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FFT ALTERNATE STRUCTURES

(OS&P 9.2-9.5)

IDFT: $x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k] W_N^{-nk}$

$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

FOR COMPUTING IDFT

FLIP SIGNS ON W_N^{nk}

ALT. METHOD FOR IDFT:

REGULAR DFT $X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

* COMPLEX CONJUGATE INPUT $X'[k] = \sum_{n=0}^{N-1} x^*[n] W_N^{nk}$

* COMPLEX CONJUGATE RESULT, DIVIDE BY N

$\frac{1}{N}\left(\sum_{n=0}^{N-1} x^*[n] W_N^{nk}\right)^*$

$\left(e^{-j\frac{2\pi nk}{N}}\right)^* = e^{j\frac{2\pi nk}{N}} = W_N^{-nk}$

$\left(x'[k]\right)^* = \frac{1}{N}\sum_{n=0}^{N-1} x[n] W_N^{-nk}$

TO COMPUTE IDFT of $X[k]$

1. COMPUTE $X^*[k]$
2. PERFORM FORWARD DFT on $X^*[k]$
3. COMPLEX CONJUGATE RESULT, DIVIDE BY N

DECIMATION IN FREQUENCY

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$$

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x[n]\, W_N^{nk} + \sum_{n=\frac{N}{2}}^{N-1} x[n]\, W_N^{nk}$$

Let $l = n - \frac{N}{2}$

$n = l + \frac{N}{2}$

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x[n]\, W_N^{nk} + \sum_{l=0}^{\frac{N}{2}-1} x[l+\tfrac{N}{2}]\, W_N^{(l+\frac{N}{2})k}$$

$$W_N^{(l+\frac{N}{2})k} = W_N^{lk} \cdot W_N^{\frac{N}{2}k} = W_N^{lk} (-1)^k$$

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} x[n]\, W_N^{nk} + (-1)^k \sum_{n=0}^{\frac{N}{2}-1} x[n+\tfrac{N}{2}]\, W_N^{nk}$$

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + (-1)^k x[n+\tfrac{N}{2}]\right) W_N^{nk}$$

For k even, let $k=2r$

<u>k EVEN</u>

$$X[k] = \underbrace{\sum_{n=0}^{\frac{N}{2}-1} \left(x[n] + x[n+\tfrac{N}{2}]\right) \cancel{W_N^{2kn}}\; W_{N/2}^{kn}}_{\frac{N}{2}\text{-PT. DFT of } x[n]+x[n+\frac{N}{2}]}$$

For k odd, let $k = 2r+1$

<u>k ODD</u>

$$X[k] = \sum_{n=0}^{\frac{N}{2}-1} \left(x[n] - x[n+\tfrac{N}{2}]\right) W_N^{n(2r+1)}$$

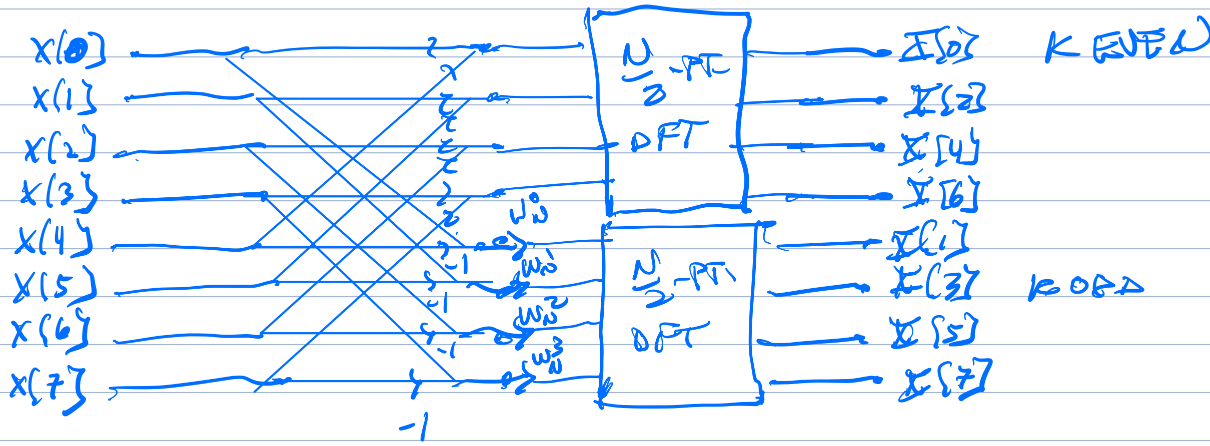
$$W_N^{n(2r+1)} = W_N^{n} \cdot W_{N/2}^{rn}$$

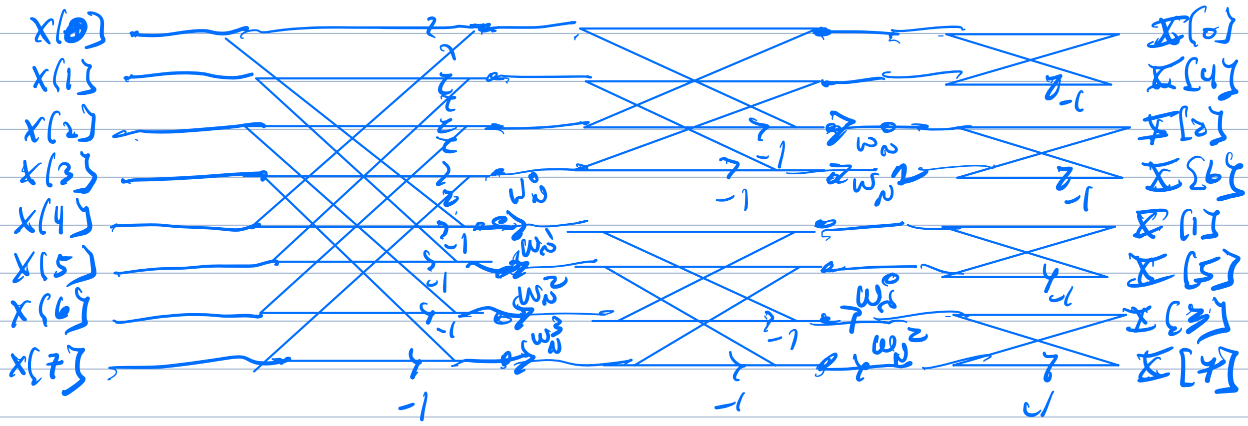
$$X[k] = \underbrace{\sum_{n=0}^{\frac{N}{2}} \left(\left(x[n] - x[n+\tfrac{N}{2}]\right) W_N^{n}\right) W_{\frac{N}{2}}^{nk}}_{\frac{N}{2}\text{-PT. DFT of } \left(x[n]-x[n+\frac{N}{2}]\right) W_N^{n}}$$

k EVEN $\qquad \frac{N}{2}$-PT. DFT of $\{x[n] + x[n+\frac{N}{2}]\}$

k ODD $\qquad \frac{N}{2}$-PT. DFT of $\left(x[n] - x[n+\frac{N}{2}]\right) W_N^n$

$N=8$





TRANSPOSES IN SIGNAL FLOW DIAGRAM

MATRIX TRANSPOSITION THEOREM (S.J. MASON)

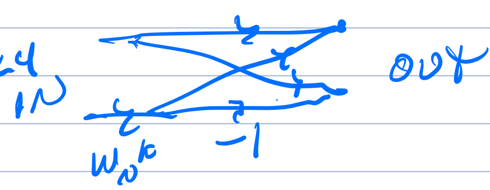
GIVEN SIGNAL FLOWGRAPH

IF 1. REVERSE ARROWS

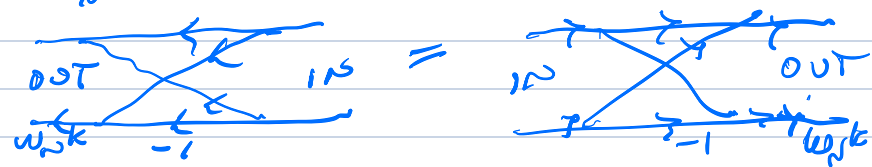
2. INTERCHANGE INPUT & OUTPUT

$\Rightarrow$ NETWORK RESPONSE IS THE SAME!

DIT BUTTERFLY



DIF BUTTERFLY



DECIMATION IN TIME FOR $N \neq 2^{\nu}$

COMPOSITE $N = p_1 p_2 p_3 \ldots p_\nu$

let $q_1 = \frac{N}{p_1}$, $q_2 = \frac{N}{p_1 p_2}$, $q_3 = \frac{N}{p_1 p_2 p_3}$ ...

$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$$

let $N = p_1 p_2$

$$X[k] = \sum_{r=0}^{q_1-1} x[p_1 r] W_N^{p_1 r k} + \sum_{r=0}^{q_1-1} x[p_1 r+1] W_N^{(p_1 r+1)k} + \ldots + \sum_{r=0}^{q_1-1} x[p_1 r + (p_1-1)] W_N^{p_1 r + (p_1 - 1)}$$

CONSIDER l^{th} SUM: $\displaystyle\sum_{r=0}^{q_1-1} x[p_1 r+l] W_N^{(p_1 r+l)k}$

$W_N^{p_1 r k} \cdot W_N^{lk}$

$= W_{q_1}^{rk} \cdot W_N^{lk}$

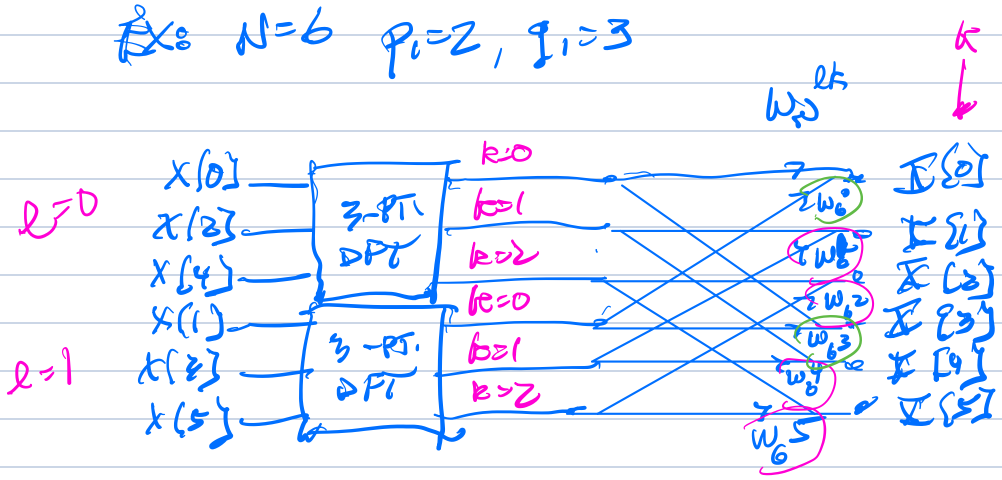
$$W_N^{lk} \sum_{r=0}^{q_1-1} x[p_1 r+l] W_{q_1}^{rk}$$

$$X[k] = \sum_{l=0}^{p_1-1} W_N^{lk} \underbrace{\sum_{r=0}^{q_1-1} x[p_1 r+l]\, W_{q_1}^{rk}}_{q_1\text{-pt. DFT } \{x[p_1 r+l]\}}$$

Ex: $N=6$ $p_1=2$, $q_1=3$

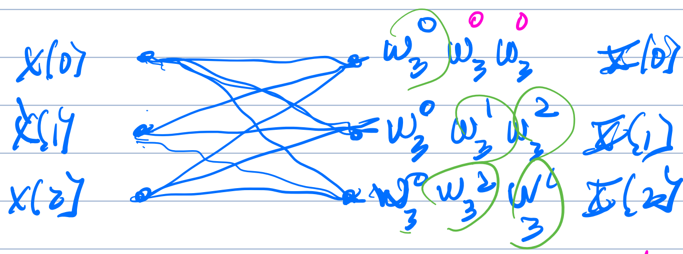
$$X[k] = \sum_{l=0}^{p_1-1} W_N^{lk} \underbrace{\sum_{r=0}^{q_1-1} x[p_1 r+l]\, W_{q_1}^{rk}}_{q_1\text{-pt. DFT of } x[p_1 r+l]}$$

Ex: $N=6$ $p_1=2,\ q_1=3$



6

4



$6^2 = 36$ MULTS

$4+4$

12 NON TRIV MULTS

ALL: $6 + 2(9)$ 24 MULTS ALL TOGETHER