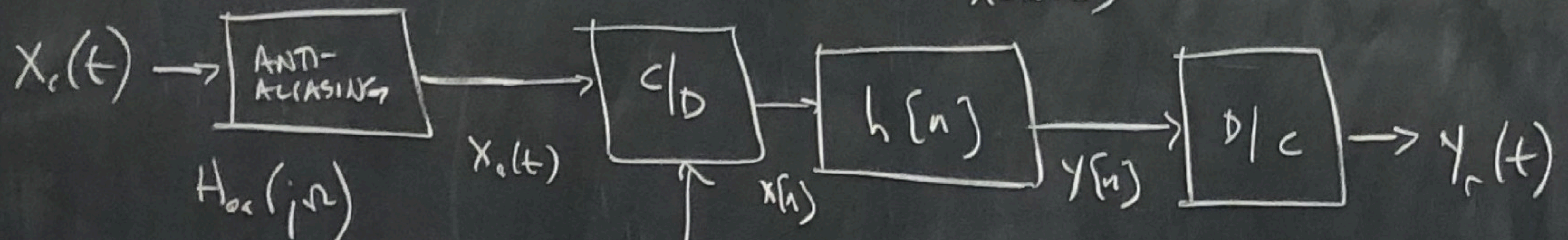
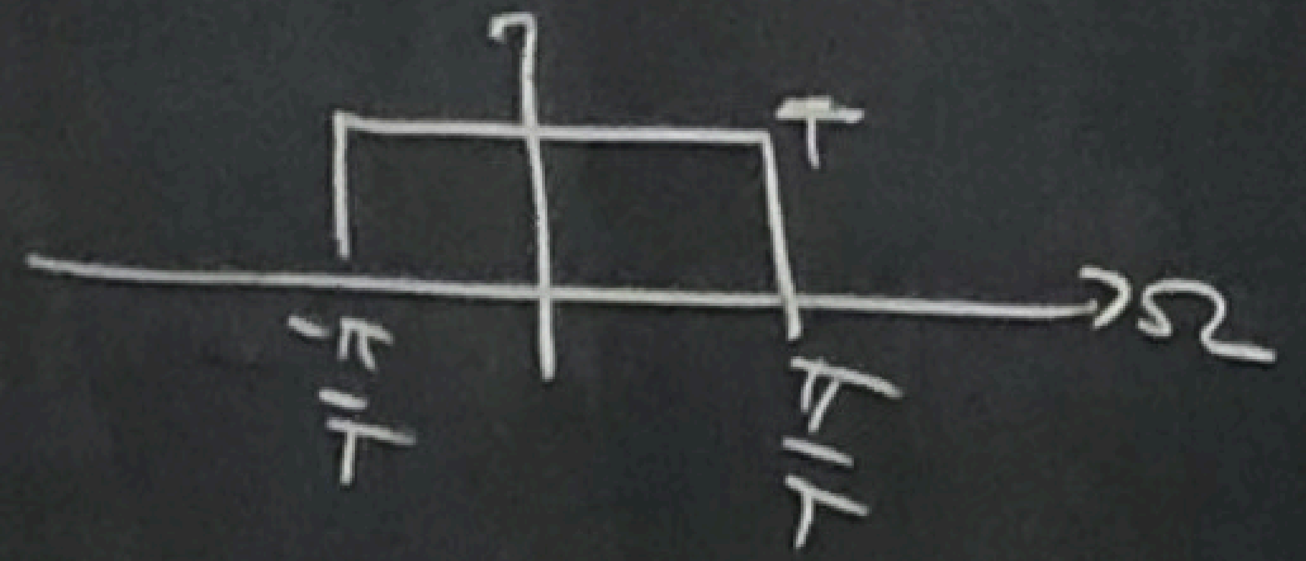
Discrete-Time Processing of Continuous-Time Signals





$H(e^{j\omega})$

$\omega = \Omega T$

$\Omega = \frac{\omega}{T}$

$$H_{eff}(j\Omega) = \frac{Y_r(j\Omega)}{X_c(j\Omega)} = H_{aa}(j\Omega)\, \underbrace{H(e^{j\Omega})}_{\Omega = \frac{\omega}{T}}$$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

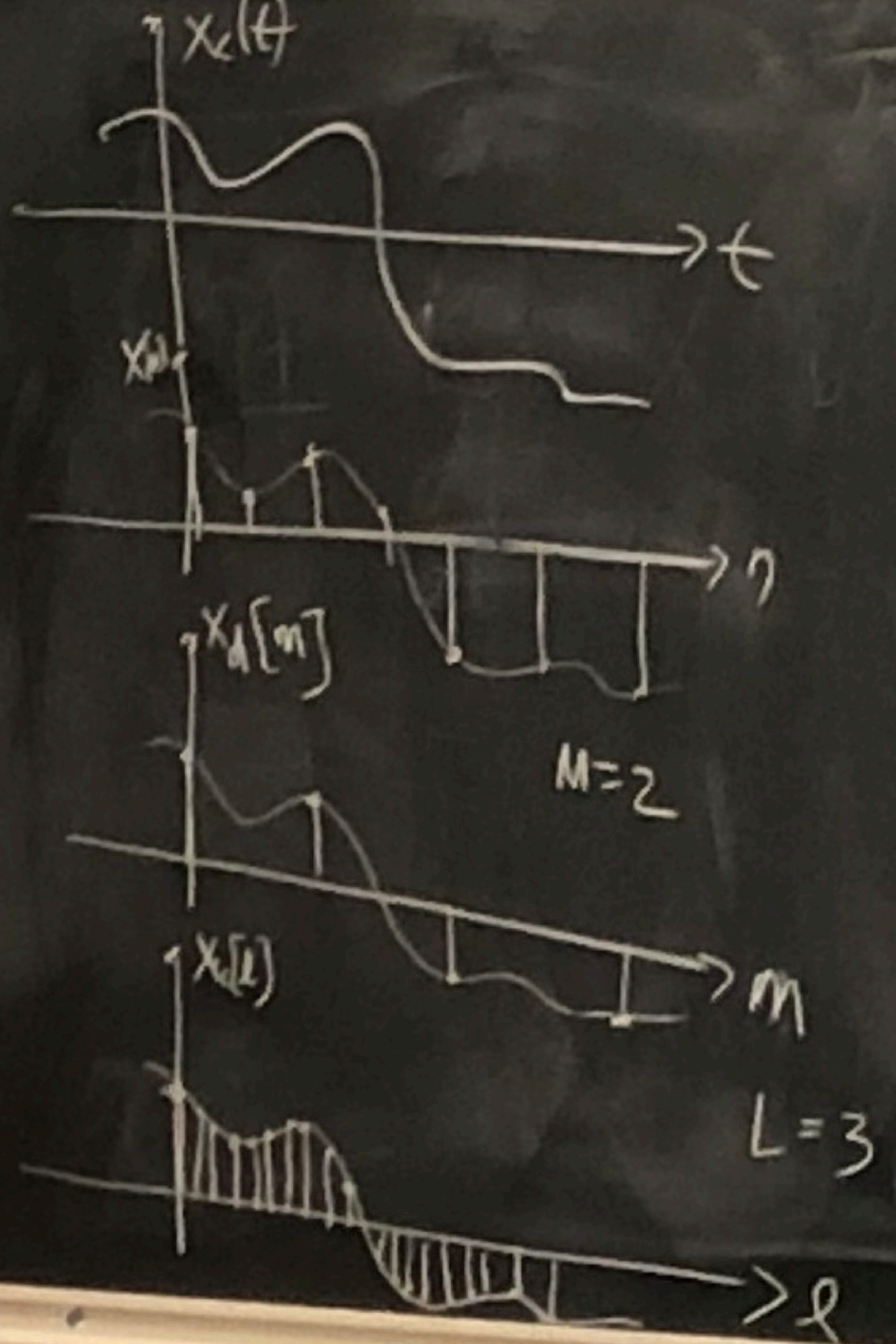
CHANGE OF DT SAMPLING RATE:

DECIMATION + INTERPOLATION

(DOWNSAMPLING + UPSAMPLING)

OS&P 4.6-4.7

EXAMPLES OF RESAMPLED SIGNALS....



DECIMATION

$$x[n] \longrightarrow \boxed{\downarrow M} \longrightarrow x_d[m] = x[mM]$$

INTERPOLATION

$$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow x_i[\ell] = x\left[\frac{\ell}{L}\right], \quad \frac{\ell}{L} \text{ INTEGER}$$

Downsampling in the frequency domain

wrong!

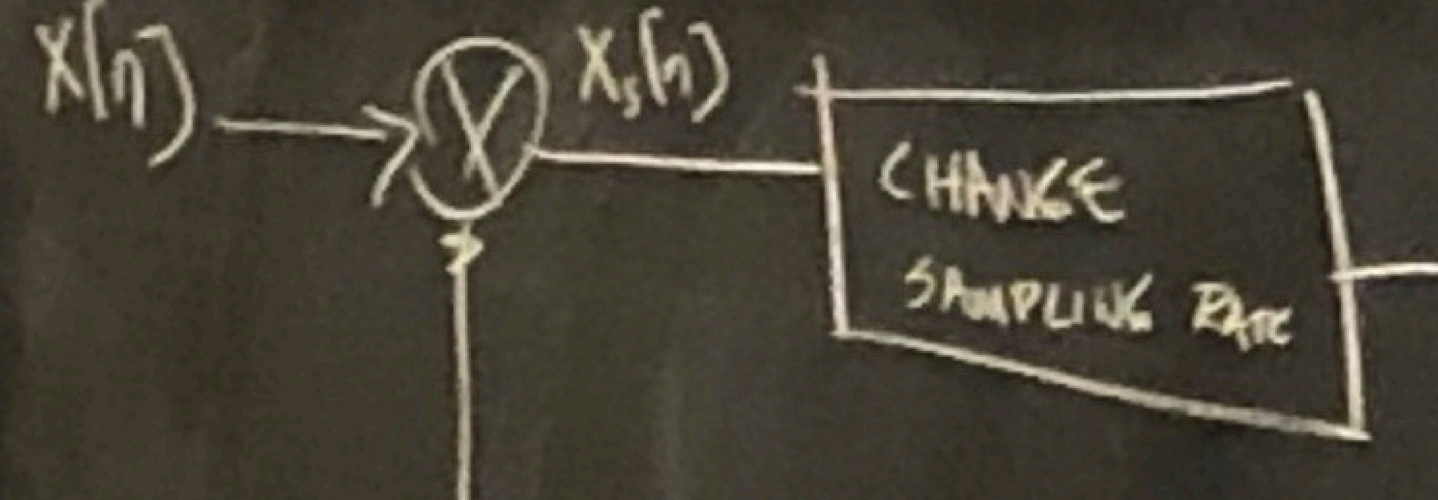
DTFT $$X_d(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_d[n]\, e^{-j\omega n} = \sum_{m=-\infty}^{\infty} x[mM]\, e^{-j\omega m}$$

let $l = mM$

$m = \frac{l}{M}$

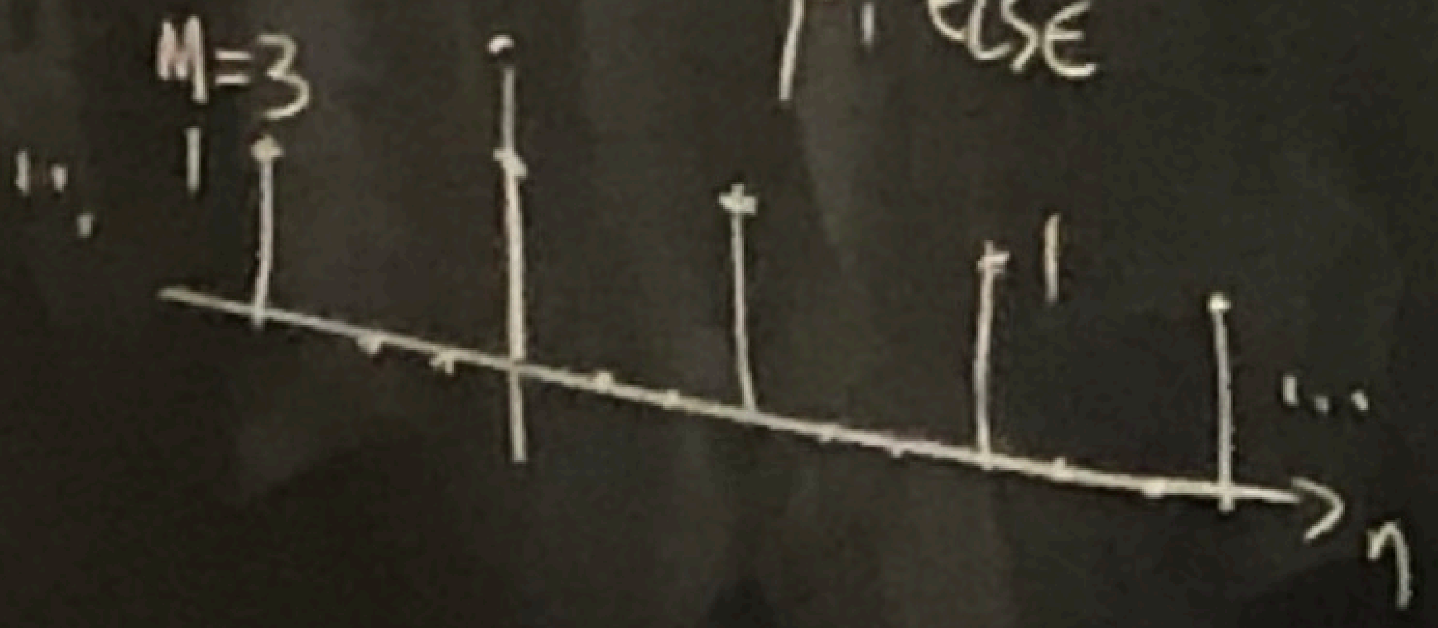
$$= \sum_{l=-\infty}^{\infty} x[l]\, e^{-j\omega \frac{l}{M}} = X\left(e^{j\frac{\omega}{M}}\right)$$

Correct Characterization of Decimation

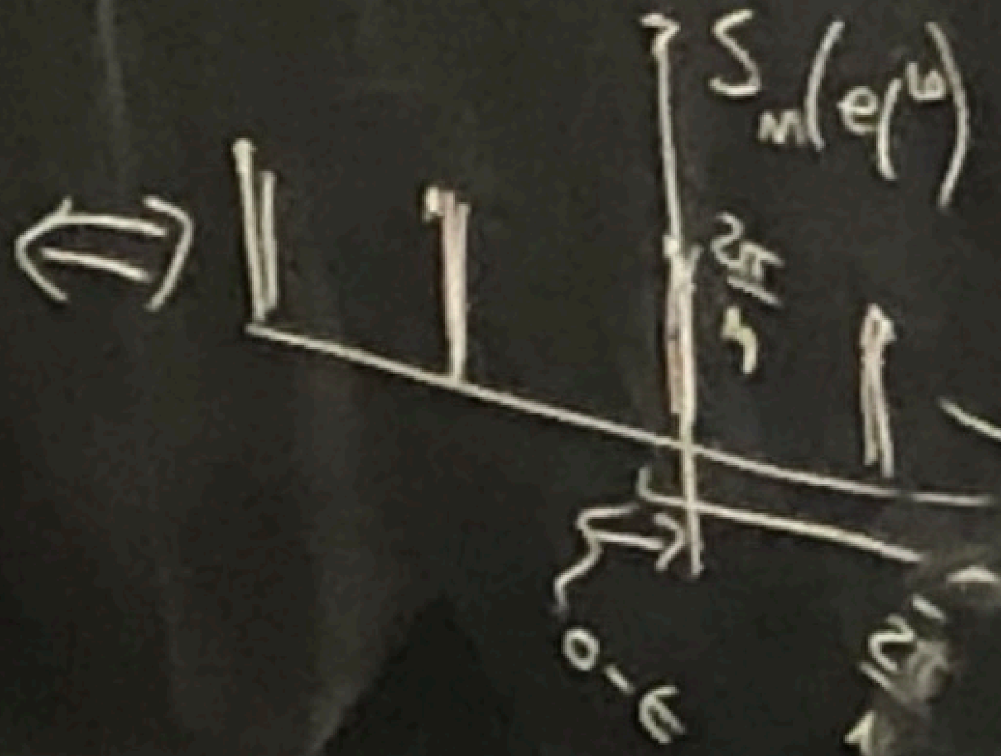


$x_d[m] = x_s[mM]$

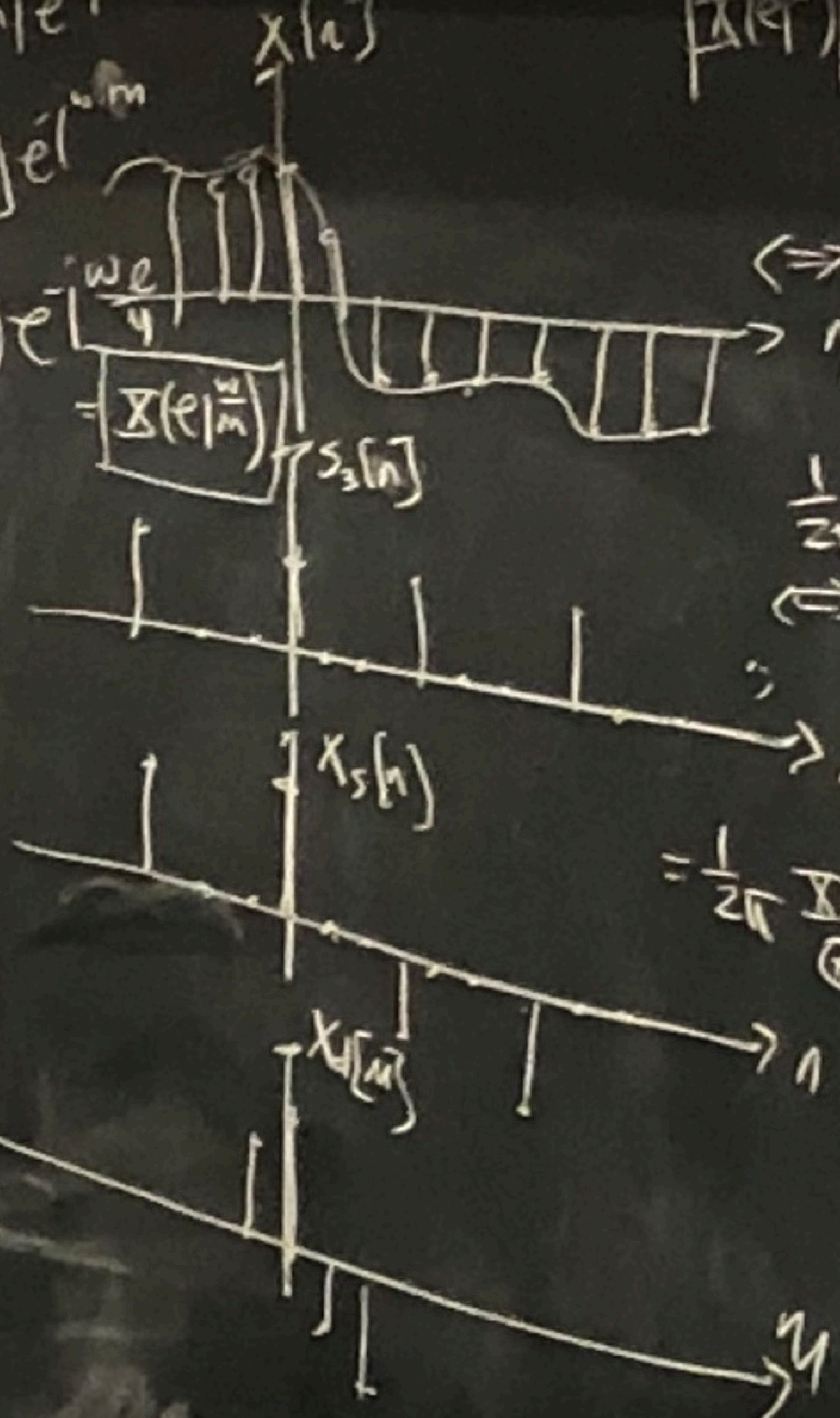
$s_M[n] = \begin{cases} 1, & n = rM \\ 0, & \text{else} \end{cases}$



$\Longleftrightarrow$



$$X_d(e^{j\omega}) = \sum_{m=-\infty}^{\infty} x_d[m] e^{-j\omega m}$$
$$= \sum_{m=-\infty}^{\infty} x[mM] e^{-j\omega m}$$
$l = mM$, $m = \frac{l}{M}$
$$= \sum_{l=-\infty}^{\infty} x[l] e^{-j\frac{\omega l}{M}}$$
$$= \boxed{X(e^{j\frac{\omega}{M}})}$$



$|X(e^{j\omega})| = 0,\ W < |\omega| \le \pi$

To avoid aliasing

$W < \frac{2\pi}{M} - W$

$\boxed{W \le \frac{\pi}{M}}$

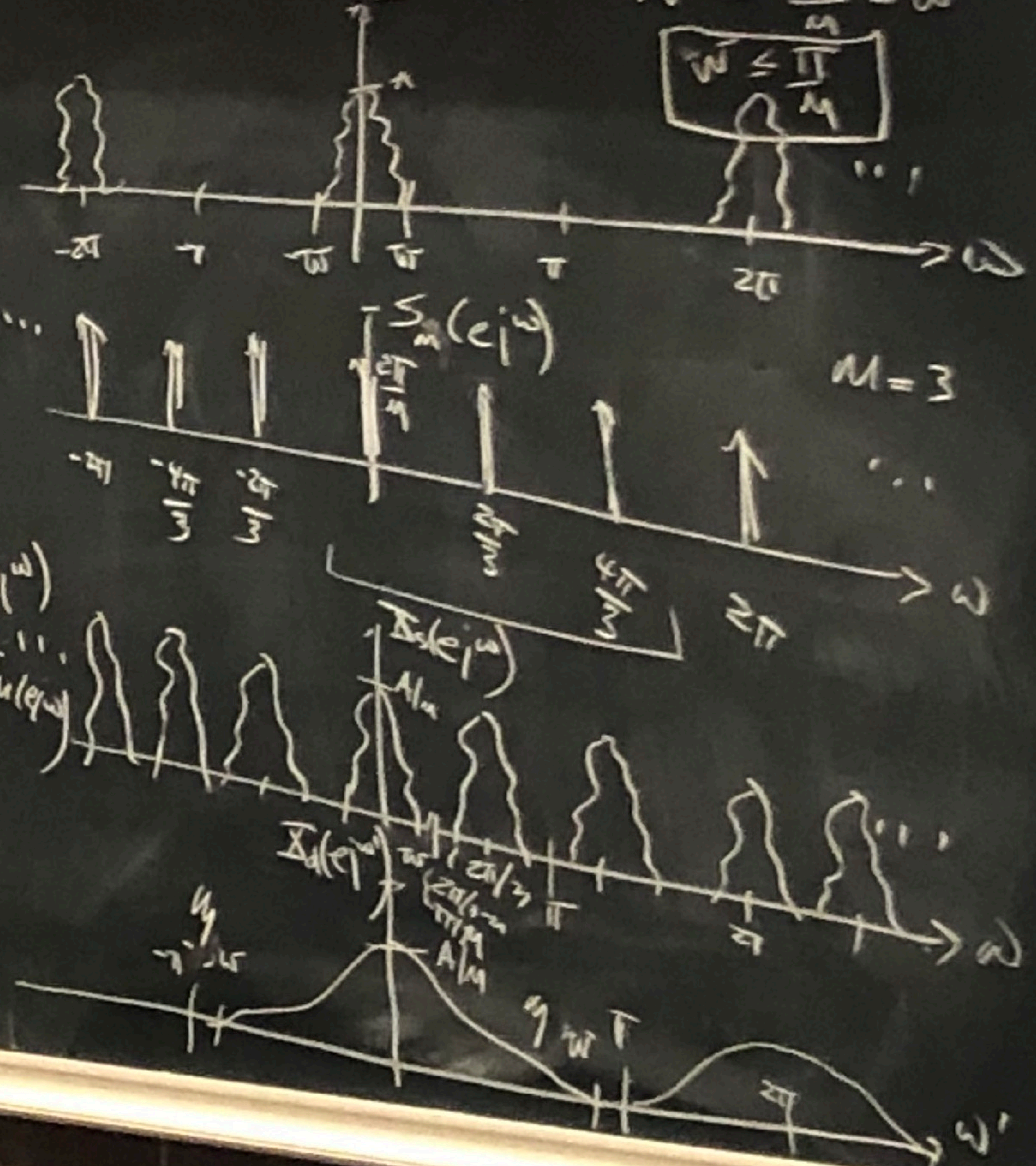
$\Longleftrightarrow$

$\frac{1}{2\pi}$

$\Longleftrightarrow$

$= \frac{1}{2\pi} X(e^{j\omega}) \circledast S_M(e^{j\omega})$

M=3



Proof of IDTFT for infinite pulse train

$$x[n] = \frac{1}{2\pi}\int_{0-\epsilon}^{2\pi-\epsilon} X(e^{j\omega})\, e^{j\omega n}\, d\omega = \frac{1}{2\pi}\int_{0-\epsilon}^{2\pi-\epsilon} \underbrace{\frac{2\pi}{M}\sum_{l=0}^{M-1} \delta\left(\omega - \frac{2\pi l}{M}\right)}_{S_M[n]} e^{j\omega n}\, d\omega$$

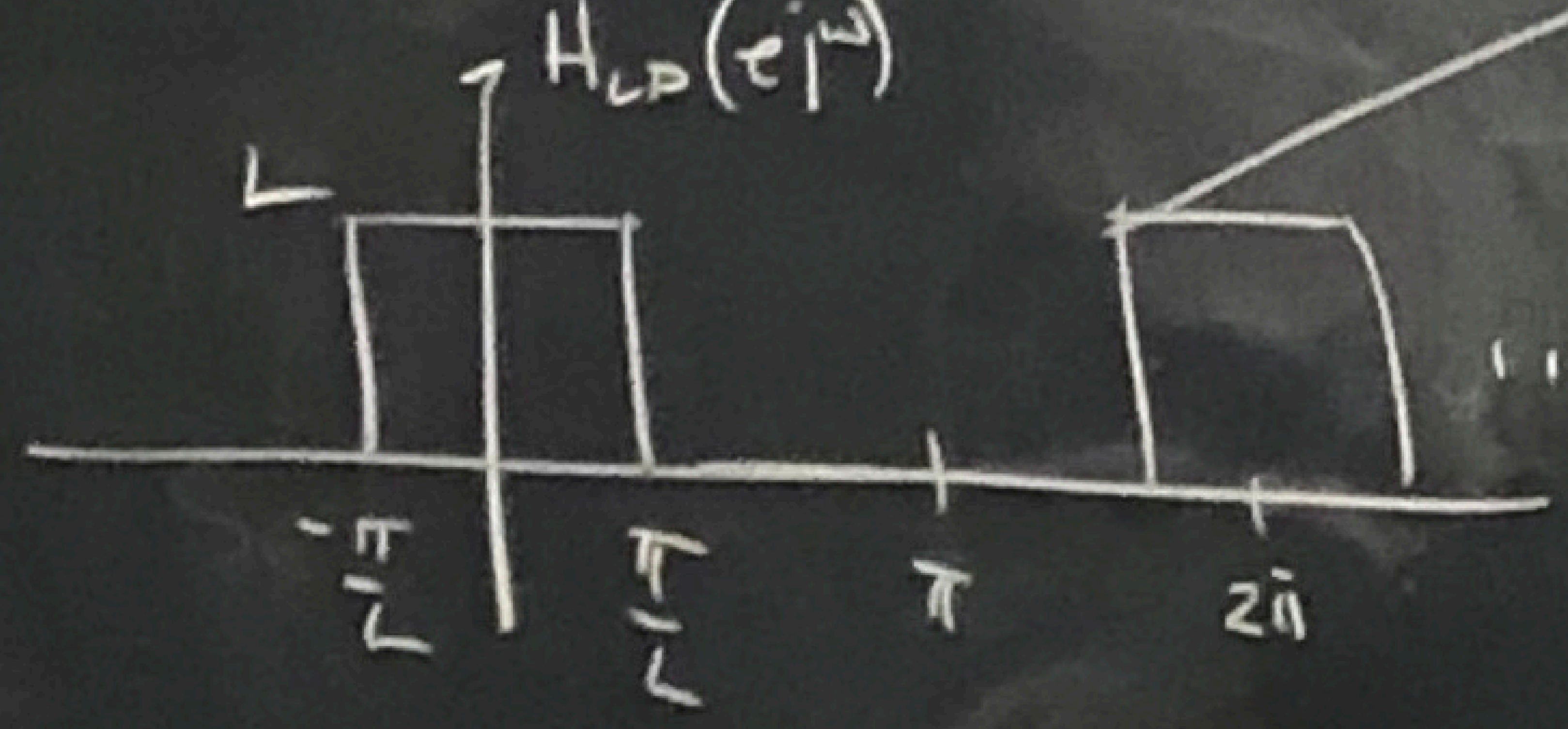
$$= \frac{1}{\cancel{2\pi}}\,\frac{\cancel{2\pi}}{M}\sum_{l=0}^{M-1} \underbrace{\int_{0-\epsilon}^{2\pi-\epsilon} \delta\left(\omega - \frac{2\pi l}{M}\right) e^{j\omega n}\, d\omega}_{e^{j\frac{2\pi l}{M} n}} = \frac{1}{M}\sum_{l=0}^{M-1}\left(e^{j\frac{2\pi n}{M}}\right)^{l}$$

$$= \frac{1}{M}\,\frac{1 - e^{j\frac{2\pi n \cancel{M}}{\cancel{M}}}}{1 - e^{j\frac{2\pi n}{M}}} = \frac{0 \text{ ALWAYS}}{0,\ n = rM} = \begin{cases} 1, & n = rM \\ 0, & \text{ELSE} \end{cases}$$

INTERPOLATION

$x[n] \longrightarrow \boxed{\uparrow L} \longrightarrow x_i[\ell]$

$x[n] \longrightarrow$ INSERT $L-1$ SAMPLES of ZERO BET. INPUT $\xrightarrow{x_e[n]}$ LPF $\longrightarrow x_i[n]$



$x_e[\ell] = x\left[\frac{\ell}{L}\right], \ \frac{\ell}{L}$ INTEGER

$$\sum_{\ell=-\infty}^{\infty} x\left[\frac{\ell}{L}\right] e^{-j\omega\ell}$$

$\frac{\ell}{L}$ INT,

Let $r = \frac{\ell}{L}$

$\ell = rL$

$$= \sum_{r=-\infty}^{\infty} x[r]\, e^{-j\omega r L}$$

$$= X(e^{jL\omega})$$

L=4