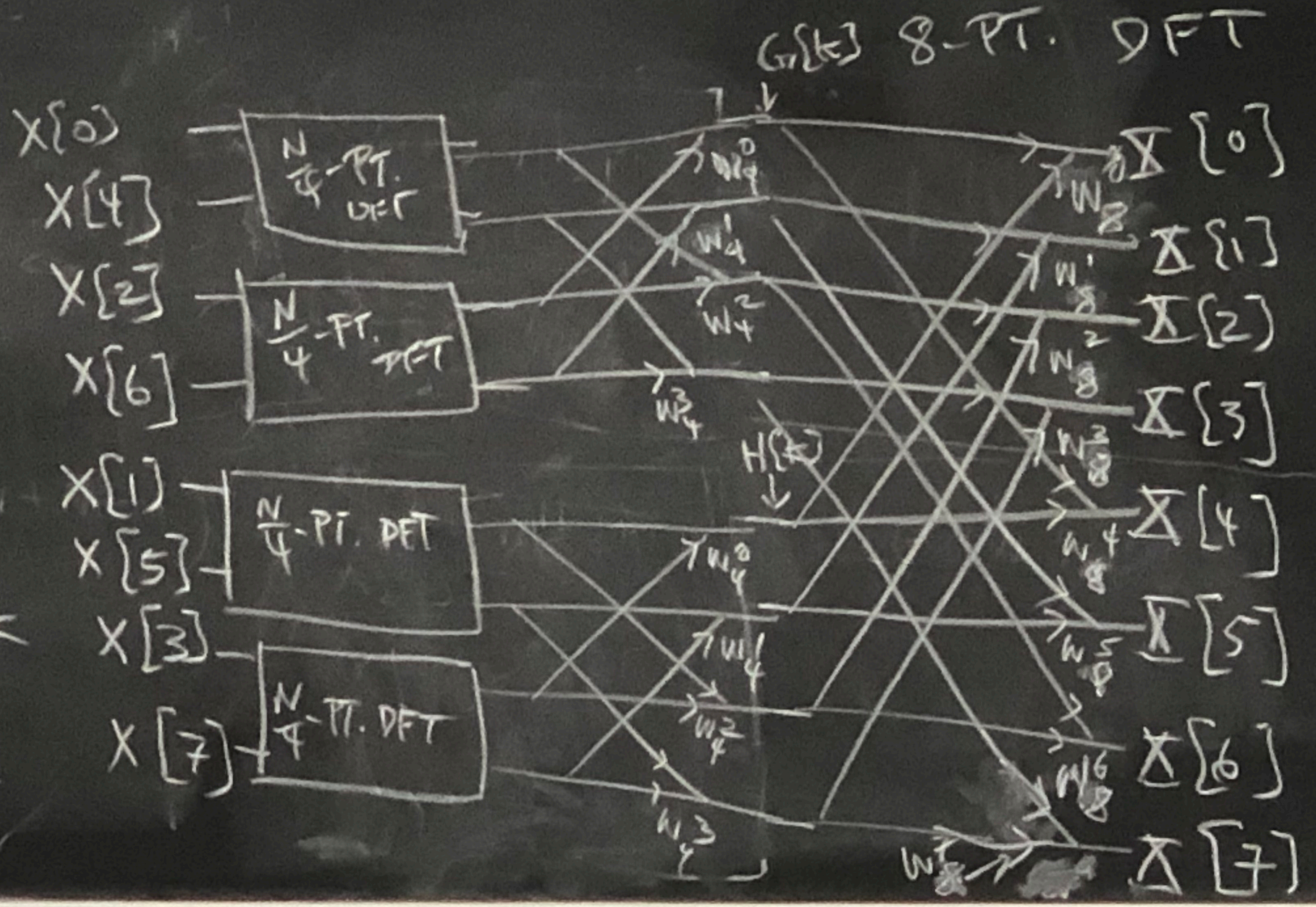
MULT BY CONST





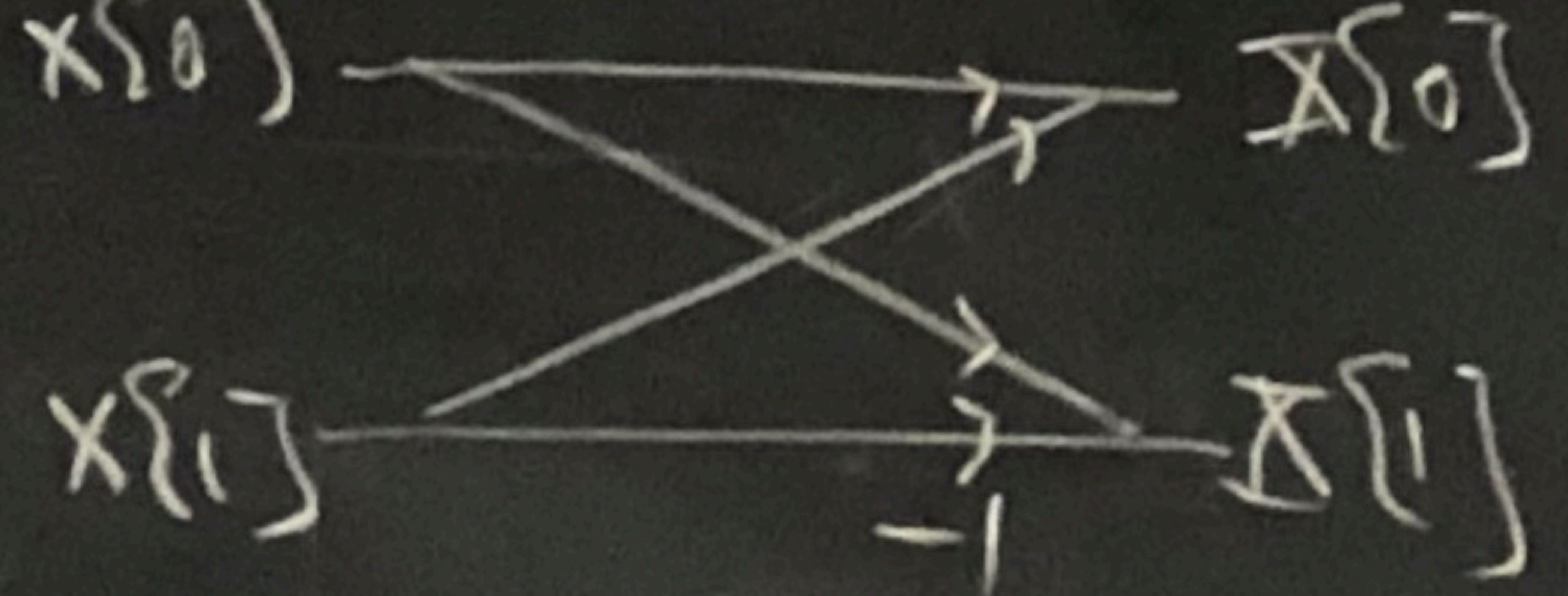
UNIT DELAY




G[k]
8-PT. DFT
x[0]
x[4]
x[2]
x[6]
x[1]
x[5]
x[3]
x[7]
N/4-PT. DFT
N/4-PT. DFT
N/4-PT. DFT
N/4-PT. DFT
W_4^0
W_4^1
W_4^2
W_4^3
W_4^0
W_4^1
W_4^2
W_4^3
H[k]
W_8^0
W_8^1
W_8^2
W_8^3
W_8^4
W_8^5
W_8^6
W_8^7
X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]

$\frac{N}{4}$ - PT. DFT (2-PT. DFT)

$$X[k] = \sum_{n=0}^{1} x[n]\, W_2^{nk} = \sum_{n=0}^{1} x[n]\, e^{-j\frac{2\pi nk}{2}} = \sum_{n=0}^{1} x[n]\, (-1)^{nk}$$



$$X[0] = x[0] + x[1]$$

$$X[1] = x[0] - x[1]$$

BASIC "BUTTERFLY"


G[k] 8-PT. DFT
x[0]
x[4]
x[2]
x[6]
x[1]
x[5]
x[3]
x[7]
W_8^0
$-1 = W_8^4$
W_8^0
$-1\ W_8^4$
W_8^0
$-1 = W_8^4$
W_8^1
$-1\ W_8^4$
$W_4^0 = W_8^0$
$W_4^1 = W_8^2$
$W_4^2 = W_8^4$
$W_4^3 = W_8^6$
H[k]
$W_4^0 = W_8^0$
$W_4^1 = W_8^2$
$W_4^2 = W_8^4$
$W_4^3 = W_8^6$
W_8^0
W_8^1
W_8^2
W_8^3
W_8^4
W_8^5
W_8^6
W_8^7
X[0]
X[1]
X[2]
X[3]
X[4]
X[5]
X[6]
X[7]

# of MULTS = # COLUMNS · MULTS / COLUMN

$N = 2^{\nu}$; $\nu = \log_2 N$

MULTS = $N \log_2 N$