IDEAL

C/D CONVERSION

$$x_c(t) \longrightarrow \boxed{C/D} \longrightarrow x[n] = x_c(nT)$$

T

IN MORE DETAIL:

C/D

$$x_c(t) \longrightarrow \otimes \xrightarrow{x_s(t)} \boxed{\text{CONVERT TO DT}} \longrightarrow x[n] = x_c(nT)$$

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$x_s(t) = s(t)\cdot x_c(t)$$

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$$

SAMPLING CT SIGNALS

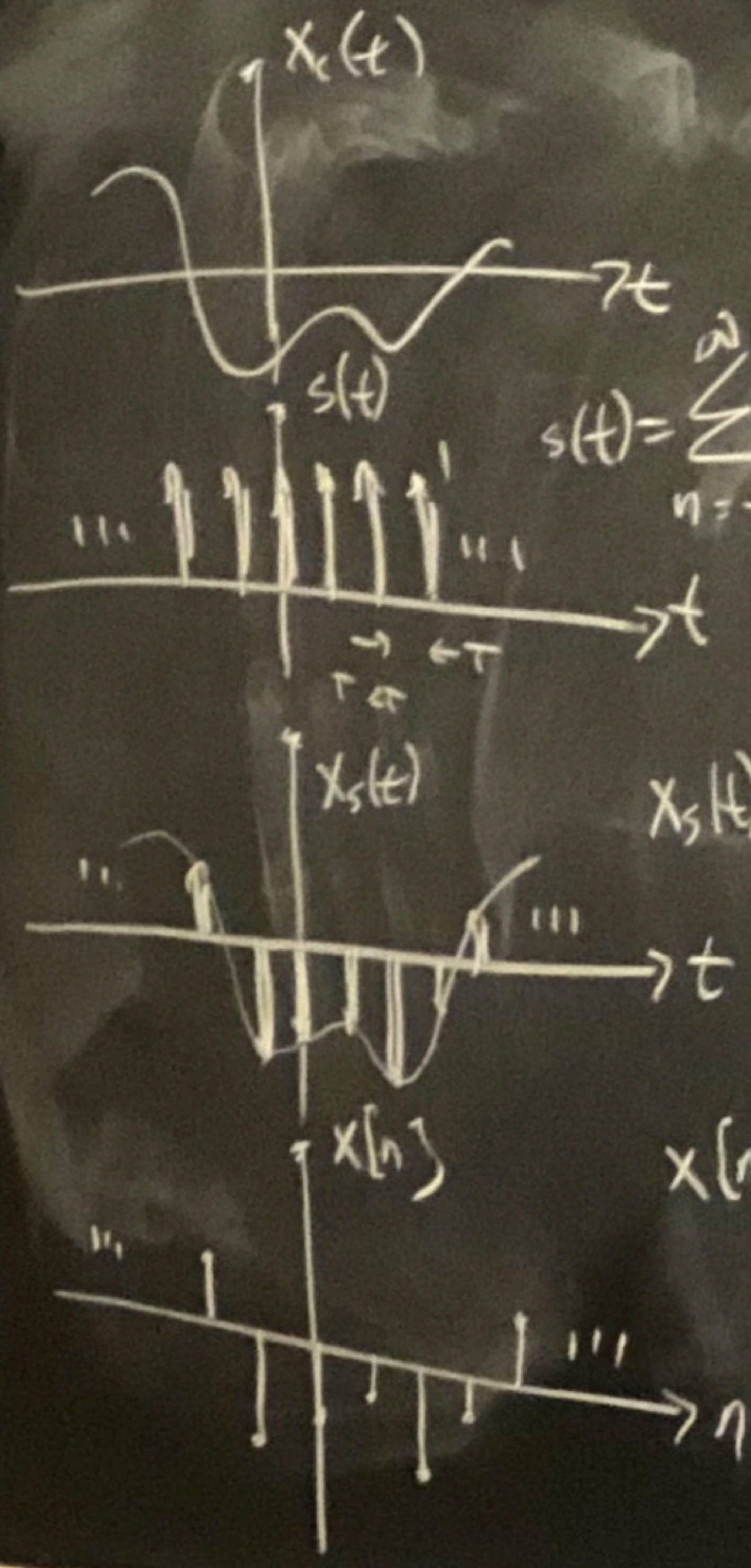
OS4P 4.0-4.5

CTFT $x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega) e^{j\Omega t}\, d\Omega$

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t) e^{-j\Omega t}\, dt$$

DTFT $x[n] = \frac{1}{2\pi}\int_{2\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$



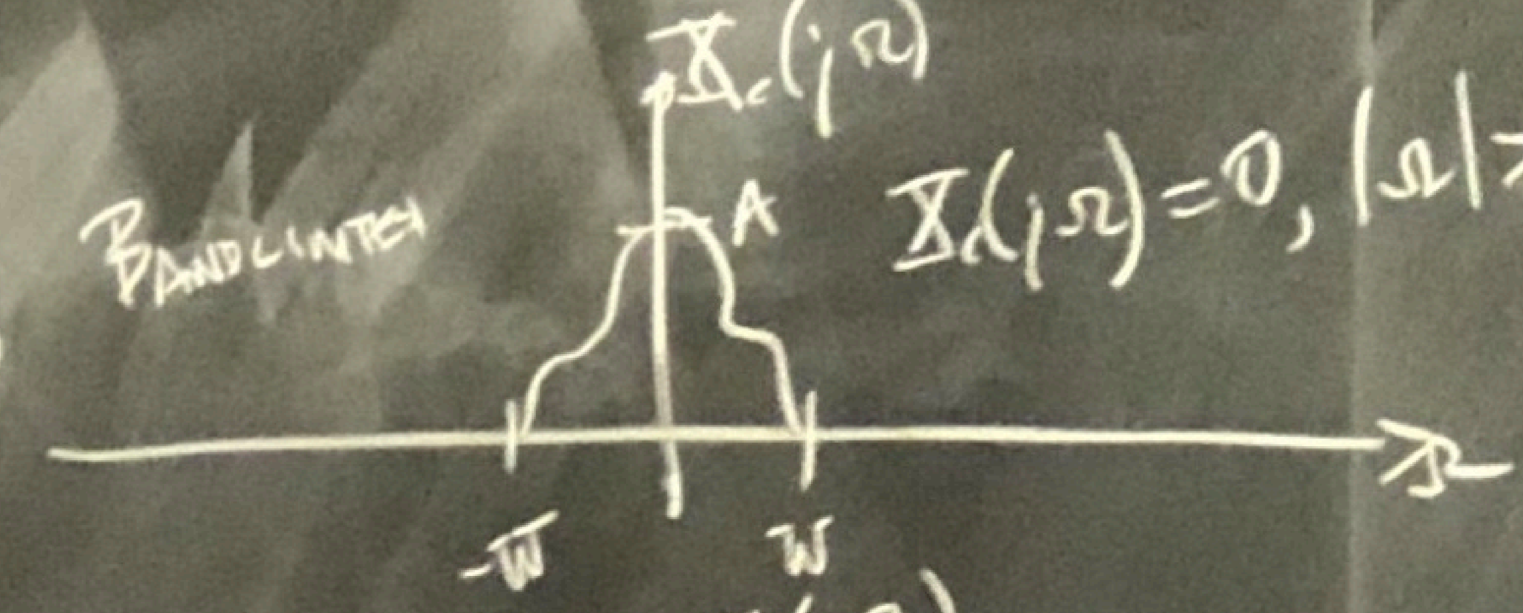
$$s(t)=\sum_{n=-\infty}^{\infty}\delta(t-nT)$$

$$x_s(t)=\sum_{n=-\infty}^{\infty}x_c(nT)\delta(t-nT)$$

$$x[n]=\sum_{\ell=-\infty}^{\infty}x_c(\ell T)\delta[n-\ell]$$

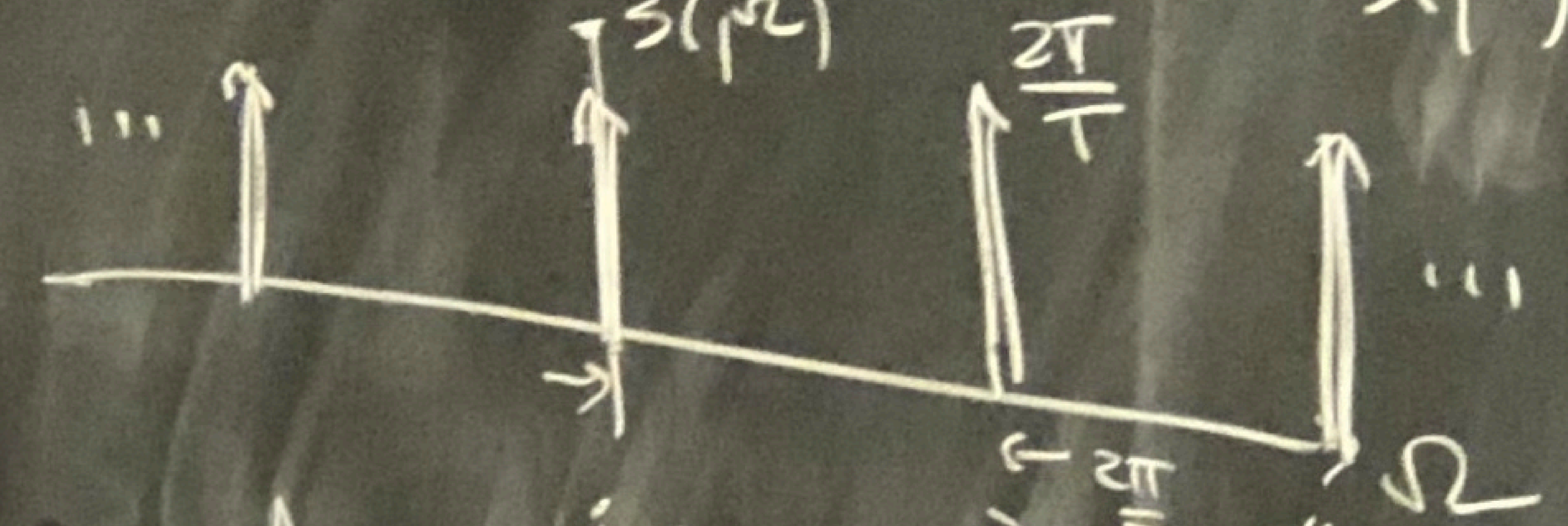
$\Longleftrightarrow$

BANDLIMITED

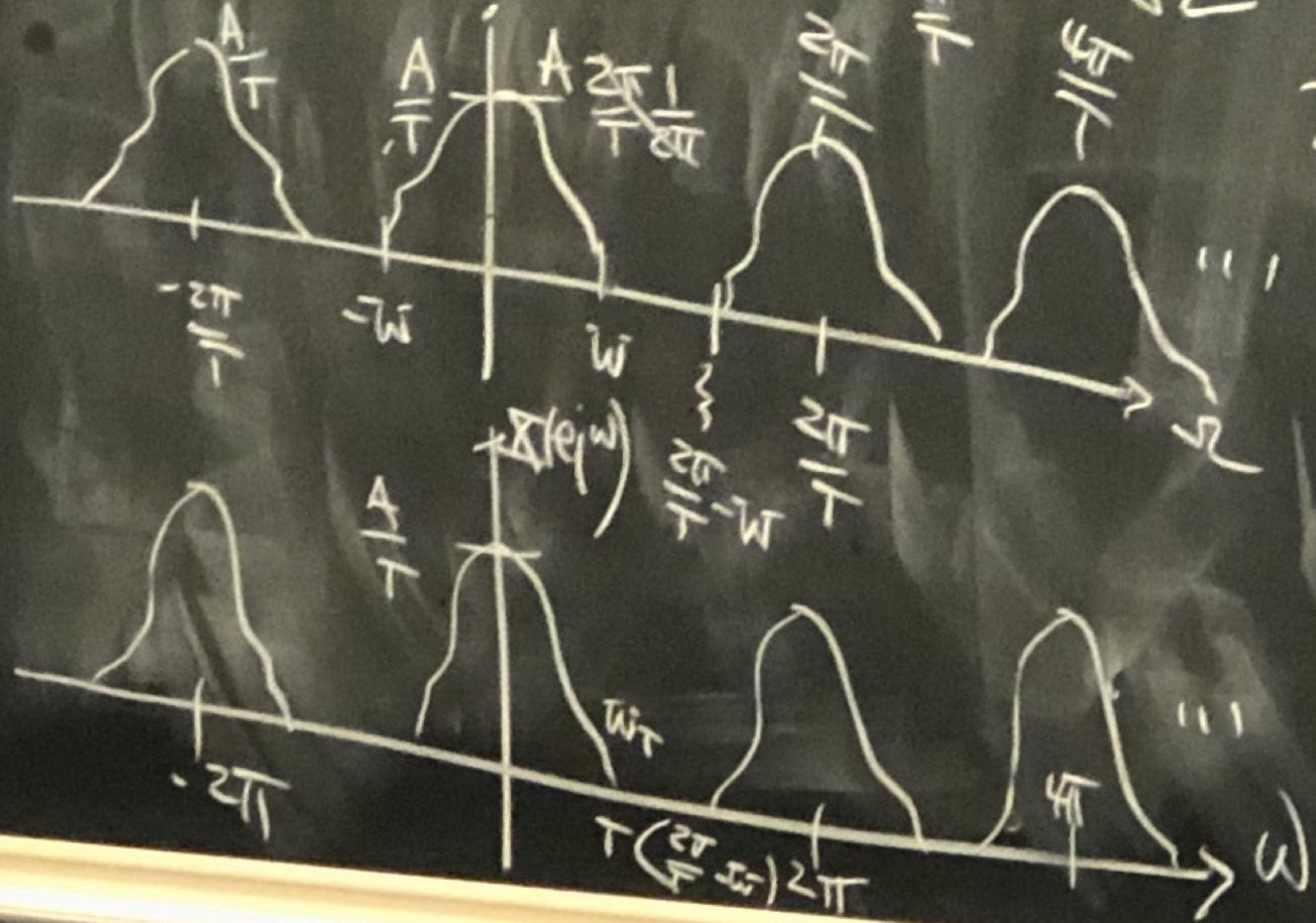


$X_c(j\Omega)=0,\ |\Omega|>W$

$\Longleftrightarrow$



$$S(j\Omega)=\sum_{k=-\infty}^{\infty}\frac{2\pi}{T}\delta\left(\Omega-k\frac{2\pi}{T}\right)$$



$$X_s(j\Omega)=\frac{1}{2\pi}X_c(j\Omega)*S(j\Omega)$$
$$=\sum_{k=-\infty}^{\infty}\frac{1}{T}X_c\left(j\left(\Omega-k\frac{2\pi}{T}\right)\right)$$

$\omega=\Omega T$

$$X(e^{j\omega})=\sum_{k=-\infty}^{\infty}\frac{1}{T}X_c\left(j\frac{\omega}{T}-k2\pi\right)$$

$\frac{2\pi}{T}=\Omega_0$

TO AVOID OVERLAP NEED

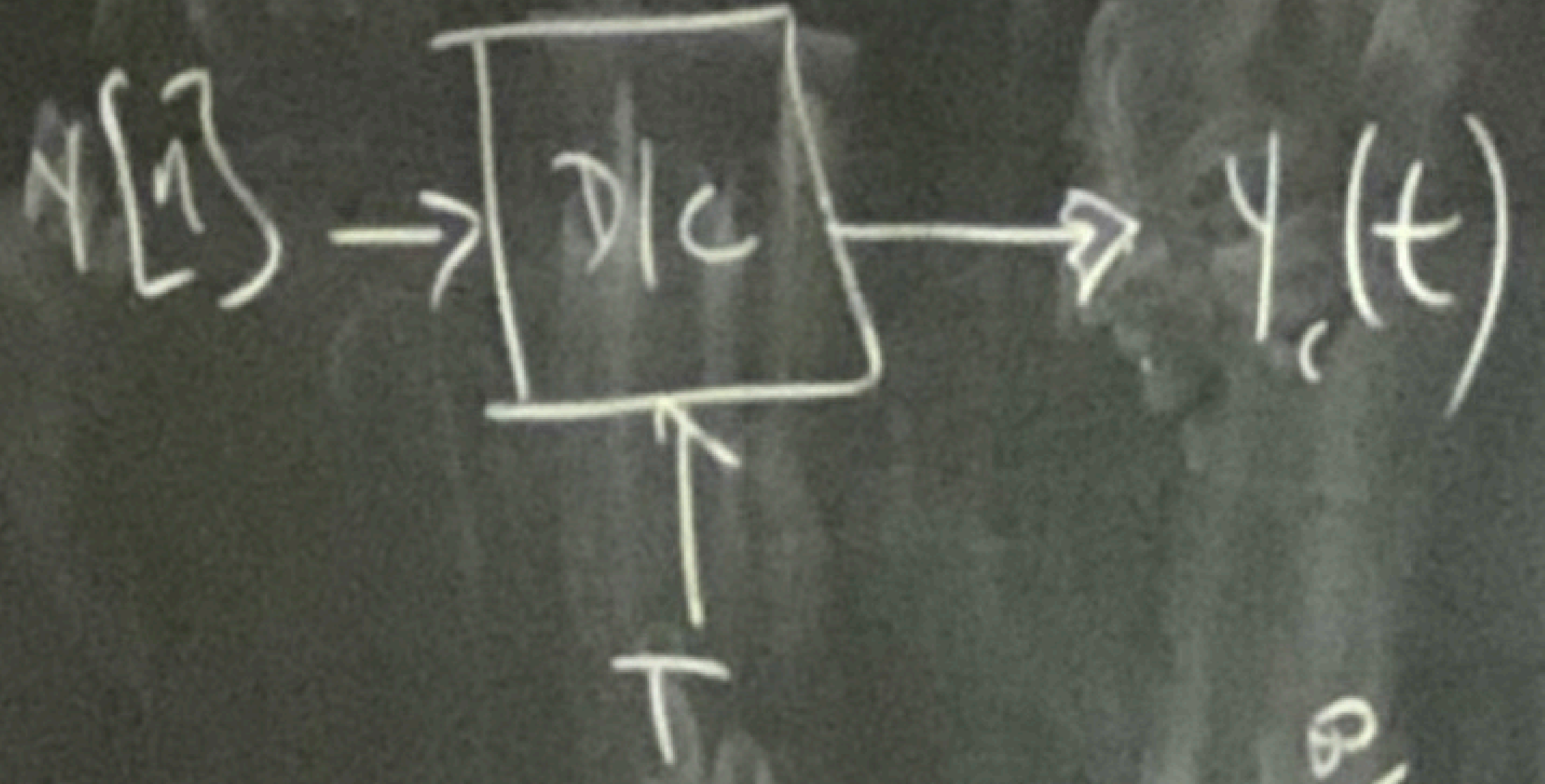
$W<\frac{2\pi}{T}-W$

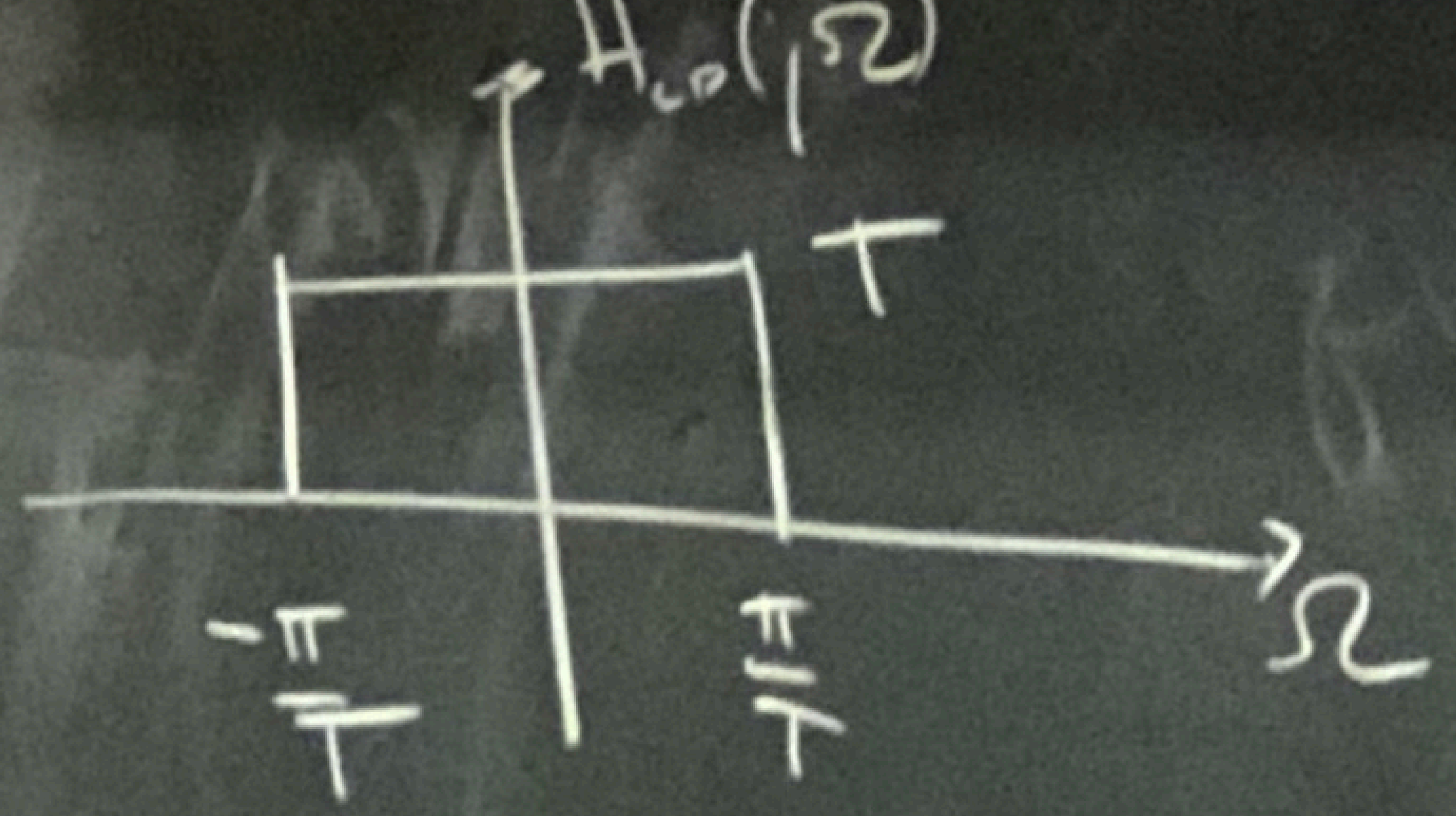
$2W<\frac{2\pi}{T}$

$$\boxed{W<\frac{\pi}{T}=\frac{1}{2}\frac{2\pi}{T}}$$

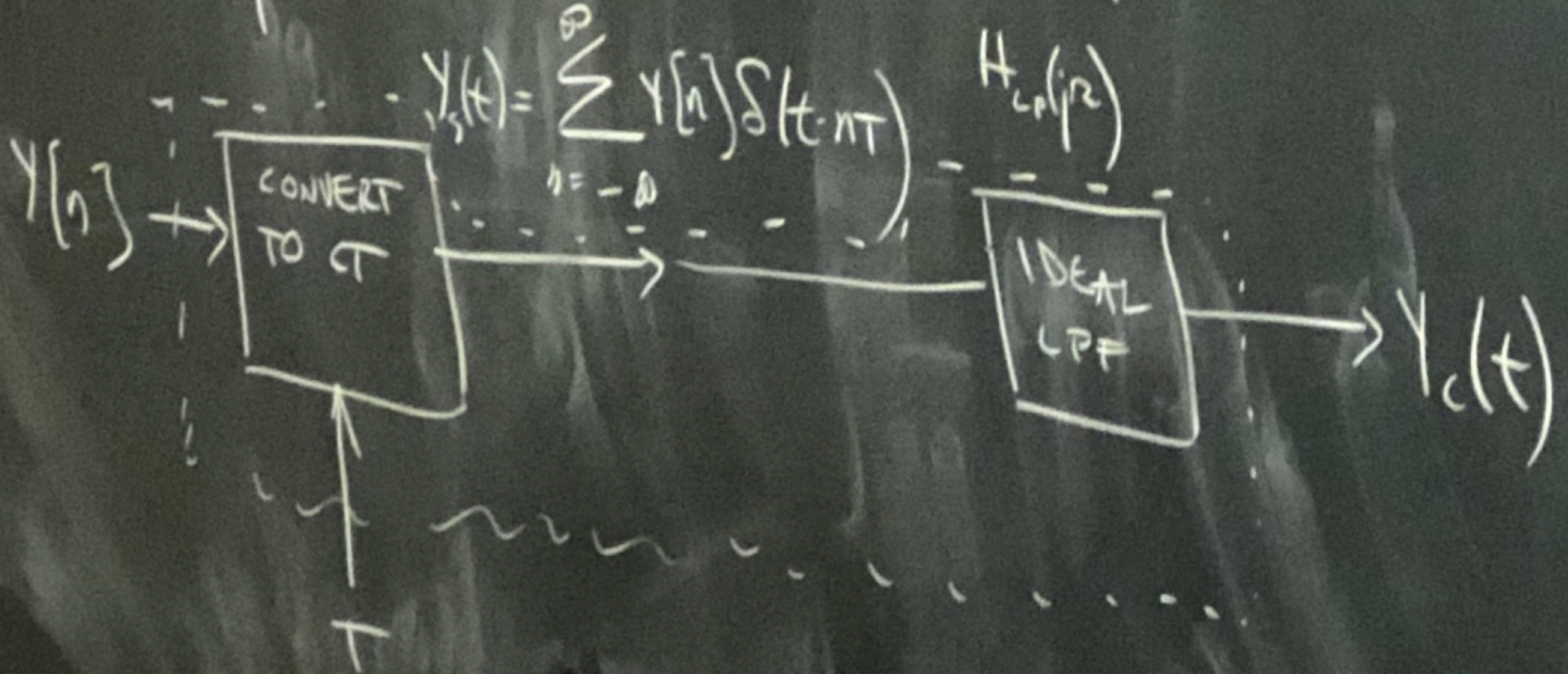
NYQUIST CONSTRAINT

IDEAL D|C CONVERSION





$$H_{LP}(j\Omega) = \begin{cases} T, & |\Omega| \le \frac{\pi}{T} \\ 0, & \text{ELSE} \end{cases}$$



COMPARING $X_s(j\Omega)$ WITH $X(e^{j\omega})$

$$X_s(j\Omega) = \int_{-\infty}^{\infty} x_s(t)\, e^{-j\Omega t}\, dt = \int_{-\infty}^{\infty} \underbrace{\sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)}_{x_s(t)}\, e^{-j\Omega t}\, dt$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT) \underbrace{\int_{-\infty}^{\infty} \delta(t-nT)\, e^{-j\Omega t}\, dt}_{e^{-j\Omega nT}} = \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-jn\Omega T} = X_s(j\Omega)$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

$$= \sum_{n=-\infty}^{\infty} x_c(nT)\, e^{-j\omega n} = X(e^{j\omega}) = X_s(j\Omega)\Big|_{\Omega = \frac{\omega}{T}}$$

$$\boxed{\omega = \Omega T}$$

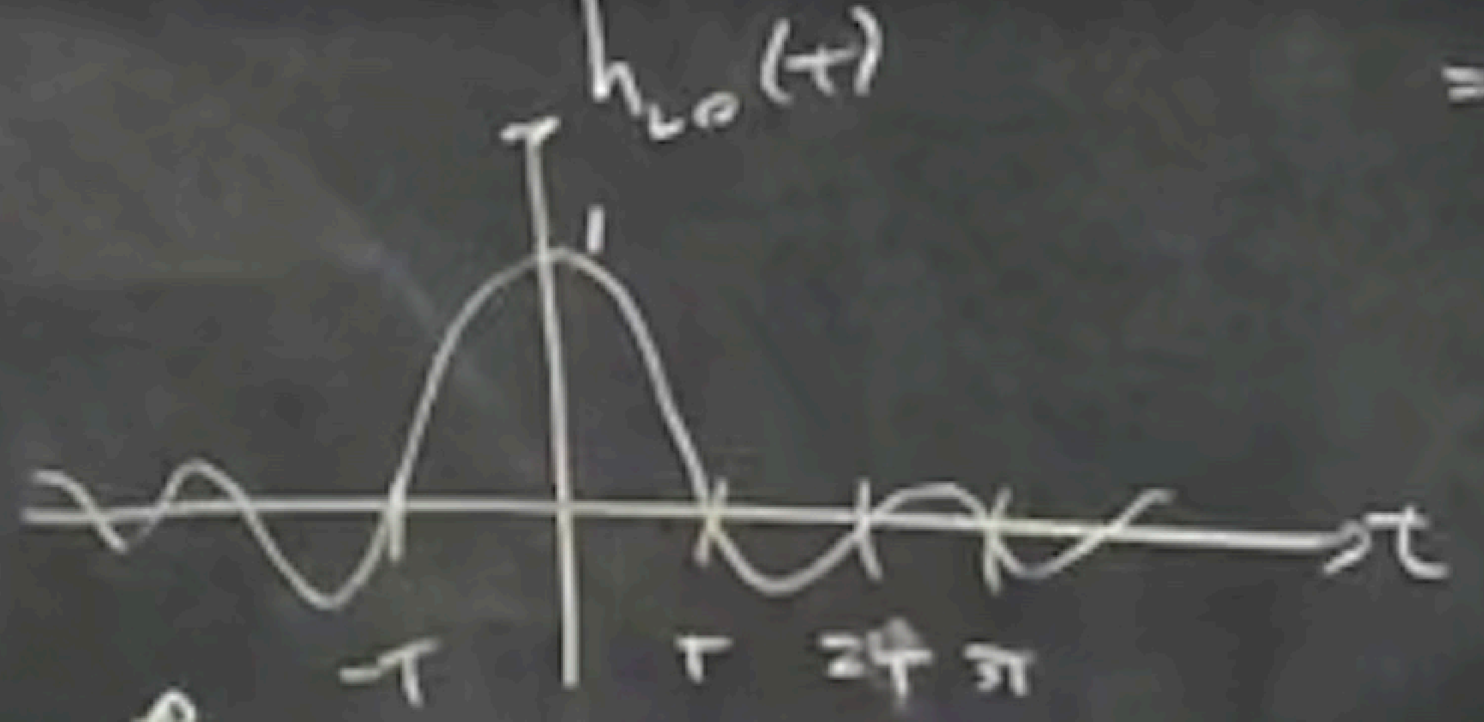
$$\Omega = \frac{\omega}{T}$$

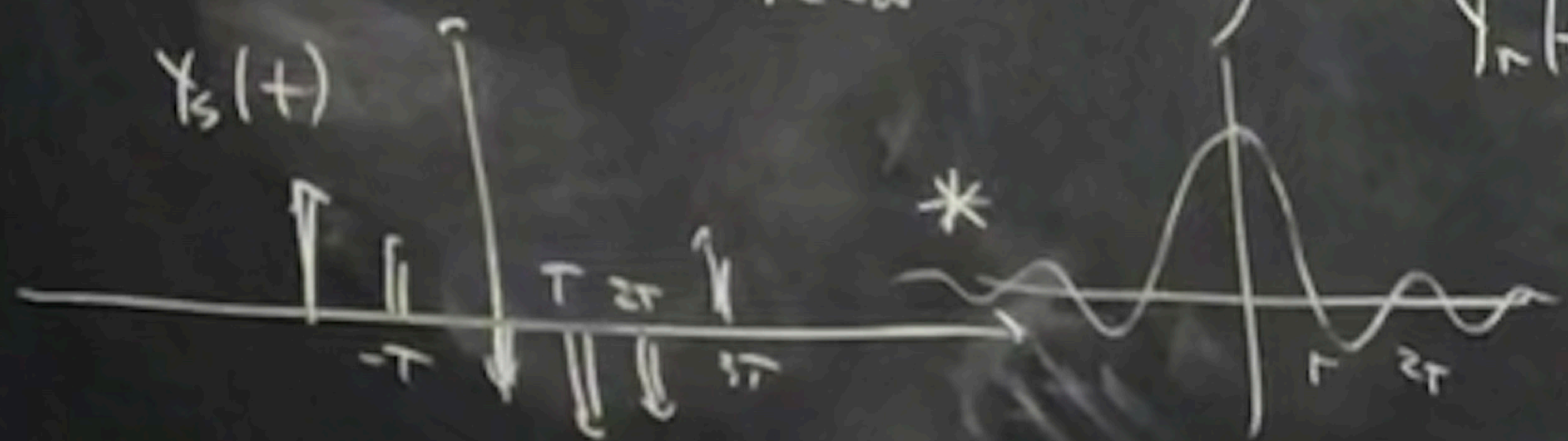
$$\text{RAD} = \frac{\text{RAD}}{\cancel{\text{SEC}}}\,\cancel{\text{SEC}}$$

RECONSTRUCTION IN THE TIME DOMAIN

$$y[n] \longrightarrow \boxed{\text{CONVERT TO CT}} \xrightarrow{\;y_s(t)\;} \underset{H_{LP}(j\Omega)}{\boxed{\text{LPF}}} \longrightarrow y_r(t)$$

$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t-nT)$$





$$y_r(t) = \sum_{n=-\infty}^{\infty} y[n] \cdot h_{LP}(t-nT)$$

$$= y_r(t) = \sum_{n=-\infty}^{\infty} y[n] \frac{\sin\left(\frac{\pi(t-nT)}{T}\right)}{\frac{\pi(t-nT)}{T}}$$

zeros @ $\frac{\pi t}{T} = k\pi$; $t = kT$

$$h_{LP}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_{LP}(j\Omega)\, e^{j\Omega t}\, d\Omega$$

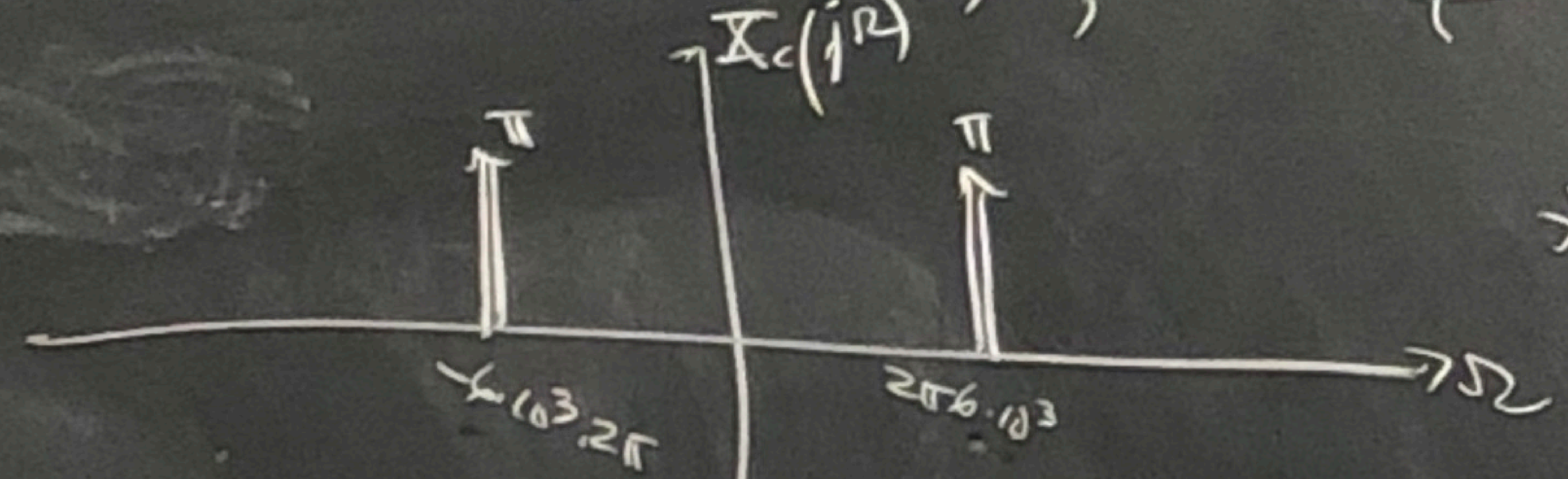
$$= \frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} T e^{j\Omega t}\, d\Omega = \frac{T}{2\pi}\, \frac{1}{jt}\, e^{j\Omega t}\Big|_{\Omega=-\frac{\pi}{T}}^{\frac{\pi}{T}}$$

$$= \frac{T}{\cancel{2}\pi \cancel{j} t} \cdot \cancel{\left(e^{j\frac{\pi}{T}t} - e^{-j\frac{\pi t}{T}}\right)} \quad \cancel{2}\cancel{j}\left(\sin\frac{\pi t}{T}\right) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}} = \frac{\lim_{t\to 0} \frac{\pi}{T}\cos\left(\frac{\pi t}{T}\right)}{\lim_{t\to 0} \frac{\pi}{T}} = 1$$

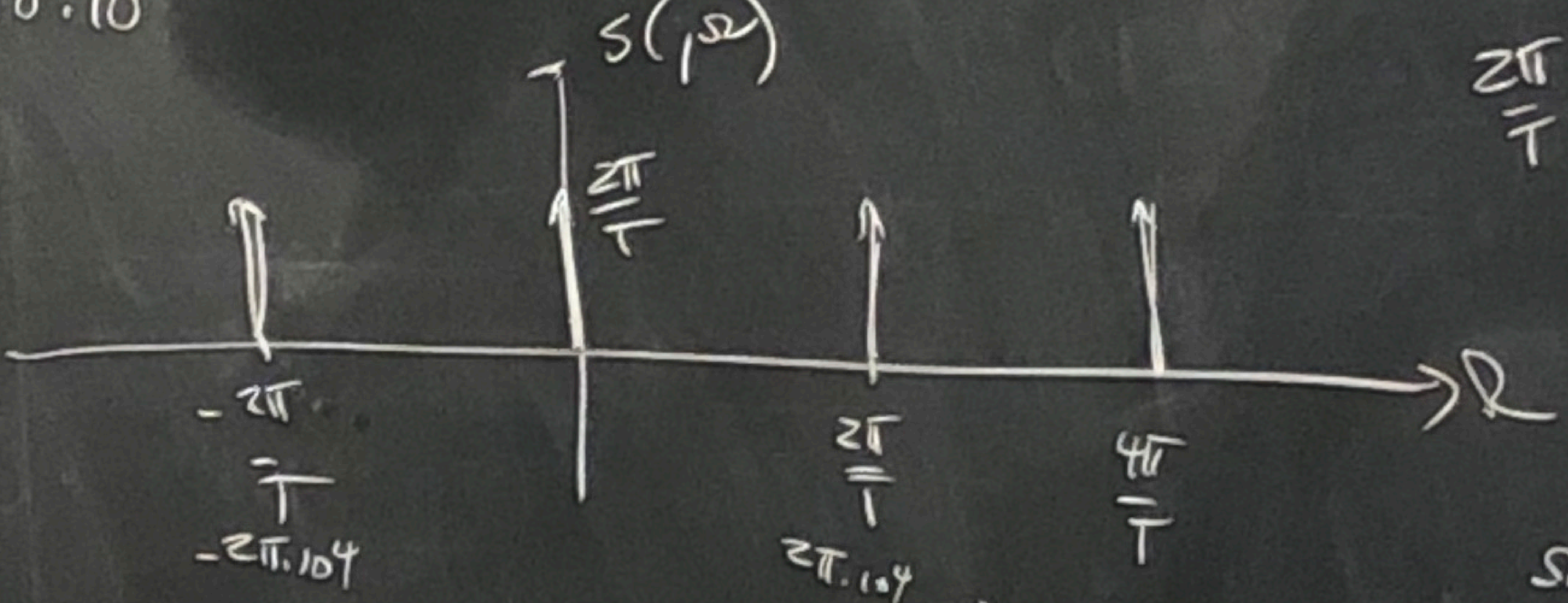
RECONSTRUCTION IN THE TIME DOMAIN

$$\delta(at) = \frac{1}{|a|}\delta(t)$$

$$x_c(t) = \cos(2\pi \cdot 6 \cdot 10^3 t), \quad \Omega_s = \frac{2\pi}{T} = 2\pi \cdot 10 \cdot 10^3$$



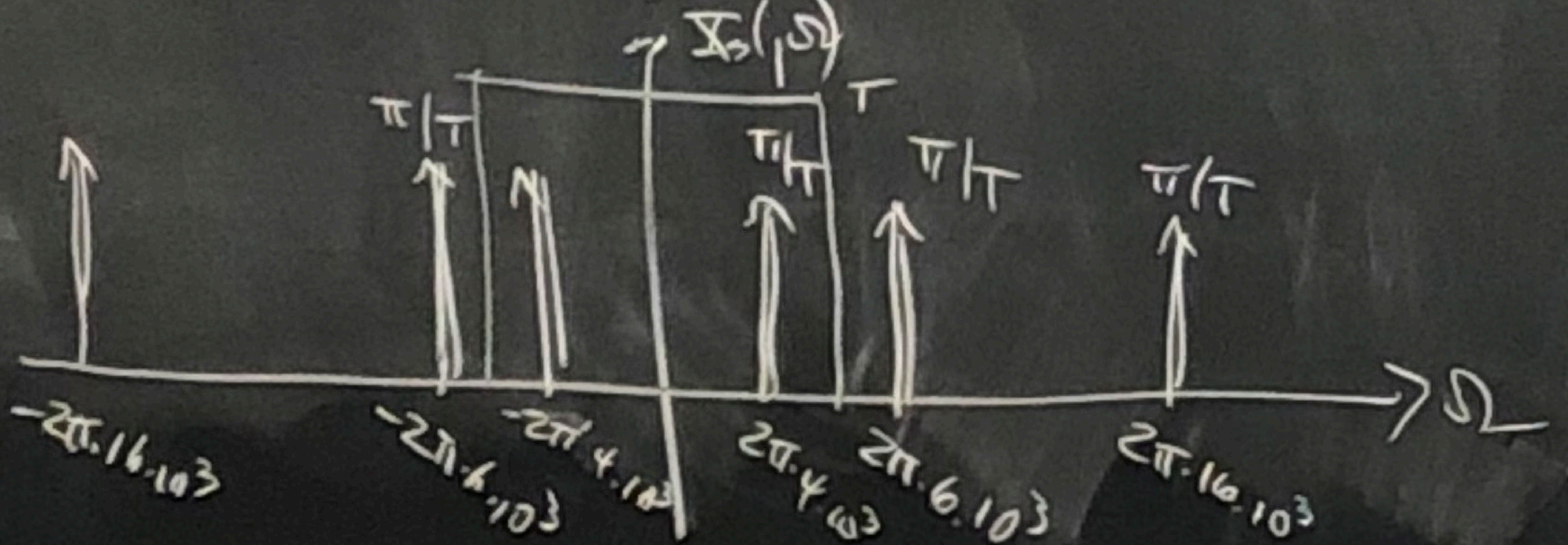
$* \ \frac{1}{2\pi}$

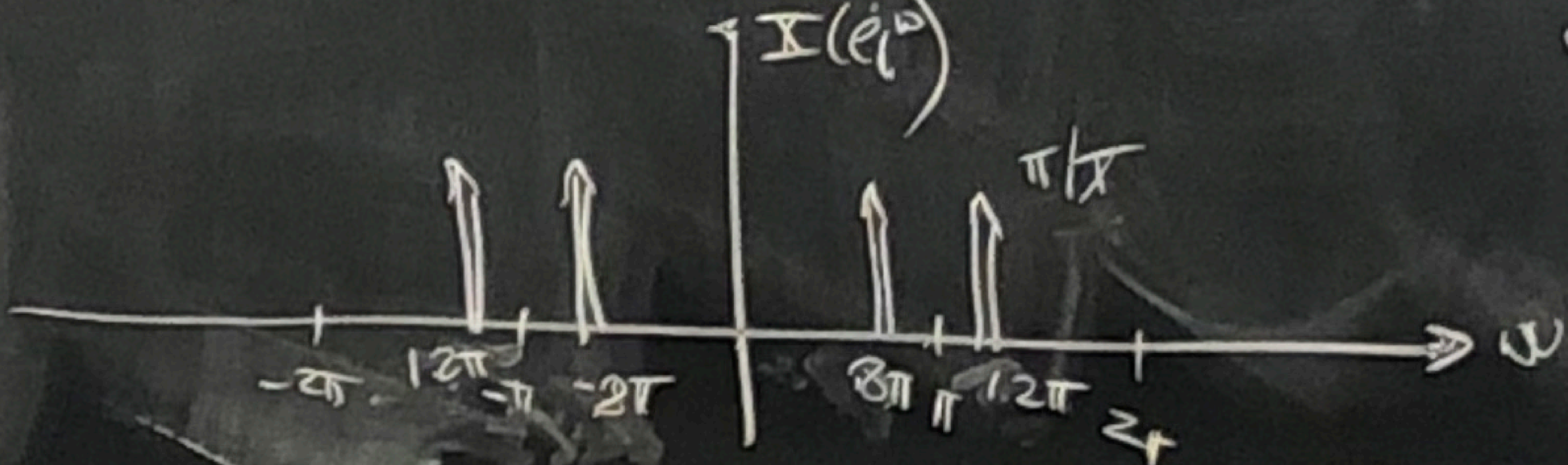


$$\frac{2\pi}{T} = \Omega_s = 2\pi \cdot 10^4$$

$$\delta(\Omega)$$

$$\delta(\omega) = \delta\left(\frac{\Omega}{T}\right)$$





$$\Omega = 2\pi \cdot 4 \cdot 10^3$$

$$\omega = \Omega T = 2\pi \cdot 4 \cdot 10^3 \cdot 10^{-4} = 2\pi(0.4) \text{ RAD}$$