REVIEW of MULTI-RATE DSP MATH + TECHNIQES;

INTRO TO THE DTFS, DTFT of PERIODIC TIME FUNCTIONS

(OS4P 4.6, 8.3–8.5)

CTFT

$$\int_{-\infty}^{\infty} |x(t)|^2 \, dt < \infty$$

$$x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t} \, d\Omega$$

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t} \, dt$$

CTFS

$$x(t) = x(t-T)$$

$$\Omega_0 = 2\pi / T$$

$$x(t) = \sum_{k=-\infty}^{\infty} X_k e^{jk\Omega_0 t}$$

$$X_k = \frac{1}{T} \int_T x(t) e^{-jk\Omega_0 t} \, dt$$

DTFT

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$$

$$x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n} \, d\omega$$

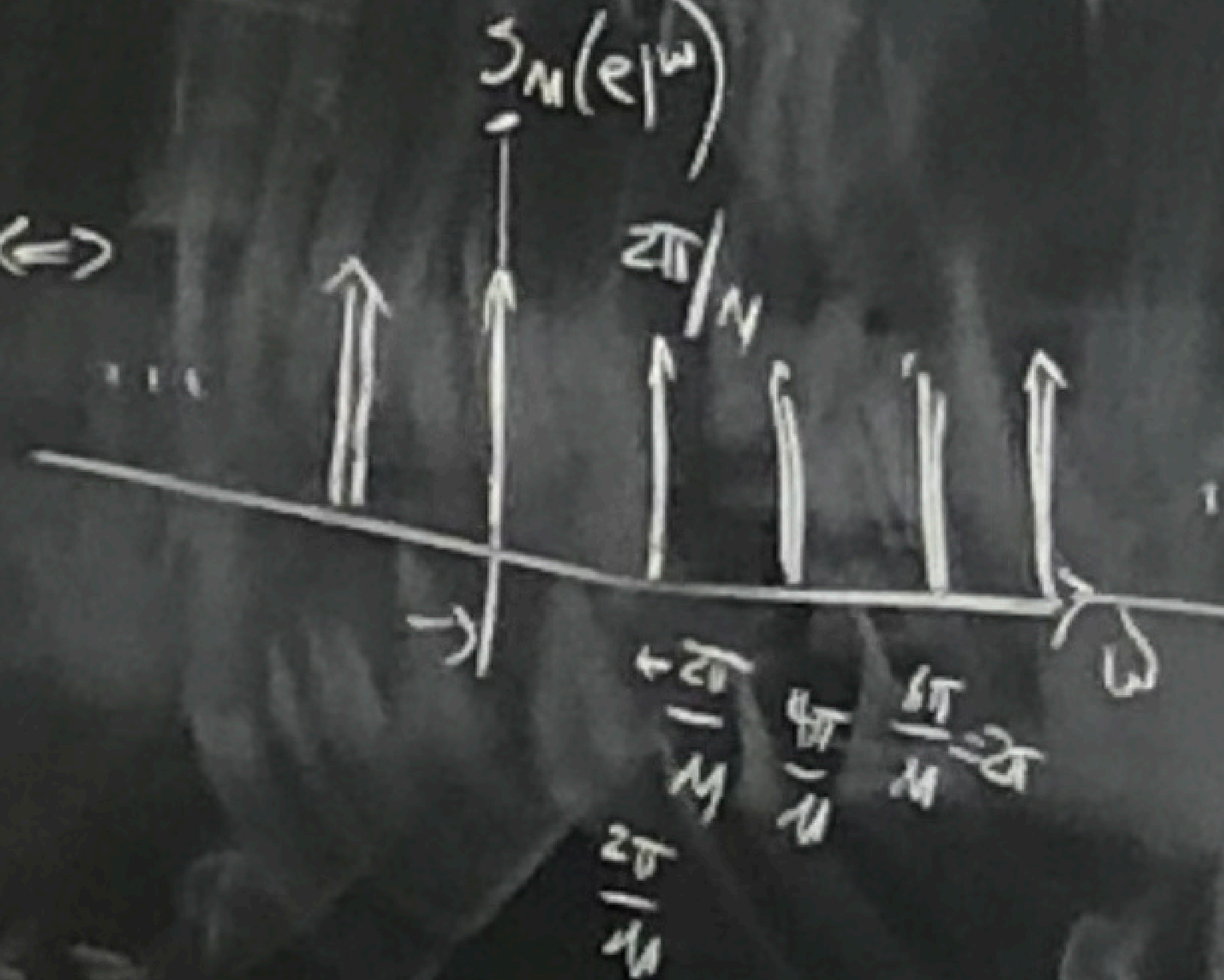
$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

IDEAL DECIMATION

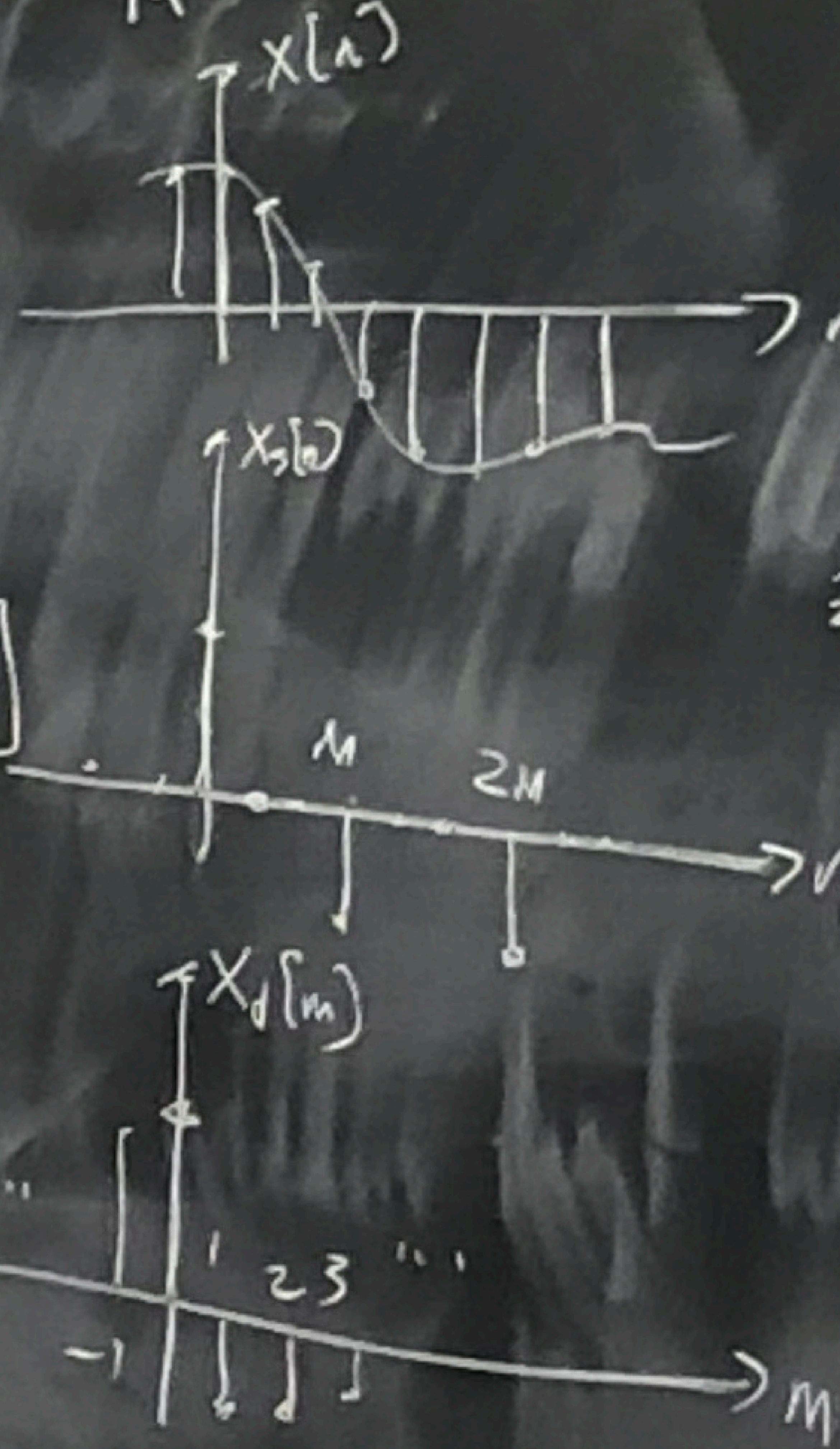
$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow x_d[m] = x[mM]$$

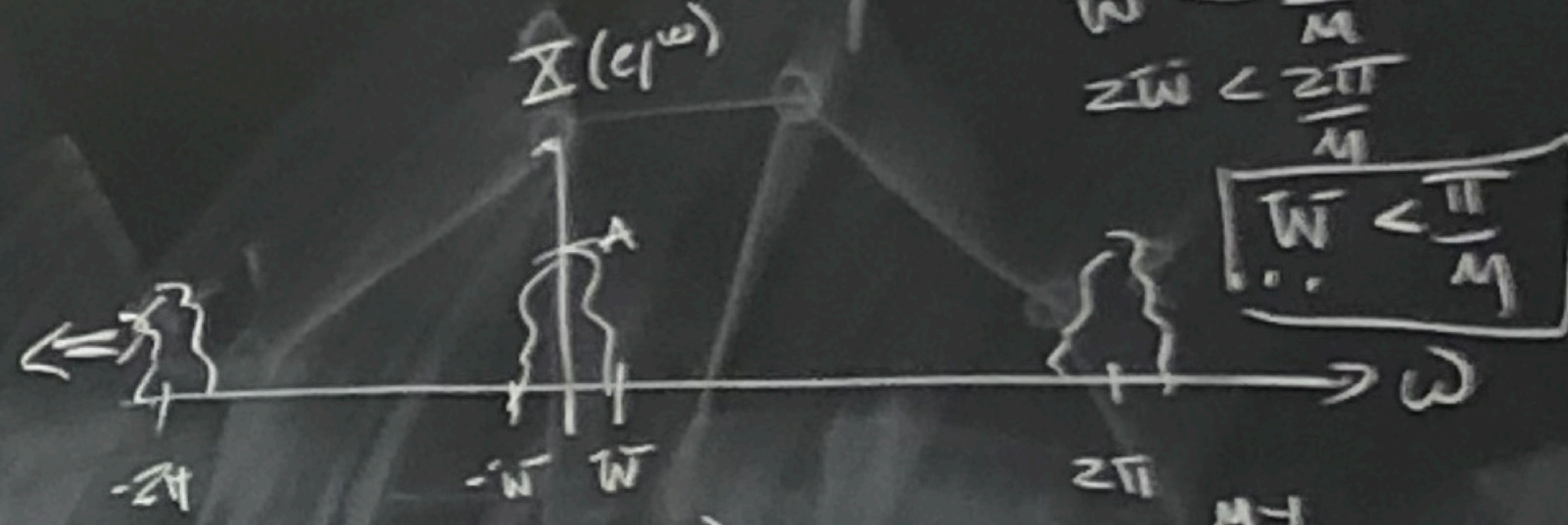
$$x[n] \longrightarrow \otimes \xrightarrow{x_s[n]} \boxed{\text{CHANGE TIME SCALE}} \longrightarrow x_d[m] = x_s[mM]$$

$$s_M[n] = \begin{cases} 1, & n = rM \\ 0, & \text{ELSE} \end{cases} \Longleftrightarrow$$

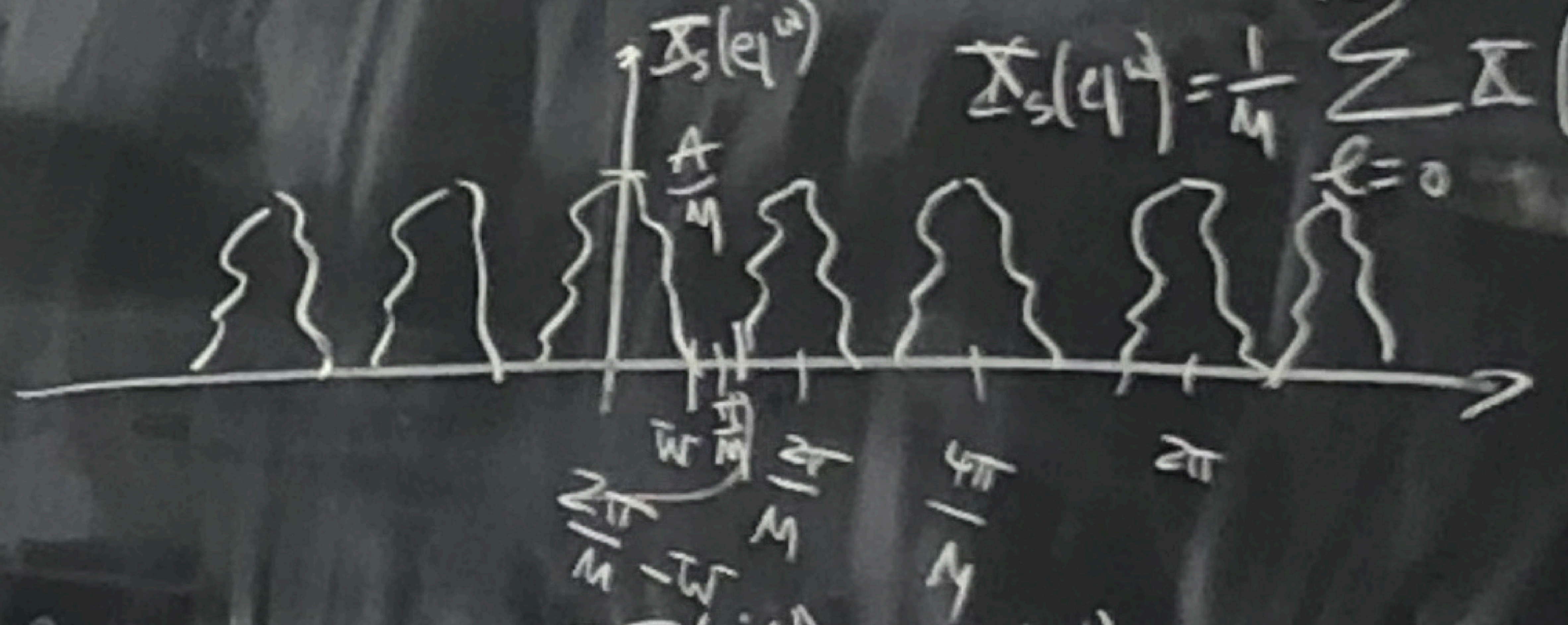


$M = 3$

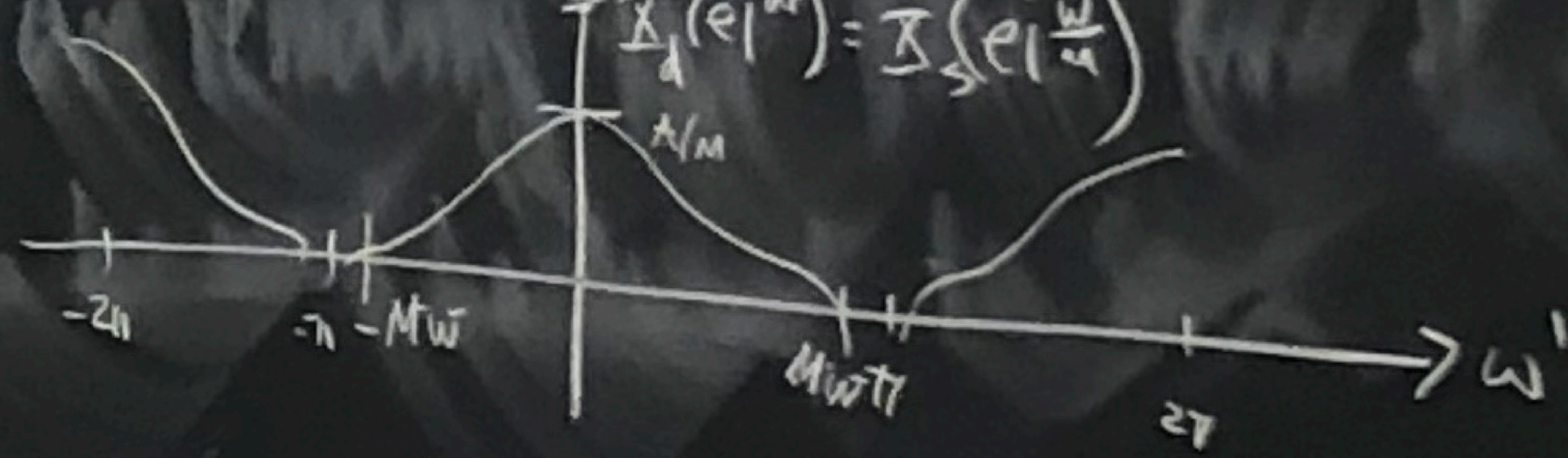




$$X_s(e^{j\omega}) = \frac{1}{M} \sum_{\ell=0}^{M-1} X\left(e^{j\left(\omega - \frac{2\pi \ell}{M}\right)}\right)$$



$$X_d(e^{j\omega'}) = X_s(e^{j\frac{\omega'}{M}})$$



TO AVOID ALIASING

$$\bar W < \frac{2\pi}{M} - \bar W$$

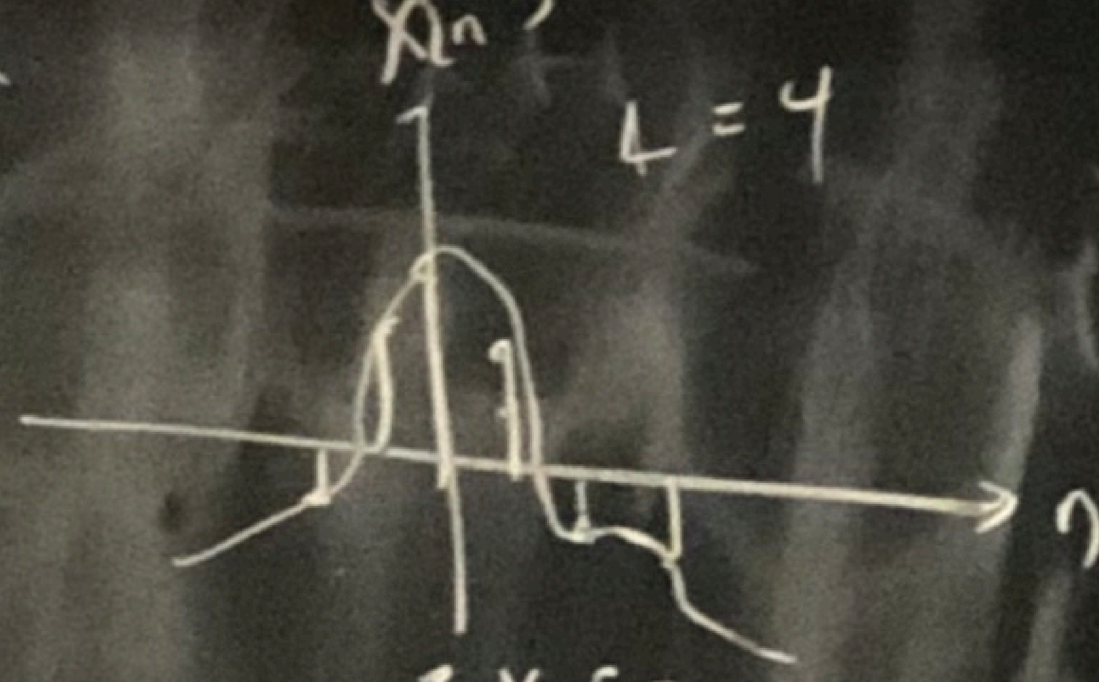
$$2\bar W < \frac{2\pi}{M}$$

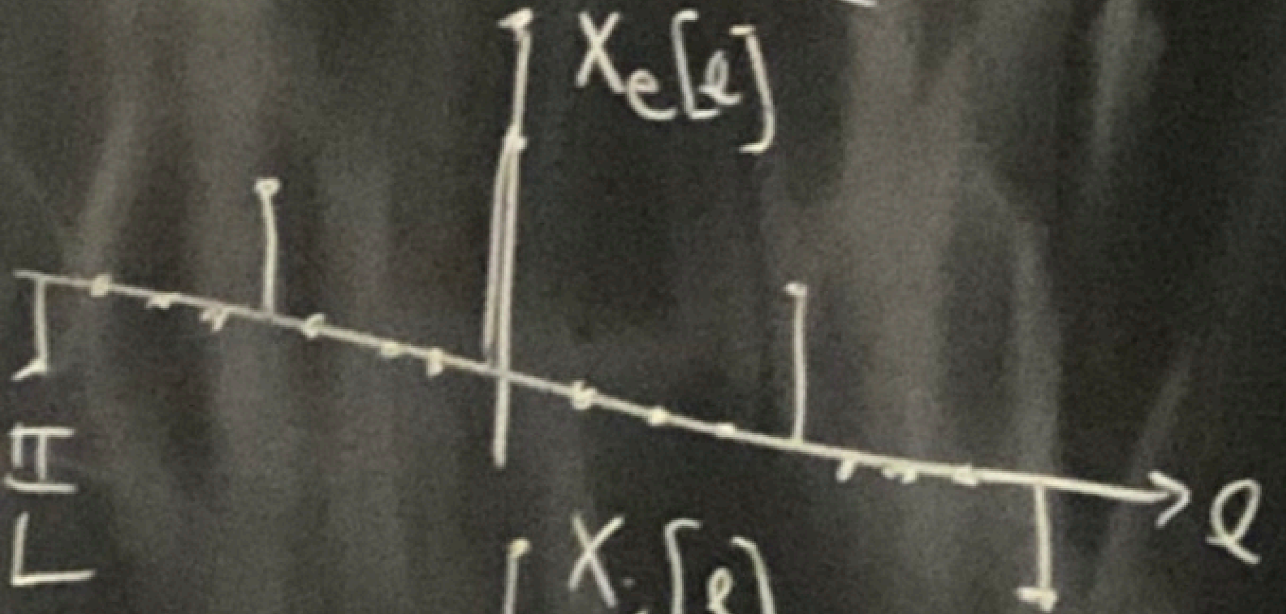
$$\boxed{\bar W < \frac{\pi}{M}}$$

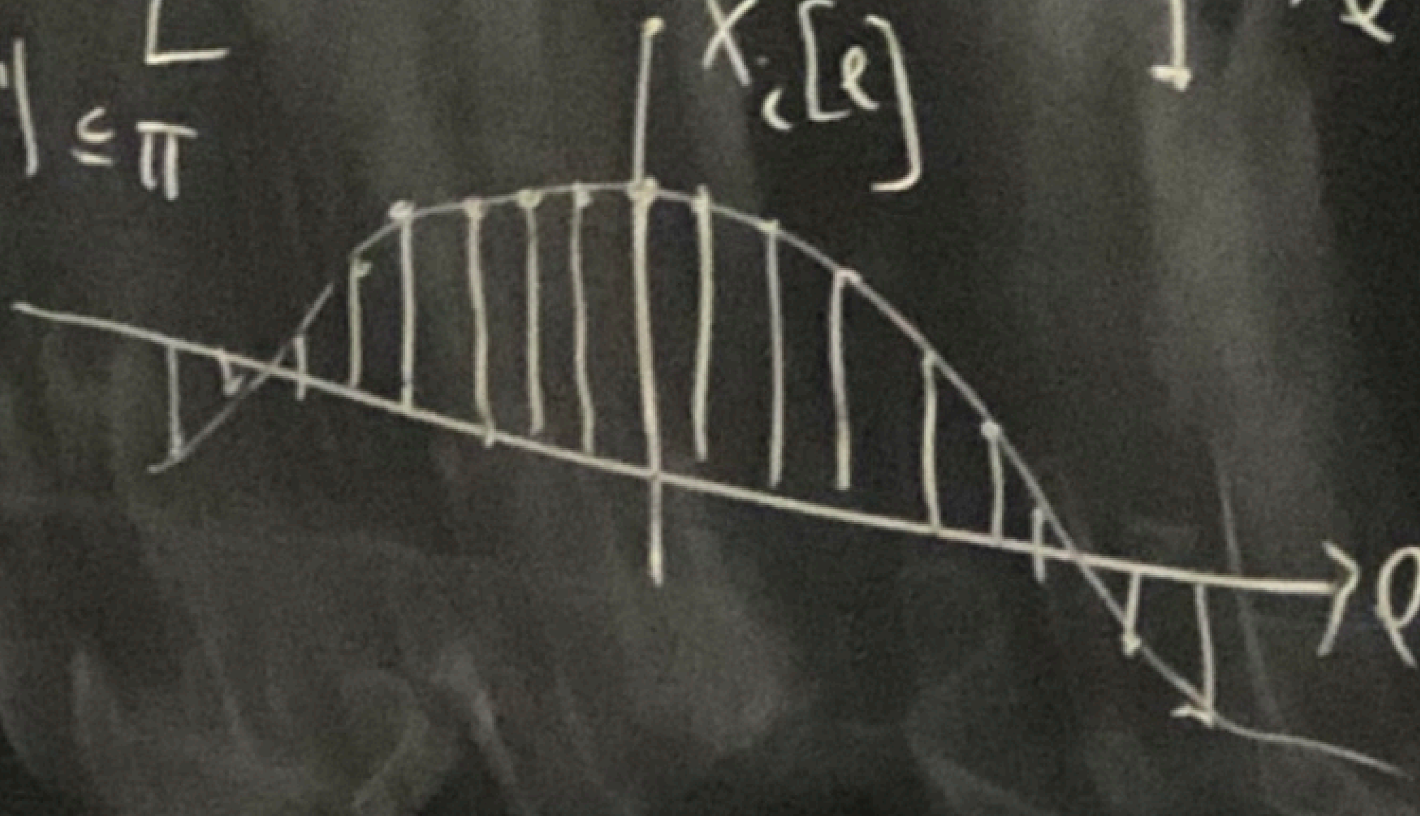
$$x[n] \to \boxed{\uparrow L} \to x_i[l] = \begin{cases} x\left[\frac{n}{L}\right], & \frac{n}{L} \text{ INTEGER} \\ \text{SOMETHING ELSE}, & \text{ELSE} \end{cases}$$

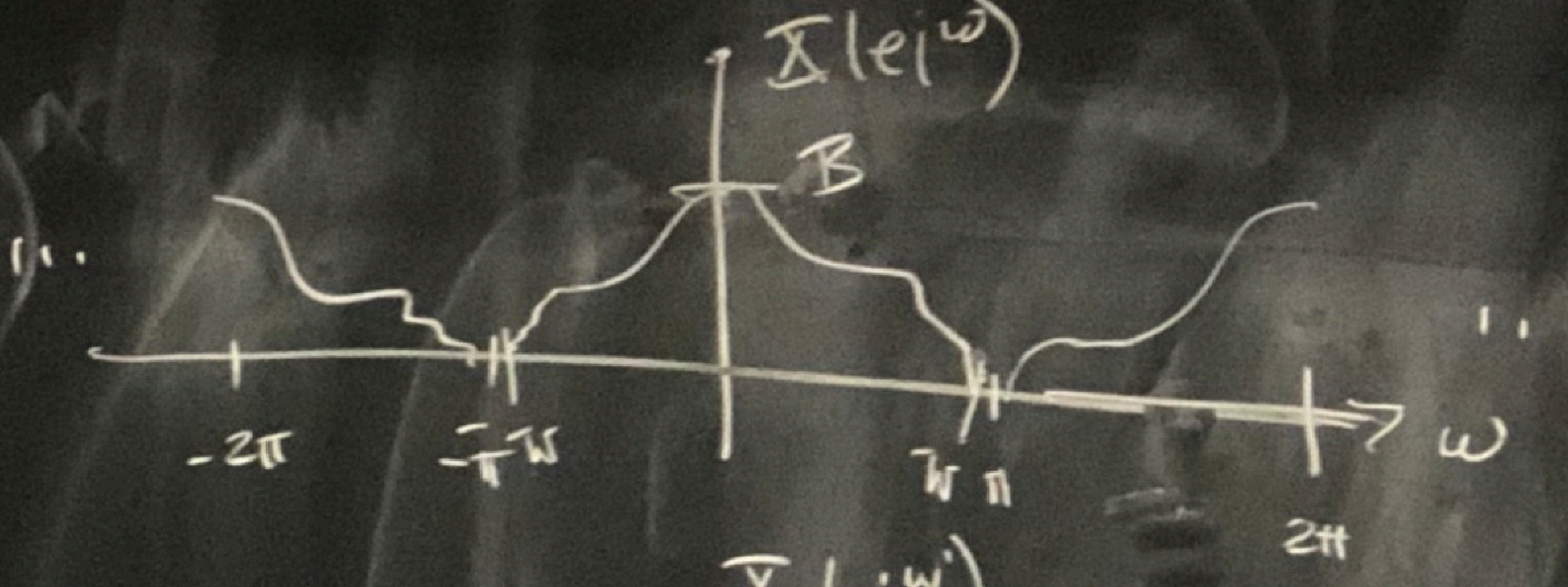
$$x[n] \to \boxed{\text{INSERTS } L-1 \text{ ZEROS}} \xrightarrow{x_e[l]} \boxed{\text{LPF}} \to x_i[l]$$

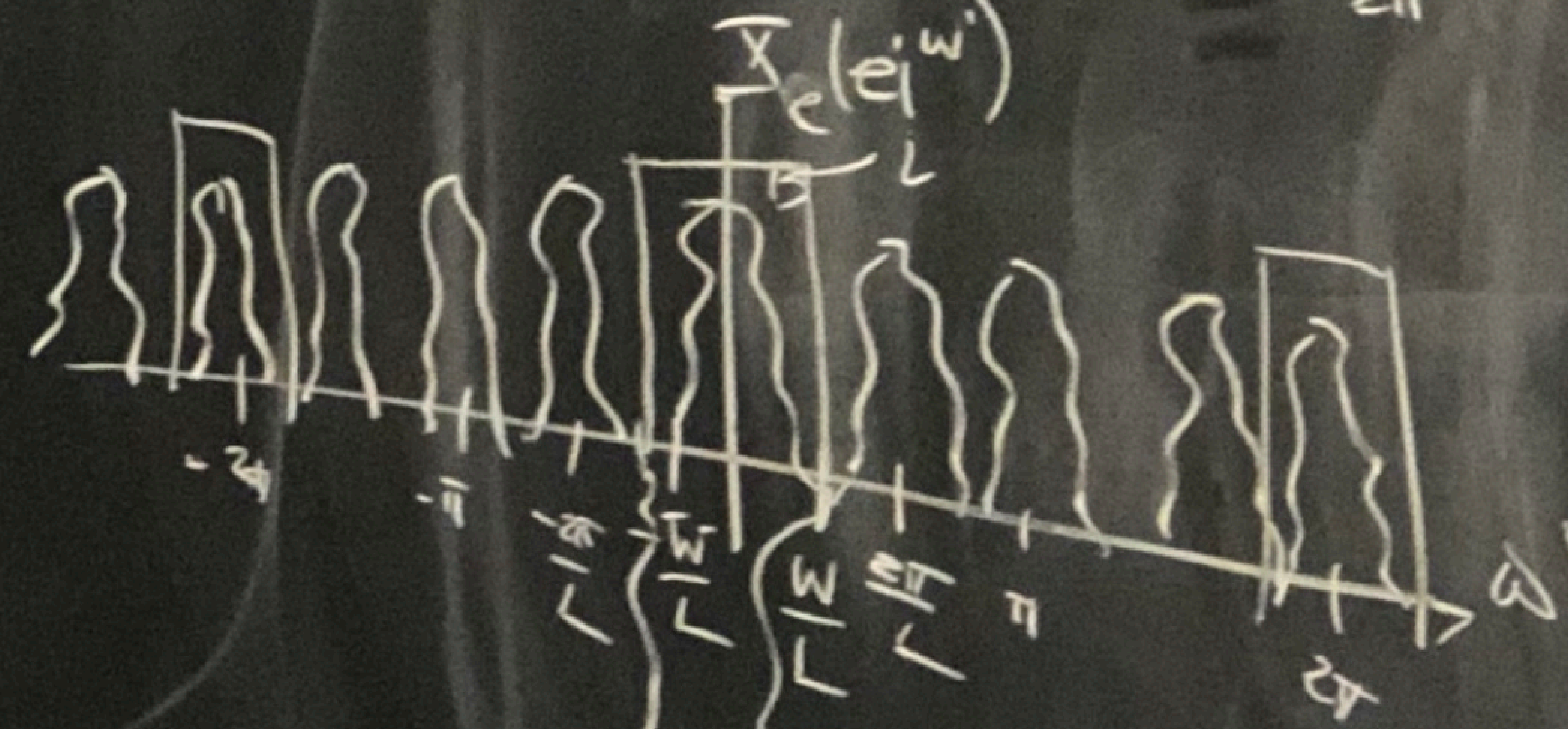
$$H_{LP}(e^{j\omega'}) = \begin{cases} L, & 0 \le |\omega'| < \frac{\pi}{L} \\ 0, & \frac{\pi}{L} \le |\omega'| \le \pi \end{cases}$$

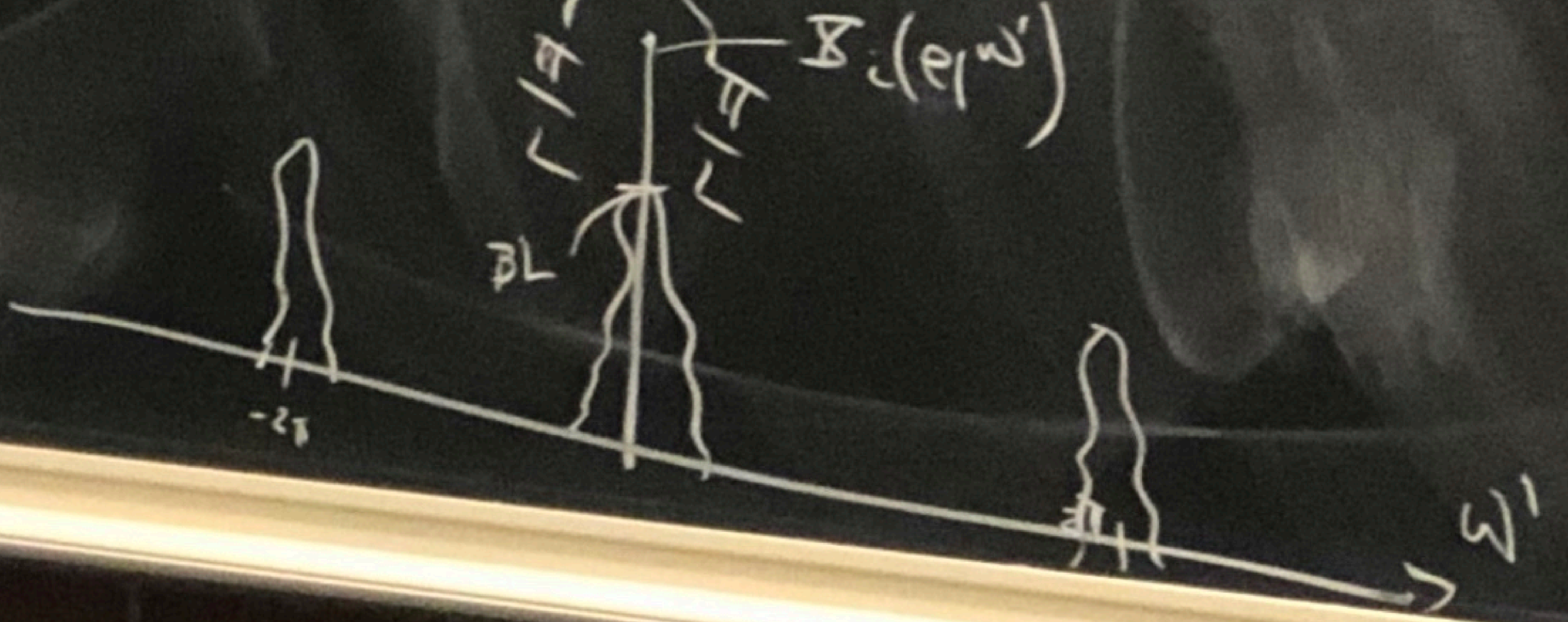












$$X_i(e^{j\omega'}) = \begin{cases} L\,X(e^{j\omega' L}), & 0 \le |\omega'| \le \frac{\pi}{L} \\ 0, & \frac{\pi}{L} < |\omega'| \le \pi \end{cases}$$

INTEGER CHANGE IN SAMPLING RATE

REPRESENTATION

PERIODIC DT TIME FUNCTIONS

IF $\tilde{x}[n] = \tilde{x}[n-N]$

CONSIDER

COEFFICENT K:

$e^{j\frac{2\pi k n}{N}}$

COEFFICIENT K+N:

$e^{j\frac{2\pi(k+N)n}{N}} = e^{j\frac{2\pi k n}{N}} \cdot \cancel{e^{j\frac{2\pi N n}{N}}}$

$\omega_0 = \frac{2\pi}{N}$

$\left\{ e^{jk\omega_0 n} \right\}$

$\left\{ e^{j\frac{2\pi k n}{N}} \right\}$

DT FS

For $\tilde{x}[n] = \tilde{x}[n-N]$

$\omega_0 = \frac{2\pi}{N}$

$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{j\frac{2\pi n k}{N}}$

$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-j\frac{2\pi n k}{N}}$

$\tilde{x}[n] = \frac{1}{N} \sum_{k=0}^{N-1} \tilde{X}[k] e^{jk\omega_0 n}$

$\tilde{X}[k] = \sum_{n=0}^{N-1} \tilde{x}[n] e^{-jk\omega_0 n}$

DTFT OF PERIODIC TIME FUNCTIONS

CONSIDER PERIODIC $\tilde{x}[n] = \tilde{x}[n-N]$

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k]\, e^{+jk\omega_0 n} \quad , \omega = \frac{2\pi}{N}$$

DTFT OF $e^{jk\omega_0 n} \Longleftrightarrow 2\pi \sum_{r=-\infty}^{\infty} \delta(\omega - k\omega_0 - 2\pi r)$

$$\tilde{X}[k] e^{jk\omega_0 n} \Longleftrightarrow 2\pi \tilde{X}[k] \sum_{r=-\infty}^{\infty} \delta(\omega - k\omega_0 - 2\pi r)$$

$$\tilde{x}[n] = \frac{1}{N}\sum_{k=0}^{N-1} \tilde{X}[k] e^{jk\omega_0 n} \Longleftrightarrow \frac{2\pi}{N} \sum_{k=0}^{N-1} \tilde{X}[k] \sum_{r=-\infty}^{\infty} \delta(\omega - k\omega_0 - 2\pi r)$$

REVIEW o

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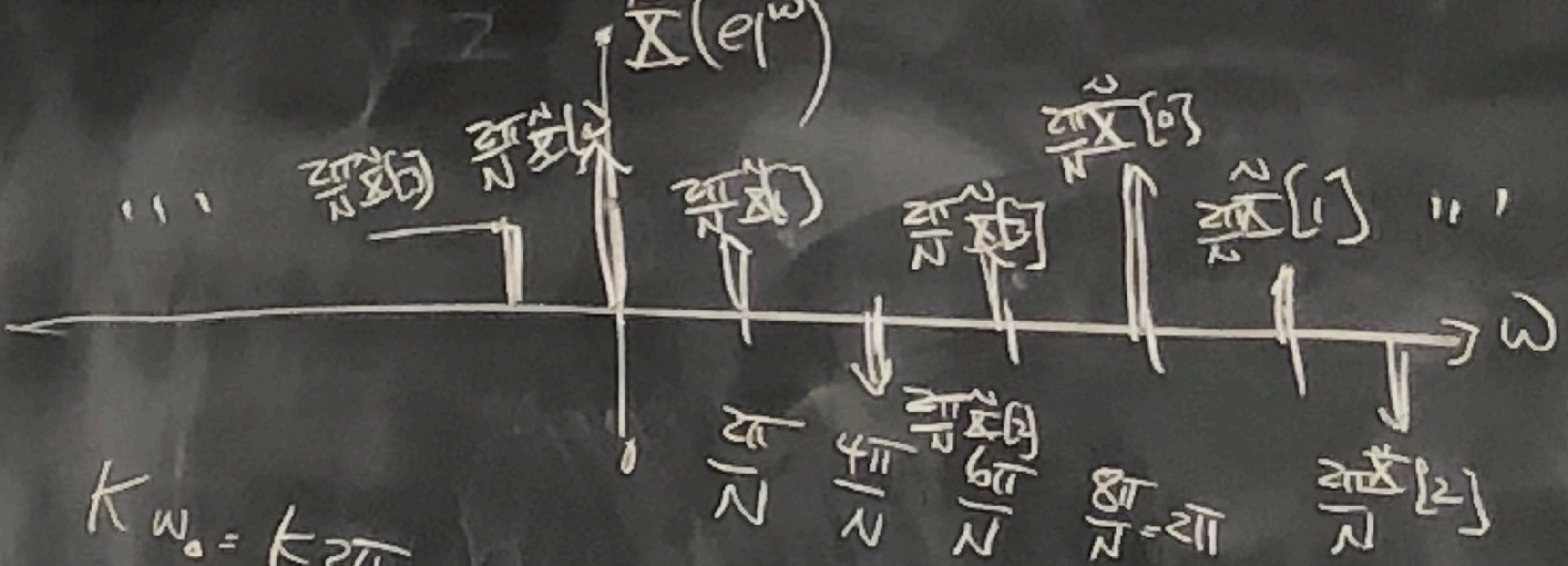
INTRO TO

of PERIODIC

(OS4P 4

IN GENERAL, N = 4



$$k\omega_0 = \frac{k2\pi}{N}$$