LENGTH N_1 $x_1[n] \overset{N}{\Longleftrightarrow} X_1[k]$

LENGTH N_2 $x_2[n] \overset{N}{\Longleftrightarrow} X_2[k]$

Let $Y[k] = X_1[k] X_2[k]$

$$y[n] = x_1[n] \overset{N}{\circledast} x_2[n] \Longleftrightarrow$$

$$= \sum_{\ell=0}^{N-1} x_1[((\ell))_N] \, x_2[((n-\ell))_N]$$

$$x_1[n] \overset{N}{\circledast} x_2[n] = \left(x_1[n] * x_2[n]\right) * S_N[n]$$

NEED $N \geq N_1 + N_2 - 1$ TO AVOID TEMPORAL ALIASING

$$S_N[n] = \begin{cases} 1, & n = rN \\ 0, & \text{ELSE} \end{cases}$$

CIRCULAR v. LINEAR CONVOLUTION

(OSYP 8.6–8.7)

REMINDER: QUIZ 1 HERE WED!

REVIEW SESSION 7 PM, BH 235A

DFT SIZE N

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j\frac{2\pi nk}{N}}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\frac{2\pi nk}{N}}$$

$W_N = e^{-j\frac{2\pi}{N}}$

$$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-nk}$$

$$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$$

$$x[n] \to \boxed{h[n]} \to y[n]$$

CONVOLVING A LONG INPUT $x[n]$ WITH $h[n]$

USING MULTIPLE CIRCULAR CONVOLUTIONS

ASSUME $x[n]$ LONG, STARTS @ $n=0$

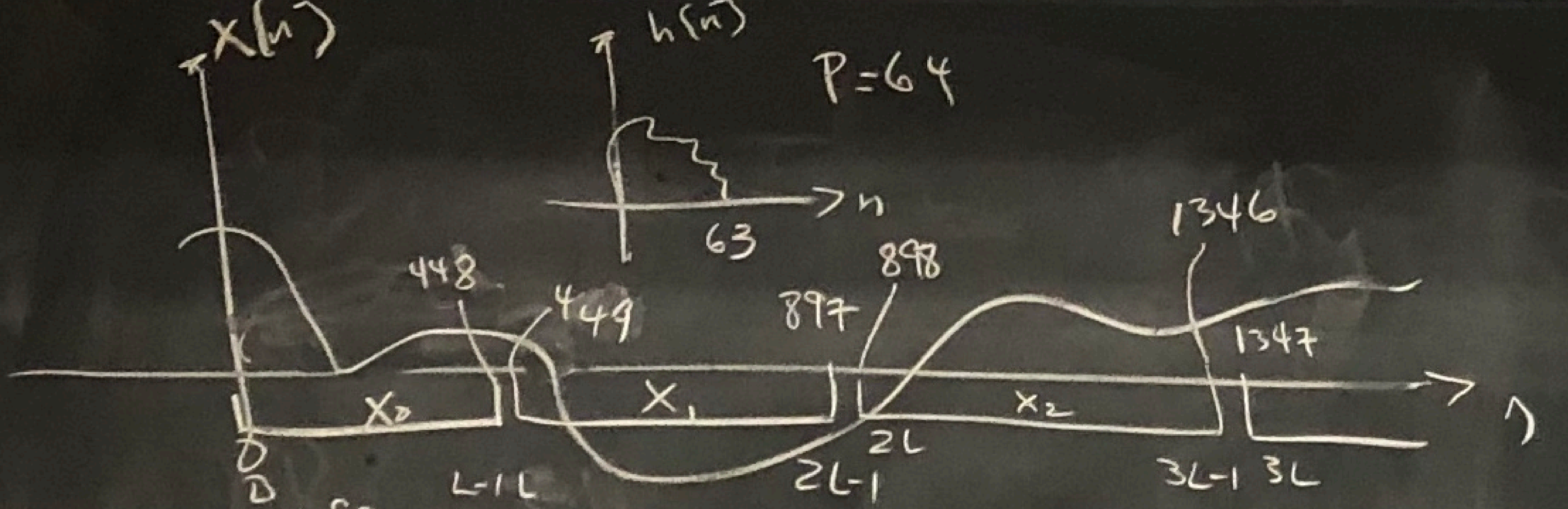
$h[n] \neq 0, \quad 0 \leq n \leq P-1$

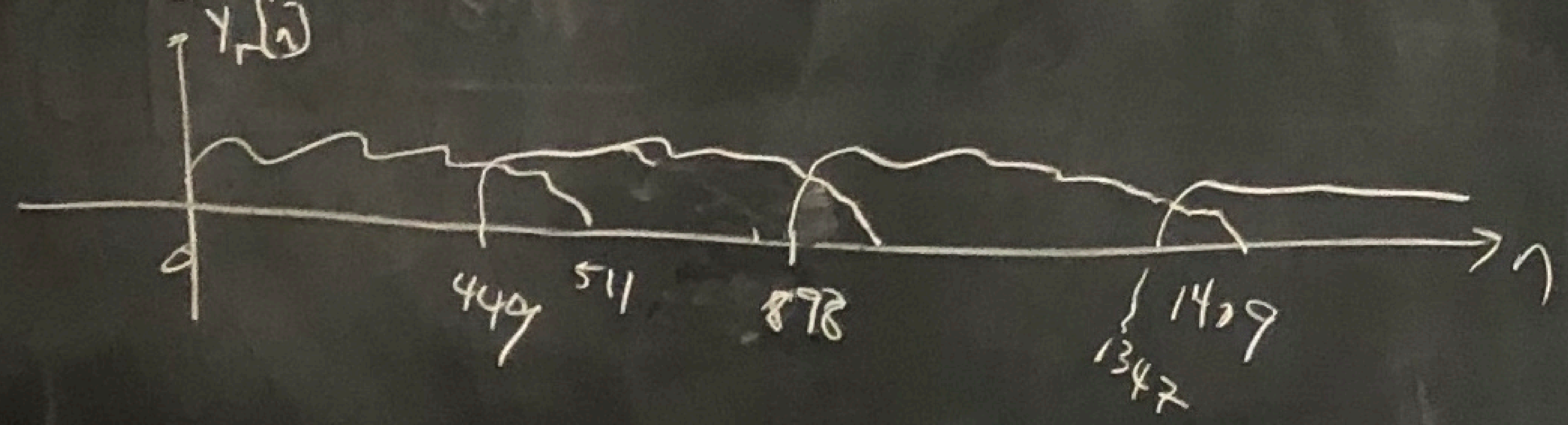
METHOD I: OVERLAP ADD (OLA)

1. BREAK INPUT INTO ABUTTING SEGMENTS $x_r[n]$ OF LENGTH L
2. CHOOSE N LARGE ENOUGH SO THAT

$$x_r[n] \overset{N}{\circledast} h[n] = x_r[n] * h[n]$$

3. OBTAIN $y[n]$ BY SUMMING OUTPUTS





CHOOSE

$N \geq L+P-1$

$N = 512$

$L = 512 - 63 = 449$

$$x[n] = \sum_{r=0}^{?} x_r[n-rL]$$

$$x_r[n] = \begin{cases} x[n+rL], & 0 \leq n \leq L-1 \\ 0, & \text{ELSE} \end{cases}$$

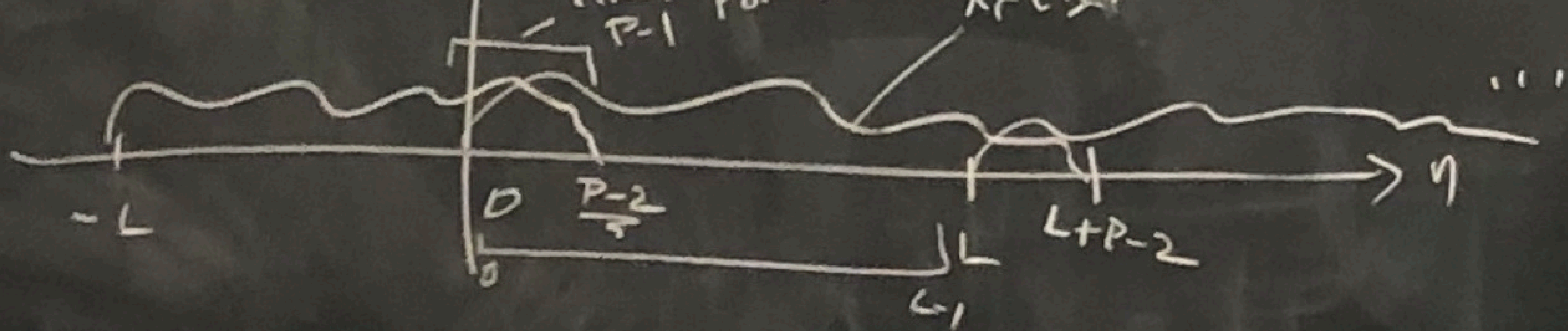
$$Y_r[n] = x_r[n] * h[n]$$

$$Y[n] = \sum_{r=0}^{?} Y_r[n-rL]$$

METHOD II: OVERLAP-SAVE (OLS)

1. BREAK $x[n]$ INTO SUBSEQUENCES LENGTH L
2. CIRCULARLY CONVOLVE WITH $h[n]$ USING DFT OF SIZE $N = L$
3. KEEP ("SAVE") UNALIASED POINTS
4. BEGIN NEXT SECTION S.T. FIRST "GOOD" POINT IS NEXT POINT NEEDED

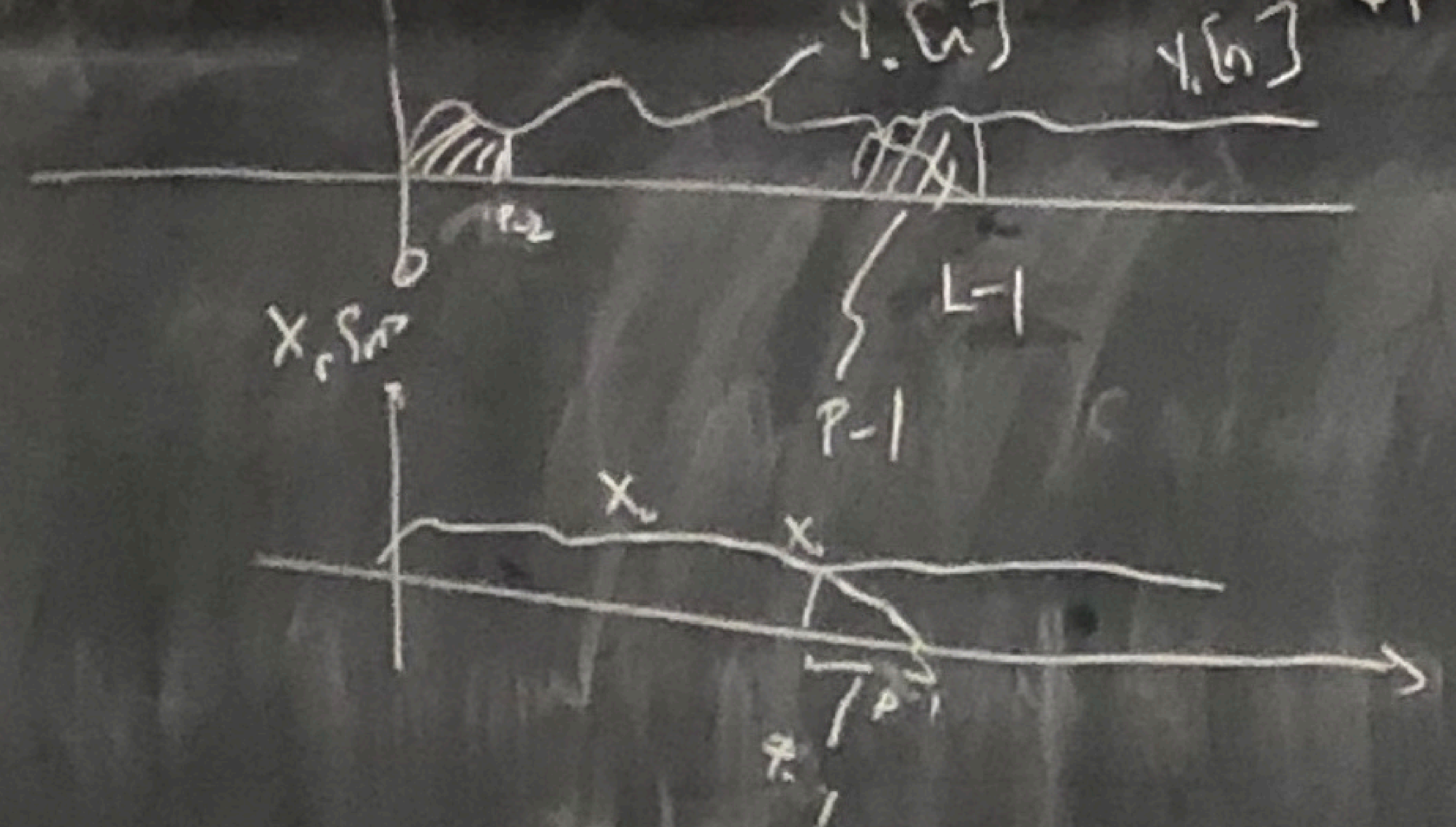
WHICH POINTS ARE ALIASED? FIRST $P-1$ POINTS ARE DISTORTED BY ALIASING



SECTIONS FOR OLS....

$$\begin{array}{r} 512 \\ -\ 63 \\ \hline 449 \end{array}$$

$L = 512,\ P = 64$



SIMPLE SECTIONS

r	INPUT INDEX	KEEP SAMPLES
0	0-511	64-511
1	449-960	512-960
2	898-1409	961-1409

r	INPUT	KEEP
0	63 ZEROS 64-448	64-448
	386-897	449-897
		898

$$x_r[n] = \begin{cases} x[n + r(L-(P-1)) - (P-1)], & 0 \le n \le L-1 \\ 0, & \text{ELSE} \end{cases}$$

$$y[n] = \sum_{n=0}^{?} y_r[n - r(L+(P-1)) + P-1]$$

$$y_{rp}[n] = x_r[n] \overset{L}{\circledast} h[n]$$

$$y_r[n] = \begin{cases} y_{rp}[n], & P-1 \le n \le L-1 \\ 0, & \text{ELSE} \end{cases}$$

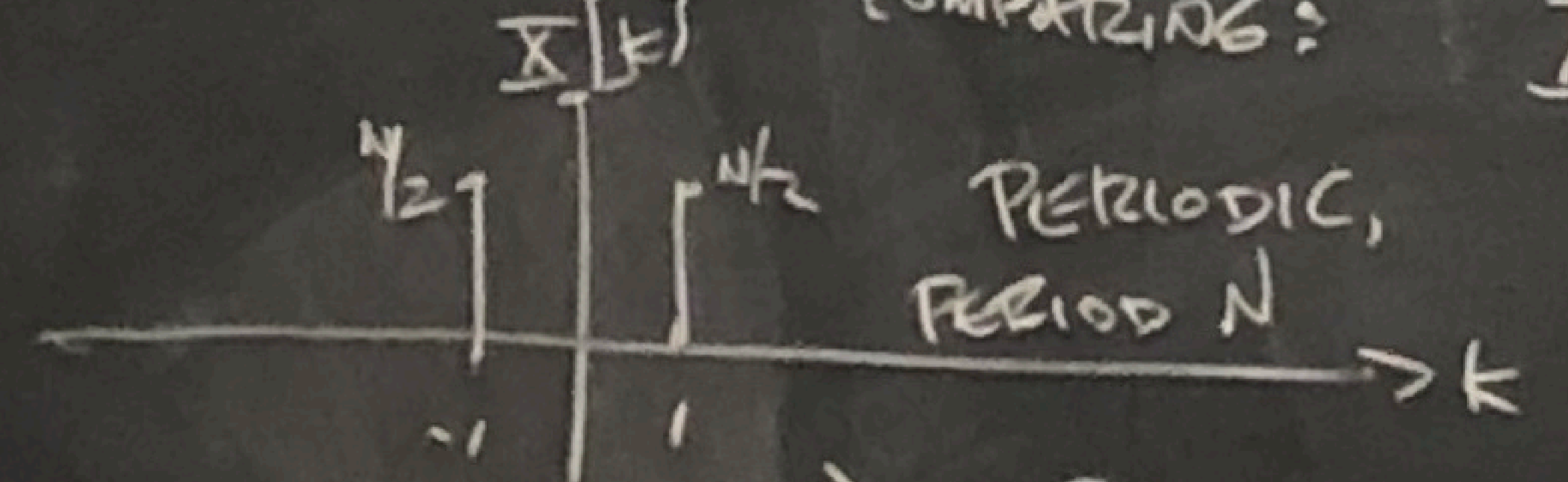
DTFTs of PERIODIC TIME FUNCTIONS

Ex. 1. $x[n] = \cos(\omega_0 n)$ $\quad \omega_0 = \frac{2\pi}{N}$, N INTEGER

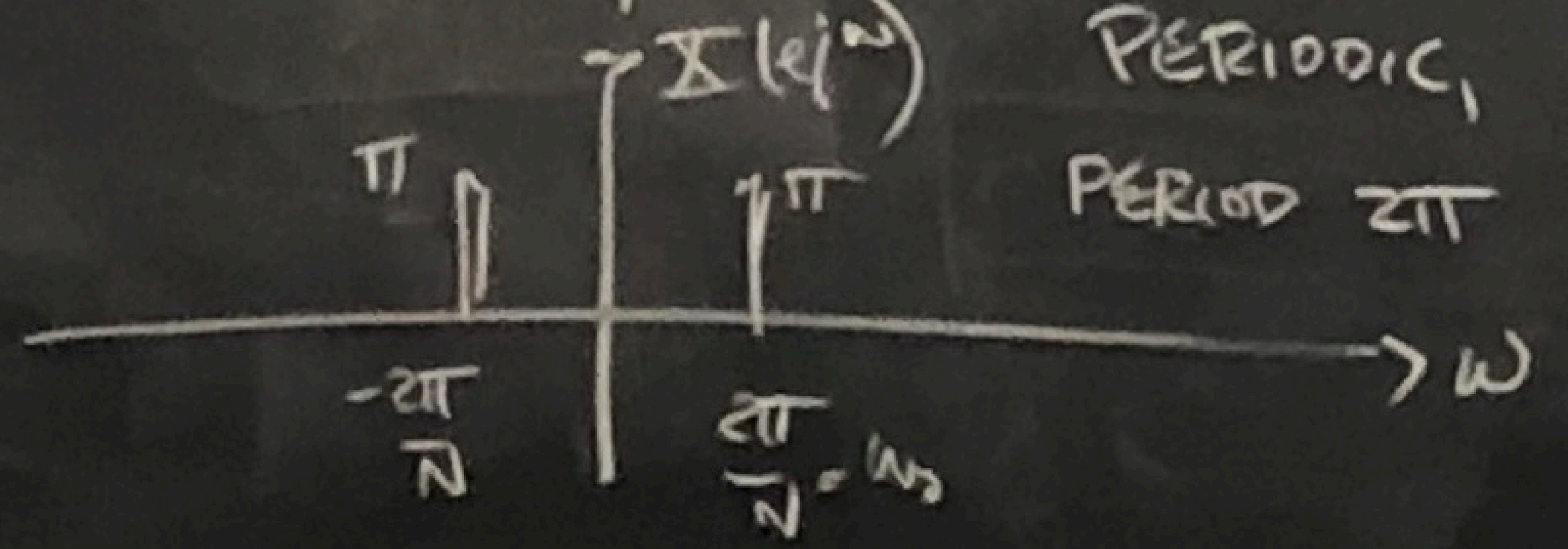
Let

$$= \frac{e^{j\omega_0 n}}{2} + \frac{e^{-j\omega_0 n}}{2} = \frac{e^{j\frac{2\pi n}{N}}}{2} + \frac{e^{-j\frac{2\pi n}{N}}}{2}$$

COMPARING: $\frac{X[\pm 1]}{N} = \frac{1}{2} \Rightarrow X[\pm 1] = \frac{N}{2}$



PERIODIC, PERIOD N



PERIODIC, PERIOD 2π

DELTA FUNCTIONS @ FREQS $\frac{2\pi k}{N}$

AREAS $\frac{2\pi}{N} X[k]$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi nk}{N}}$$

DFT SIZE N

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi nk}{N}}$$

$W_N = e^{-j\frac{2\pi}{N}}$

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, W_N^{-nk}$$

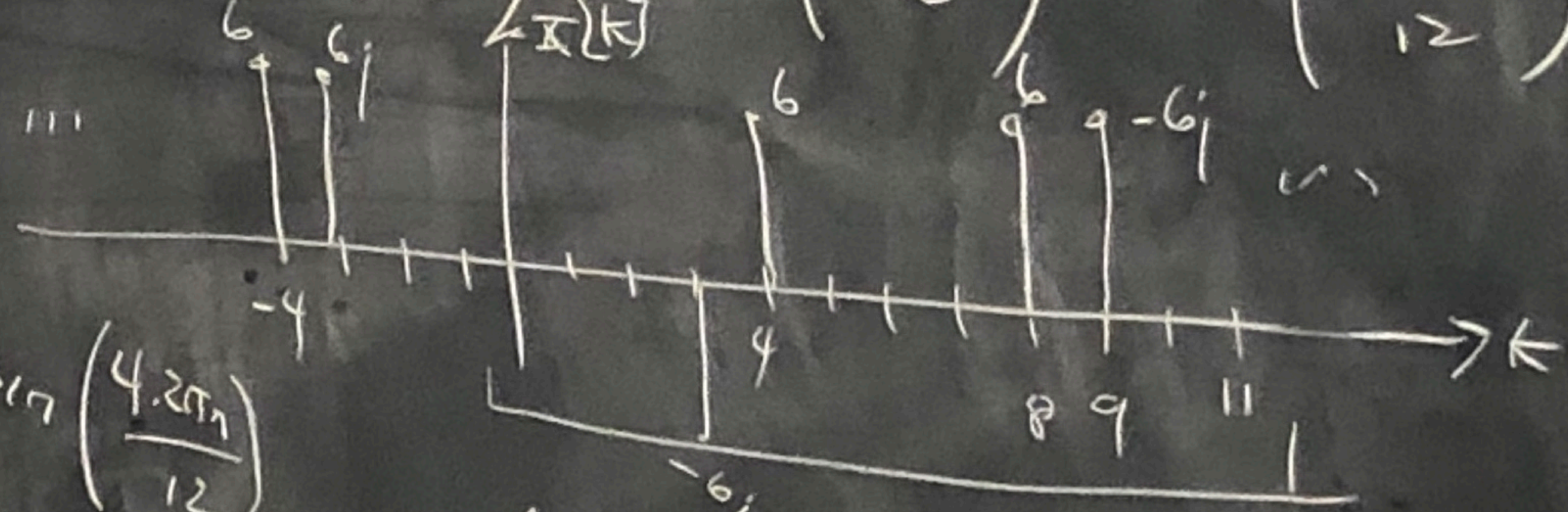
$$X[k] = \sum_{n=0}^{N-1} x[n]\, W_N^{nk}$$

Ex. 3. $\cos\left(\frac{2\pi n}{2}\right) + \sin\left(\frac{2\pi n}{3}\right)$

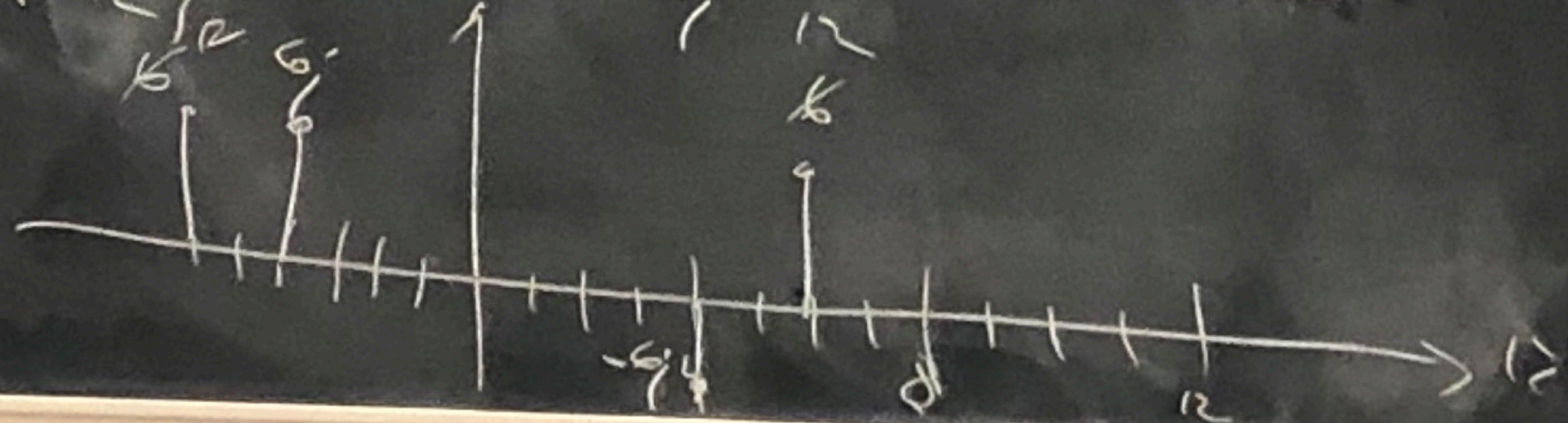
$$\frac{e^{j\omega_0 n}}{2j} - \frac{e^{-j\omega_0 n}}{2j}$$

CONSIDER $\cos\left(\frac{2\pi n}{3}\right) + \sin\left(\frac{2\pi n}{4}\right) = \cos\left(\frac{4 \cdot 2\pi n}{12}\right) + \sin\left(\frac{3 \cdot 2\pi n}{12}\right)$

$$\frac{2\pi \cdot 4}{12} = \frac{2\pi}{3}$$



$\cos\left(\frac{6 \cdot 2\pi n}{12}\right) + \sin\left(\frac{4 \cdot 2\pi n}{12}\right)$



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