DFT

$$x[n] = \frac{1}{N}\sum_{k=0}^{N-1} X[k]\, e^{j\frac{2\pi nk}{N}}$$

$x[n] = 0,\ n < 0$

$n \geq N$

$$X[k] = \sum_{n=0}^{N-1} x[n]\, e^{-j\frac{2\pi nk}{N}}$$

PROPERTIES OF

THE DFT: CIRCULAR

VERSUS LINEAR CONVOLUTION

(OSYP 8.5-8.7)

REVIEW: SELECTED PROPERTIES:

$$x[n] \overset{N}{\Longleftrightarrow} X[k]$$

$$W_N = e^{-j\frac{2\pi}{N}}$$

LINEARITY: $a x_1[n] + b x_2[n] \overset{N}{\Longleftrightarrow} a X_1[k] + b X_2[k]$

TIME SHIFT $x[((n-n_0))_N] \overset{N}{\Longleftrightarrow} e^{-j\frac{2\pi k n_0}{N}} X[k] = W_N^{k n_0} X[k]$

MULT BY COMPLEX EXP. $x[n] e^{j\frac{2\pi r n}{N}} = x[n] W_N^{-rn} \overset{N}{\Longleftrightarrow} X[((k-r))_N]$

Consider:

$$x_1[n] \overset{N}{\Longleftrightarrow} X_1[k]$$

$$x_2[n] \overset{N}{\Longleftrightarrow} X_2[k]$$

$x_1[n], x_2[n]$ NONZERO ONLY FOR $0 \le n \le N-1$

$$X_1[k] = \sum_{n=0}^{N-1} x_1[n] e^{-j\frac{2\pi nk}{N}} = \sum_{n=0}^{N-1} x_1[((n))_N] e^{-j\frac{2\pi nk}{N}}$$

$$y[n] = \sum_{r=0}^{N-1} x_1[((r))_N]\, x_2[((n-r))_N]$$

$$= x_1[n] \overset{N}{\circledast} x_2[n] \overset{N}{\Longleftrightarrow} X_1[k]\, X_2[k]$$

LET $Y[k] = X_1[k] \cdot X_2[k]$

WHAT IS $y[n]$?

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} Y[k]\, e^{j\frac{2\pi nk}{N}} = \frac{1}{N}\sum_{k=0}^{N-1} X_1[k]\, X_2[k]\, e^{j\frac{2\pi nk}{N}}$$

$$y[n] = \frac{1}{N}\sum_{k=0}^{N-1} \underbrace{\sum_{r=0}^{N-1} x_1[((r))_N]\, e^{-j\frac{2\pi rk}{N}}}_{X_1[k]} \cdot \underbrace{\sum_{s=0}^{N-1} x_2[((s))_N]\, e^{-j\frac{2\pi sk}{N}}}_{X_2[k]} \cdot e^{j\frac{2\pi nk}{N}}$$

$$y[n] = \sum_{r=0}^{N-1} x_1[((r))_N] \sum_{s=0}^{N-1} x_2[((s))_N] \cdot \underbrace{\frac{1}{N}\sum_{k=0}^{N-1} e^{+j\frac{2\pi}{N}(n-r-s)k}}_{\begin{cases} 1, & n = r+s \text{ OR } s = n-r \\ 0, & \text{ELSE} \end{cases}}$$

$$y[n] = \sum_{r=0}^{N-1} x_1[((r))_N]\, x_2[((n-r))_N]$$

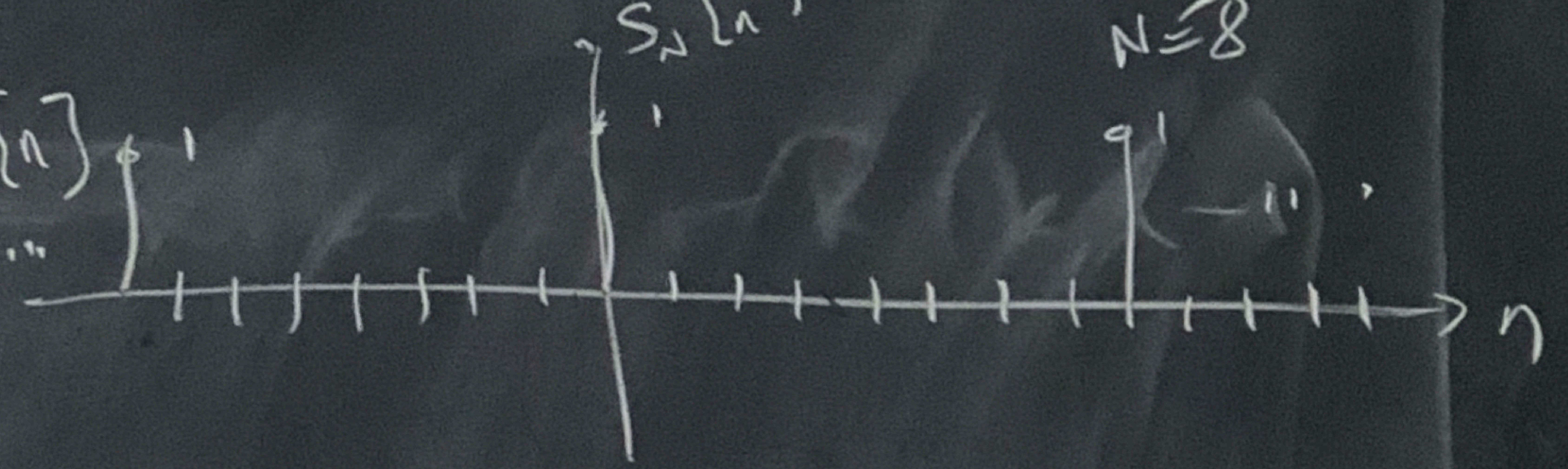
$$W_N = e^{-i\frac{2\pi}{N}}$$

$$\frac{1}{N}\sum_{k=0}^{N-1}\left(e^{i\frac{2\pi}{N}(n-r-s)}\right)^{k} = \frac{1}{N}\,\frac{\left(1-e^{i\frac{2\pi}{\cancel{N}}(n-r-s)\cancel{N}}\right)}{1-e^{i\frac{2\pi}{N}(n-r-s)}}\quad \frac{=0}{\begin{matrix}0, & n=r+s\\ \neq 0, & \text{ELSE}\end{matrix}}$$

$$= \begin{cases} 1, & n = r+s \\ 0, & \text{OTHERWISE} \end{cases}$$

IF $x_1[n], x_2[n]$ ARE FINITE, NONZERO FOR $0 \le n \le N-1$

$$x_2[((n))_N] = x_2[n] * S_N[n]$$



$$y[n] = \sum_{r=0}^{N-1} x_1[((r))_N]\, x_2[((n-r))_N]$$

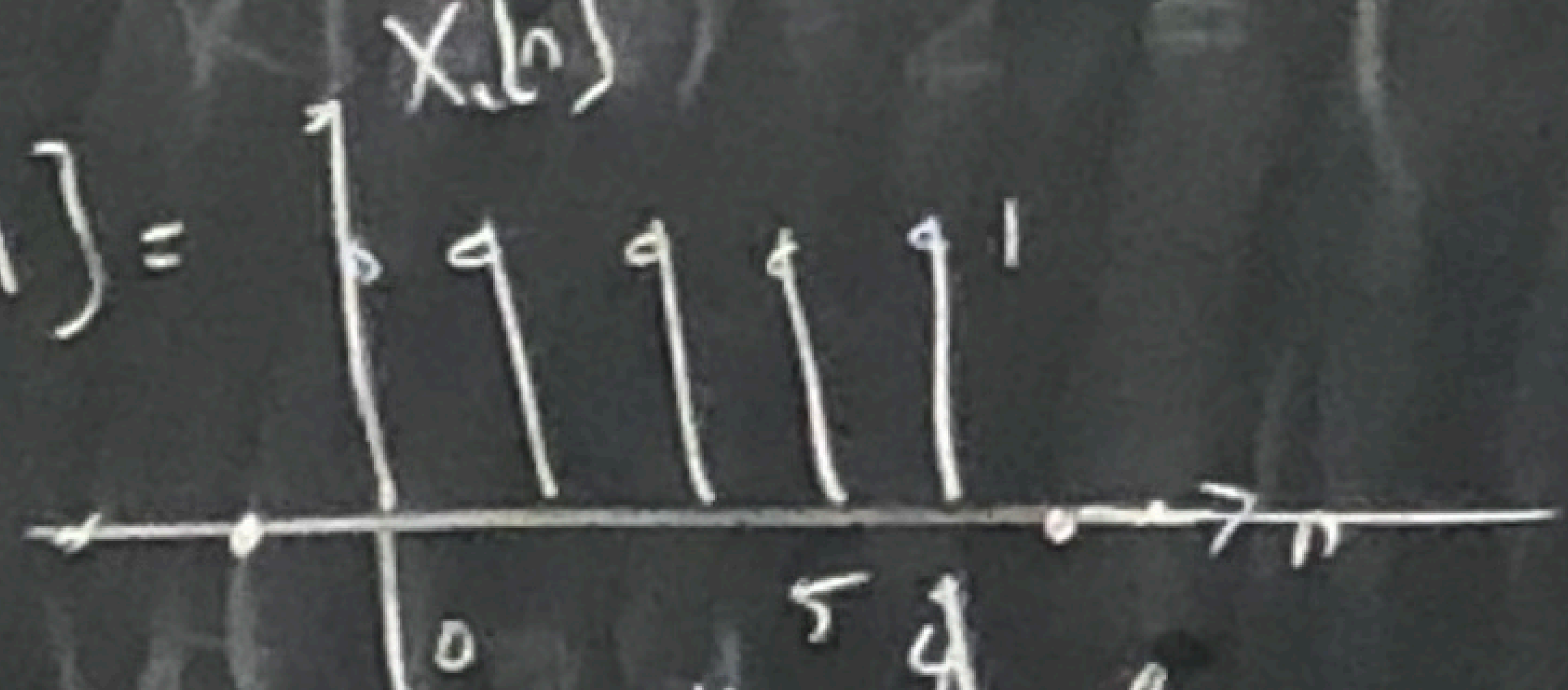
$$= x_1[n] * \left(x_2[n] * S_N[n]\right)$$

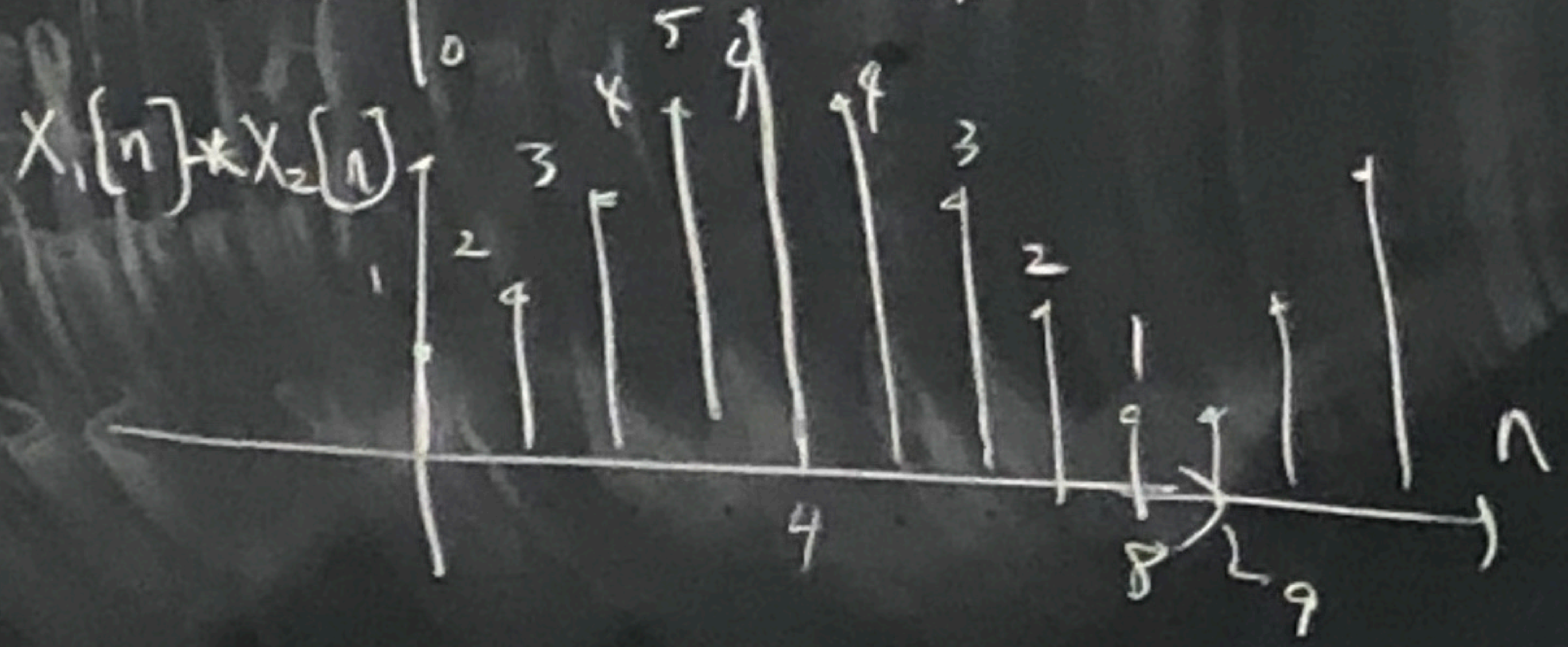
$$= \left(x_1[n] * x_2[n]\right) * S_N[n]$$

CIRCULAR CONVOLUTION = LINEAR CONVOLUTION FOLLOWED BY N-POINT ALIASING

CIRCULAR CONVOLUTION: LINER CONVOLUTION FOLLOWED BY ALIASING

Ex. LET $x_1[n] = x_2[n] =$





CONSIDER FINITE DURATION SEQUENCES...

$x_1[n]$ LENGTH N_1

$x_2[n]$ LENGTH N_2

$N \geq N_1, N_2$

FOR WHAT N IS $\underbrace{x_1[n]*x_2[n]}_{\text{LENGTH } N_1+N_2-1} = x_1[n] \overset{N}{\circledast} x_2[n]$

$\boxed{\text{FOR } x_1[n] \overset{N}{\circledast} x_2[n] = x_1[n]*x_2[n] \text{ MUST HAVE } N \geq N_1+N_2-1}$

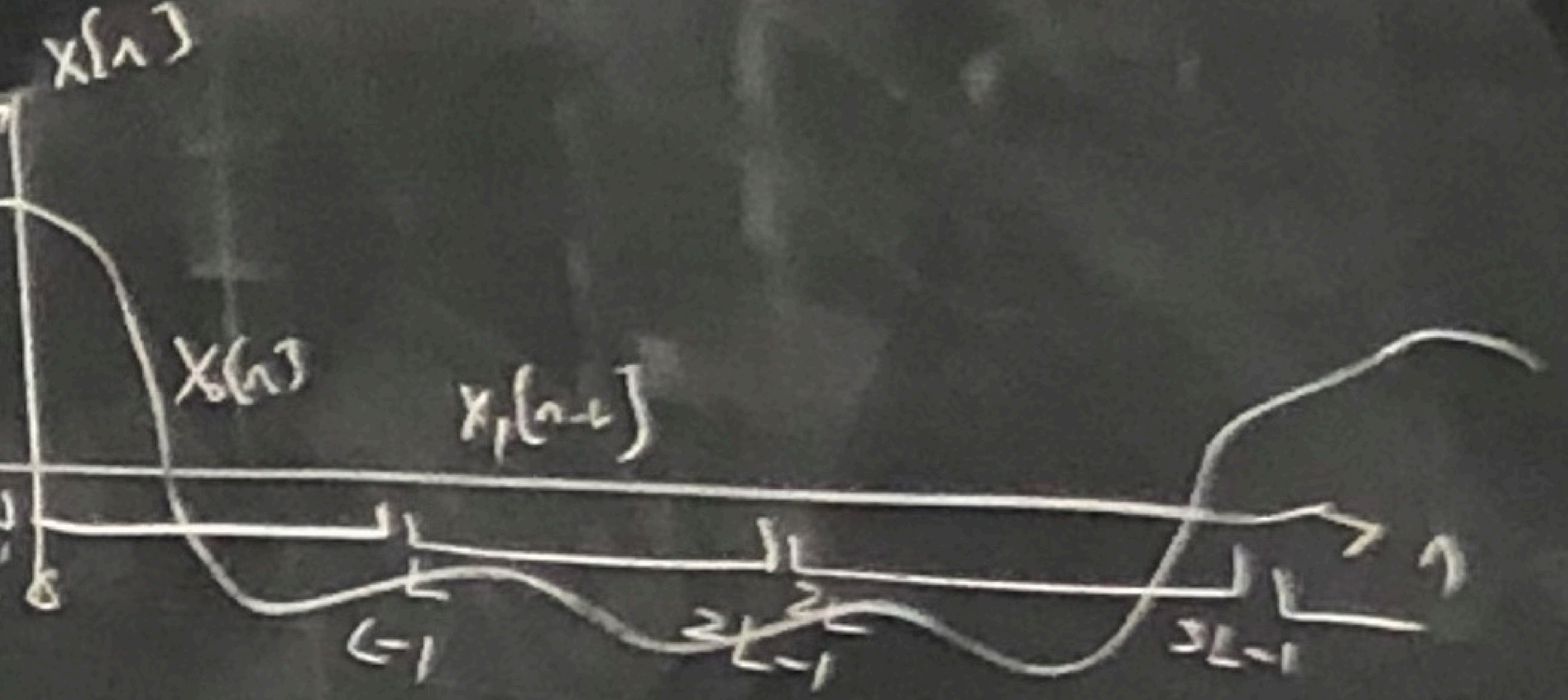
CONSIDER

$x[n] \neq 0, \quad 0 \le n \le \cancel{\text{LARGE}}\ 10^6$

$h[n] \neq 0, \quad 0 \le n \le P-1$

OVERLAP ADD METHOD

BREAK INTO ABUTTING SECTIONS OF LENGTH L



1. $x[n] = \sum_{r=0}^{\infty} x_r[n-rL]$; $x_r[n] = \begin{cases} x[n+rL], & 0 \le n \le L-1 \\ 0 & ; \text{ELSE} \end{cases}$

2. LET $y_r[n] = x_r[n] \circledast h[n]$ — $L+P-1$ pt. CIRC CONV

3. $y[n] = \sum_{r=0}^{\infty} y_r[n-rL]$

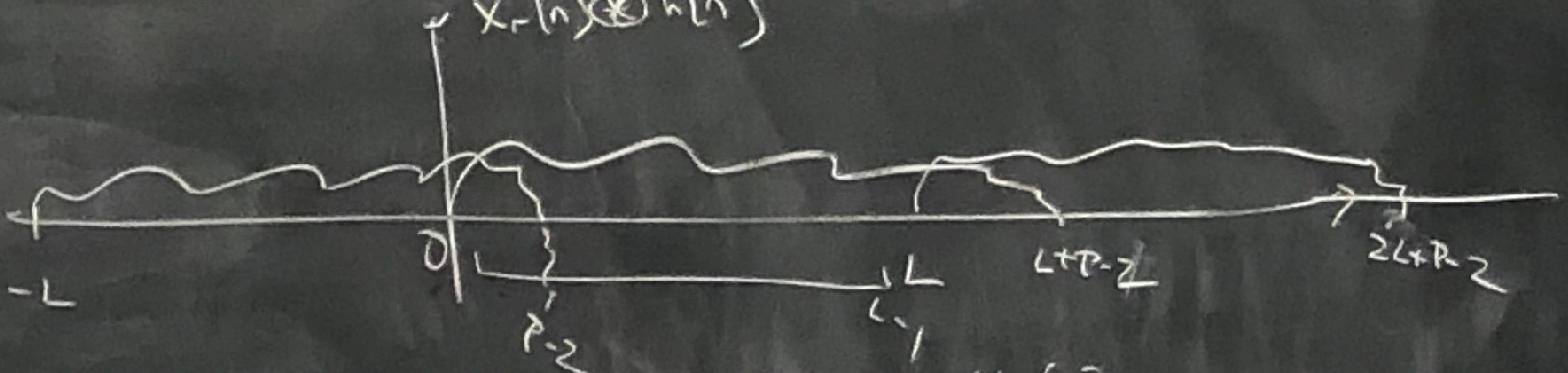
OVERLAP-SAVE ALGORITHM

1. DEVELOP SUBSEQUENCES of INPUT $x[n]$ of LENGTH L, $x_r[n]$
2. PERFORM L-PT. CIRCULAR CONVOLUTION of $x_r[n]$ with $h[n]$
3. DISCARD OUTPUT POINTS CORRUPTED BY ALIASING; KEEP THE "GOOD" ONES

   RECONSTRUCT OUTPUT

WHICH POINTS ARE "GOOD"?

$x_r[n]$ LENGTH L, $h[n]$ LENGTH P, L-PT. CIRC. CONV



THROW AWAY $y_r[n]$, $0 \le n \le P-2$

KEEP $y_r[n]$, $P-1 \le n \le L-1$