2/26/24

LINEAR VERSUS CIRCULAR CONVOLUTION

[Cosp 8.6-8.8]

REMINDER: Q1 WEDNESDAY

IF $x[n] \neq 0, \ 0 \leq n \leq N-1$

IDFT $x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] e^{j \frac{2\pi nk}{N}}$

DFT $X[k] = \sum_{n=0}^{N-1} x[n] e^{-j \frac{2\pi nk}{N}}$

let $W_N = e^{-j \frac{2\pi}{N}}$

$x[n] = \frac{1}{N} \sum_{k=0}^{N-1} X[k] W_N^{-nk}$

$X[k] = \sum_{n=0}^{N-1} x[n] W_N^{nk}$

CONVOLUTION/MULTIPLICATION PROPERTY

let $x_1[n] \overset{N}{\Longleftrightarrow} X_1[k]$

$x_2[n] \overset{N}{\Longleftrightarrow} X_2[k]$

$Y[k] = X_1[k] X_2[k]$

THEN $y[n] \overset{N}{\Longleftrightarrow} Y[k]$

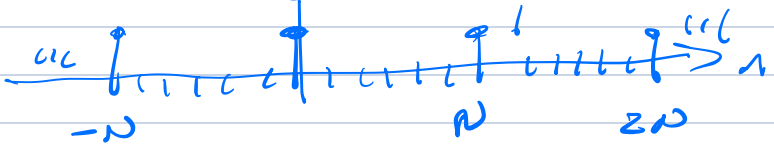
$y[n] = \sum_{k=0}^{N-1} x_1[((n-k))_N] \, x_2[((k))_N] = x_1[n] \overset{N}{\circledast} x_2[n]$

DOUBLY INFINITE

N-POINT CIRCULAR CONVOLUTION

$x_1[n] \overset{N}{\circledast} x_2[n] = x_2[n] * \left[x_1[n] * \delta_N[n] \right]$

$\delta_N[n] = \sum_{r=-\infty}^{\infty} \delta[n - rN]$



$$x_1[n] \overset{N}{\circledast} x_2[n] = x_2[n] * \left[x_1[n] *_N \delta_N[n] \right] = \left(x_2[n] * x_1[n] \right) *_N \delta_N[n]$$

CIRCULAR CONVOLUTION

LINEAR CONVOLUTION

N-POINT ALIASING

CONVOLUTION OF FINITE-DURATION SEQUENCES

$x_1[n] * x_2[n] = y[n]$

$x_1[n]$: LENGTH N_1; $x_2[n]$: LENGTH N_2; $y[n]$: LENGTH N_1+N_2-1

$$x_1[n] * x_2[n] = x_1[n] \overset{N}{\circledast} x_2[n] \quad \text{IF } N \geq N_1+N_2-1$$

FILTERING USING CIRCULAR CONVOLUTION

$x[n] \rightarrow \boxed{h[n]} \rightarrow y[n]$

1 HOUR OF SPEECH, 16 kHz: $(16)(10^3)(60)(60) = (57.6)(10^6)$ SAMPLES $\approx 2^{25.8} \approx 2^{26}$ (67.1 MILLION)

$h[n]$ IS OF LENGTH 1024

POSSIBLE SOLUTION: 67.1-MILLION-PT. DFT of x; 67.1-MILLION-PT. DFT of h; MULT; 67.1 MILLION PTS IDFT of $X[k]H[k]$ FOR RESULT

SOLUTIONS:

1. BREAK UP INPUT INTO SUBSEQUENCES
2. CONVOLVE SUBSEQUENCES INDIVIDUALLY WITH $h[n]$
3. REASSEMBLE OUTPUT

OVERLAP-ADD ALGORITHM OLA

[OLS] OVERLAP-SAVE ALGORITHM

THE OVERLAP-ADD PROCEDURE (OLA)

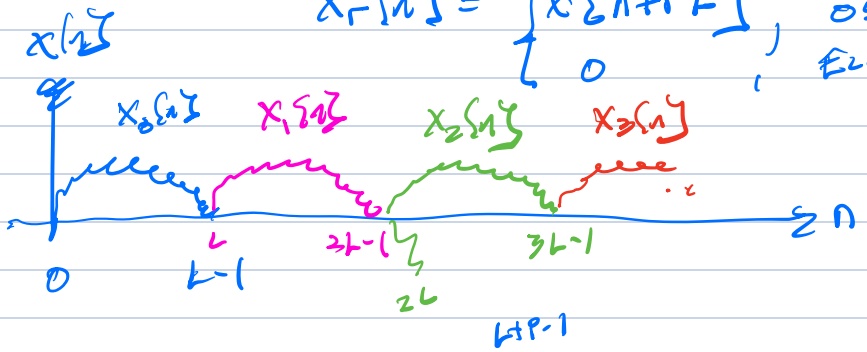
ASSUME $h[n] \neq 0, \quad 0 \le n \le P-1$

OVERLAP-ADD

1. BREAK INPUT INTO ABUTTING SEQUENCES of LENGTH L
2. PERFORM CIRCULAR CONVOLUTION OF SAMPLE RESPONSE USING DFTs OF SIZE $L+P-1$
3. COMBINE OUTPUTS

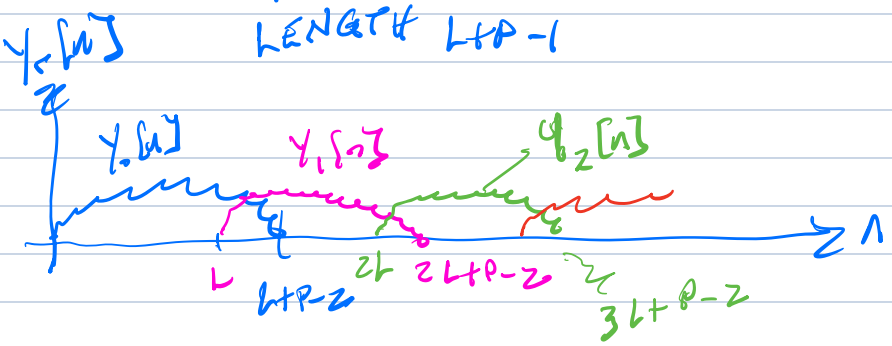
Let $x[n] = \sum_{r=0}^{\infty} x_r[n - rL], \quad x_r[n] \neq 0, \quad 0 \le n \le L-1$

$$x_r[n] = \begin{cases} x[n+rL], & 0 \le n \le L-1 \\ 0, & \text{ELSE} \end{cases}$$



OUTPUTS $y_r[n] = x_r[n] \circledast h[n] = x_r[n] * h[n]$

$y_r[n]$ LENGTH $L+P-1$



$$y[n] = \sum_{r=0}^{\infty} y_r[n - rL] = \sum_{r=0}^{\infty} x_r[n - rL] \circledast h[n]$$

EX: 1024-PT. DFT

$P = 256$

$L+P-1 = 1024$

$L = 769$

	$x_r[n]$		$y_r[n]$	
r	FIRST	LAST	FIRST	LAST
0	0	768	0	1023
1	769	1537	769	1792
2	1538	2306	1538	2561

THE OVERLAP SAVE METHOD (OLS)

$h[n]$ P SAMPLES

1. OBTAIN SUBSEQUENCES OF INPUT, LENGTH L $x_r[n]$

2. PERFORM L-PT. CIRCULAR CONV of $x_r[n]$ AND $h[n]$

3. SAVE THE "GOOD" POINTS THAT ARE NOT AFFECTED BY ALIASING

(COMBINE GOOD POINTS, DISCARD BAD POINTS)

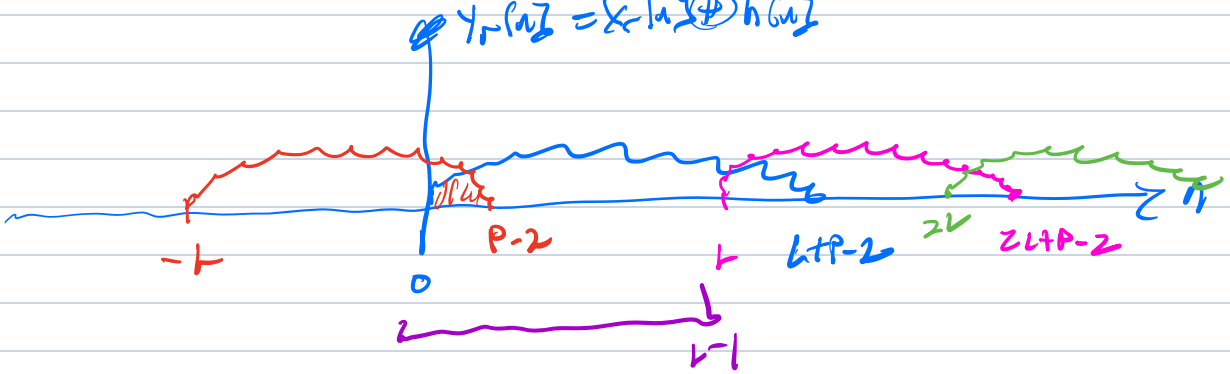
BUT WHICH POINTS of $y_r[n]$ ARE GOOD?

ASSUME $x_r[n]$ IS of LENGTH L
$h[n]$ IS of LENGTH P
<IR. CONV. (DFT) SIZE IS ALSO L

$$y_r[n] = x_r[n] \overset{L}{\circledast} h[n]$$

LENGTH L LENGTH P

$y_r[n] = x_r[n] \overset{L}{\circledast} h[n]$



$y_r[n]$ AFFECTED BY ALIASING FOR $0 \le n \le P-2$
OK, $P-1 \le n \le L-1$

4. ADD THE ADJACENT "GOOD" POINTS

5. START NEXT INPUT SEQUENCE P-1 POINTS EARLIER

ANALYTICALLY (AS PER OSAP),

$$x_r[n] = x[n + r(L-(P-1)) - (P-1)], \quad 0 \le n \le L-1$$

$$y[n] = \sum_{r=0}^{\infty} y_{rp}[n], \quad P-1 \le n \le L-1$$

$$y_{rp}[n] = x_r[n] \overset{L}{\circledast} h[n]$$

	$x_r[n]$		$y_{rp}[n]$	
r	LOWER SEQ.		SAMPLES	
0	0	1023	255	1023
1	769	1792	1024	1792
2	1538	2561	1793	2561

EX: 1024-PT. DFTs

$P = 256$