2/12/24

SAMPLING CONTINUOUS-TIME SIGNALS

(OSDP 4.0–4.5, CLASS NOTES FROM ADSP)

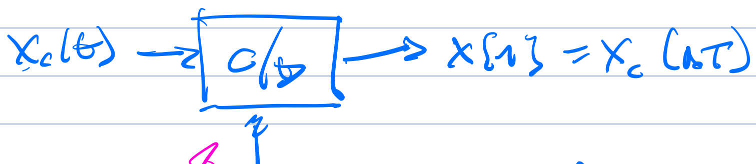
CTFT
$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega$$

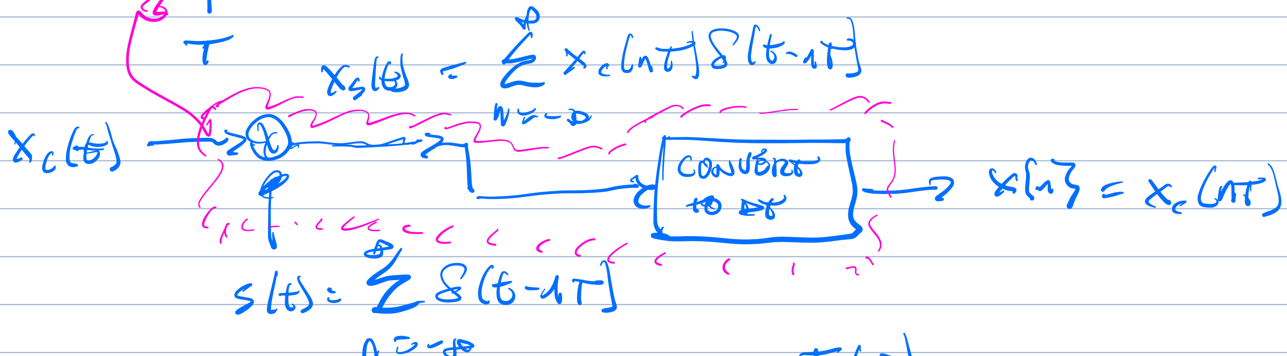
$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$

DTFT
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

IDEAL C/D CONVERSION

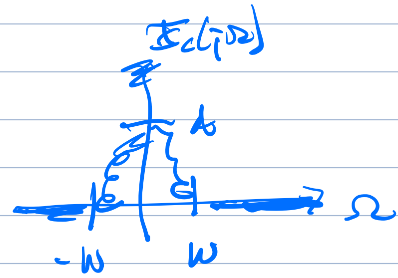


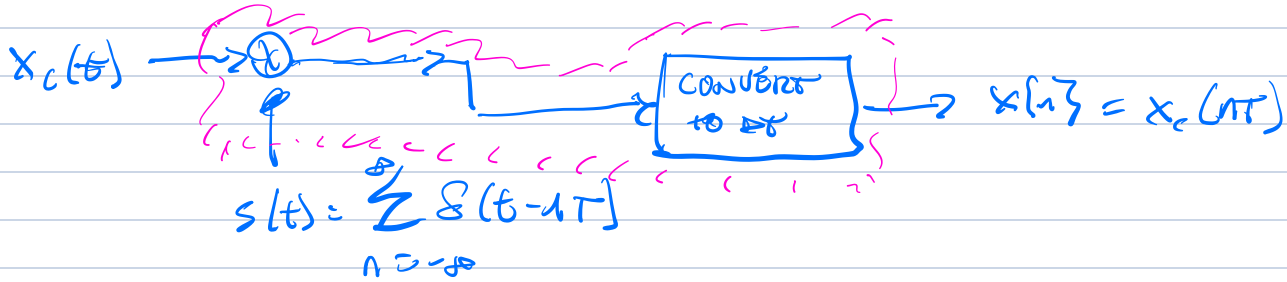


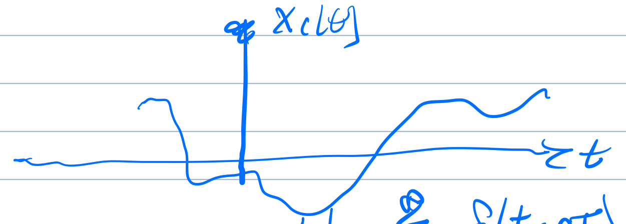
$x_c(t)$ IS BANDLIMITED

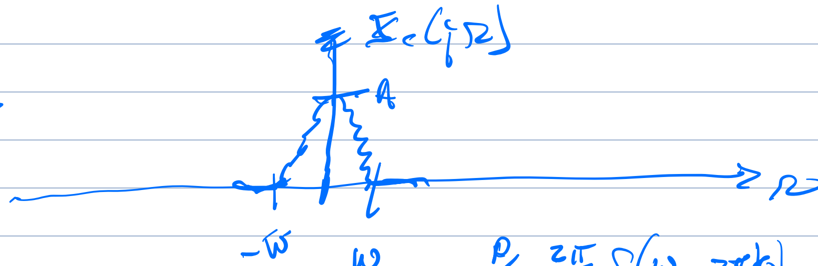
$X_c(j\Omega) = 0,$

$|\Omega| > W$

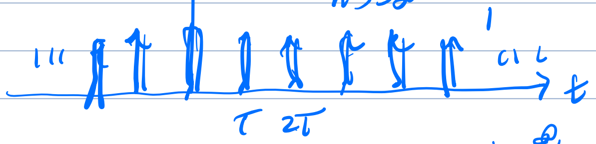




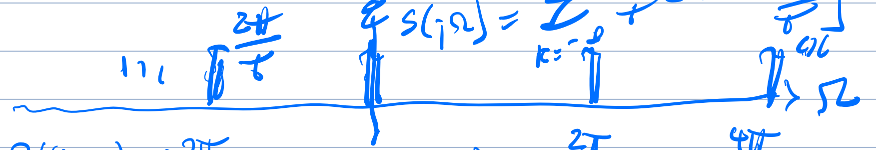




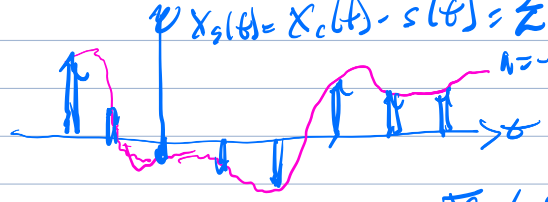
$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

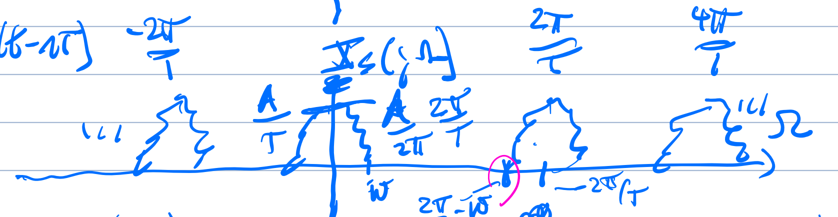


$$S(j\Omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\delta\left(\omega - \frac{2\pi k}{T}\right)$$

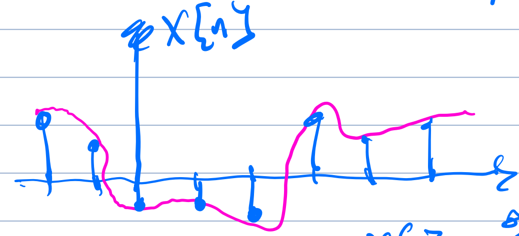


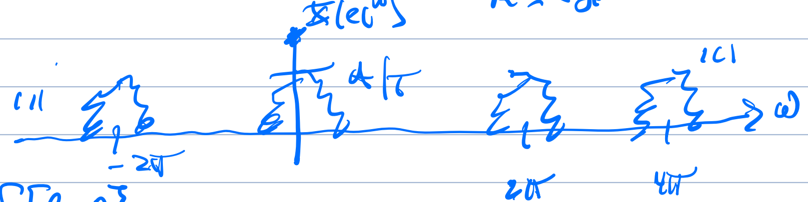
$$x_s(t) = x_c(t)\cdot s(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)$$





$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega) = \frac{1}{T}\sum_{k=-\infty}^{\infty} X_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right)$$





$$x[n] = \sum_{l=-\infty}^{\infty} x_c(lT)\,\delta[l-n]$$

$$X(e^{j\omega}) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X_c\left(j(\Omega T - 2\pi k)\right)$$

DTFT of $x[n]$?

From CTFT of $x_s(t)$

$$X_s(j\Omega) = \int_{-\infty}^{\infty} x_s(t)\,e^{-j\Omega t}\,dt = \int_{-\infty}^{\infty} \underbrace{\sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t-nT)}_{x_s(t)}\,e^{-j\Omega t}\,dt$$

$$X_s(j\Omega) = \sum_{n=-\infty}^{\infty} x_c(nT)\underbrace{\int_{-\infty}^{\infty}\delta(t-nT)\,e^{-j\Omega t}\,dt}_{e^{-j\Omega nT}} = \sum_{n=-\infty}^{\infty} x_c(nT)\,e^{-jn\Omega T}$$

DTFT of $x[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\,e^{-j\omega n} = \sum_{n=-\infty}^{\infty} x_c(nT)\,e^{-j\omega n}$$

$$\boxed{\omega = \Omega T}$$

$$X(e^{j\omega}) = X_s(j\Omega)\Big|_{\Omega = \frac{\omega}{T}}$$

RADIANS = $\frac{\text{RADIANS}}{\cancel{\text{SEC}}} \cdot \cancel{\text{SEC}}$

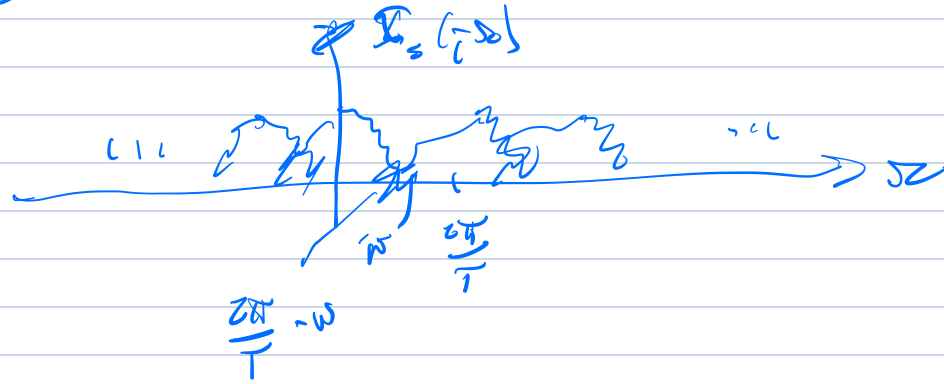
To avoid overlap of $X(j(\Omega - \frac{2\pi k}{T}))$ terms

Require $\frac{2\pi}{T} - W > W$

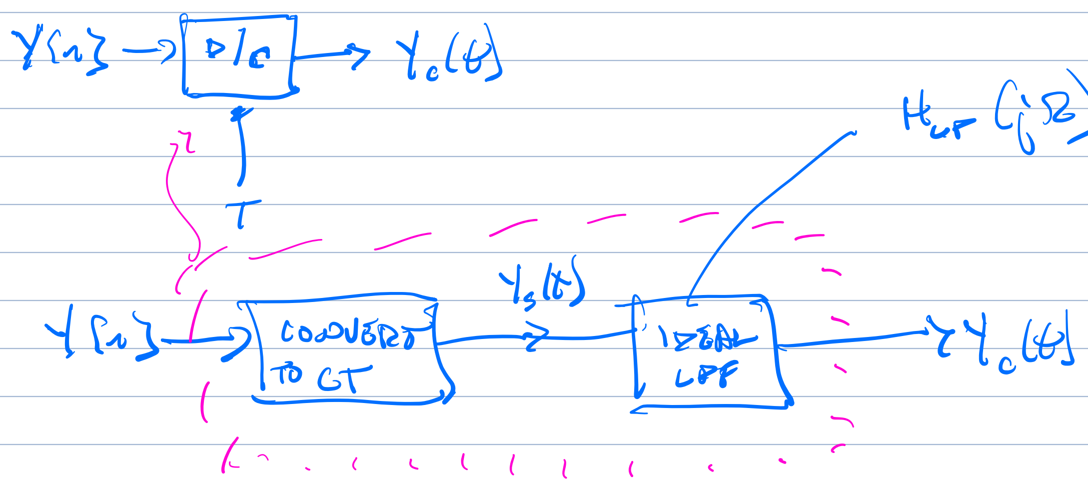
$$2W < \frac{2\pi}{T}$$

$$\boxed{W < \frac{\pi}{T}}$$ NYQUIST CRITERION

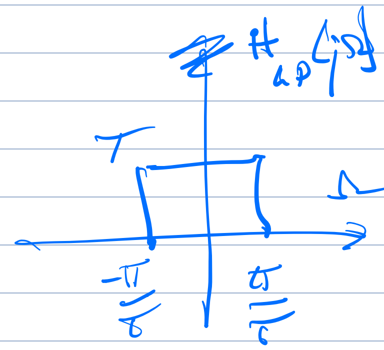
ALIASING IN FREQUENCY



IDEAL D/C CONVERSION



$$H_{LP}(j\Omega) = \begin{cases} T, & |\Omega| \le \frac{\pi}{T} \\ 0, & \text{else} \end{cases}$$

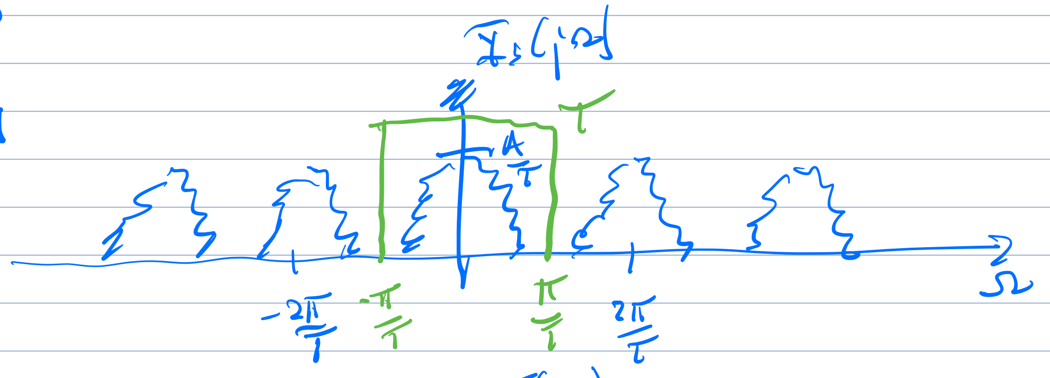


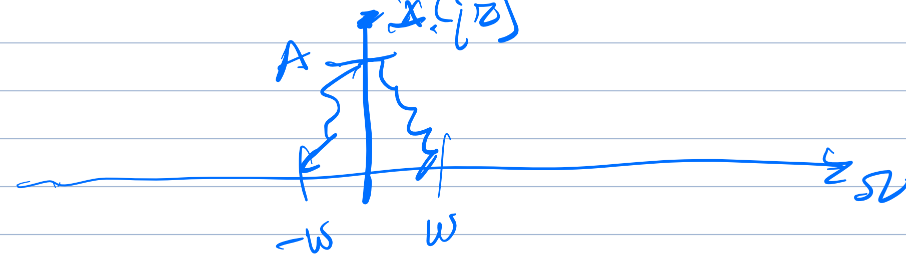
Let $y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\, \delta(t - nT)$

If $y[n] = x[n]$

And T for C/D

is same as T for D/C





LOOKING AT THIS IN THE TIME DOMAIN

CONSIDER $$h_{LP}(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} H_{LP}(j\Omega)\, e^{j\Omega t}\, d\Omega$$

$$= \frac{1}{2\pi}\int_{-\frac{\pi}{T}}^{\frac{\pi}{T}} T\, e^{j\Omega t}\, d\Omega$$

$$= \frac{T}{2\pi}\,\frac{1}{jt}\left[e^{j\Omega t}\right]_{\Omega=-\frac{\pi}{T}}^{\frac{\pi}{T}}$$

$$= \frac{T}{2\pi}\,\frac{1}{jt}\underbrace{\left(e^{j\frac{\pi}{T}t} - e^{-j\frac{\pi}{T}t}\right)}_{2j\sin\left(\frac{\pi t}{T}\right)}$$

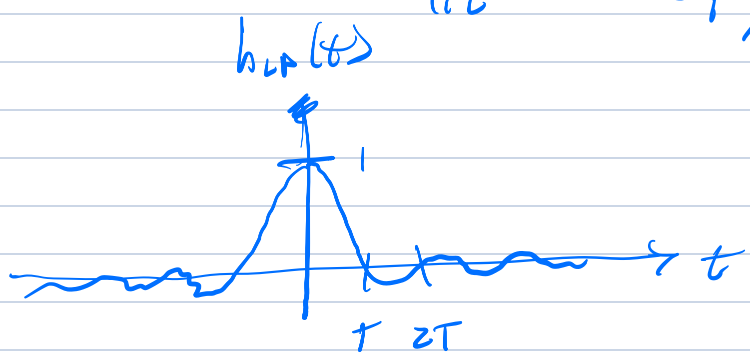
$$h_{LP}(t) = \frac{T}{\cancel{2}\pi \cancel{j} t}\,\cancel{2}\cancel{j}\,\sin\left(\frac{\pi t}{T}\right)$$

$$= \frac{T}{\pi t}\cdot\sin\left(\frac{\pi t}{T}\right) = \frac{\sin\left(\frac{\pi t}{T}\right)}{\frac{\pi t}{T}}$$

ZEROS

$\frac{\pi t}{T} = k\pi$

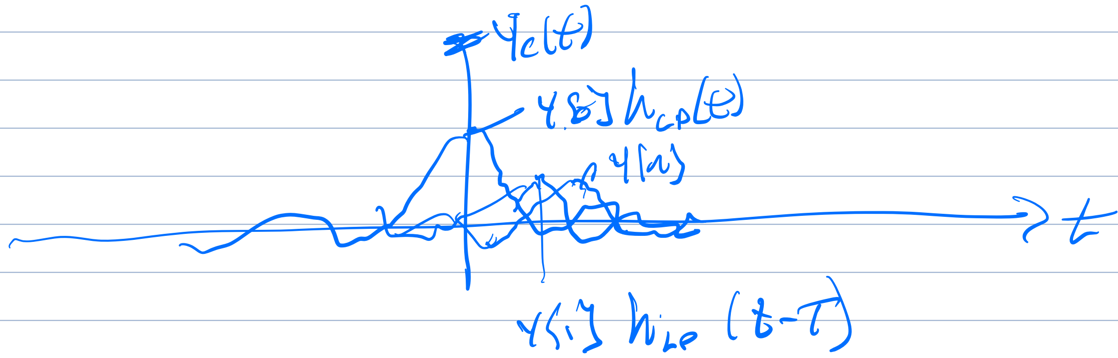
$t = kT$



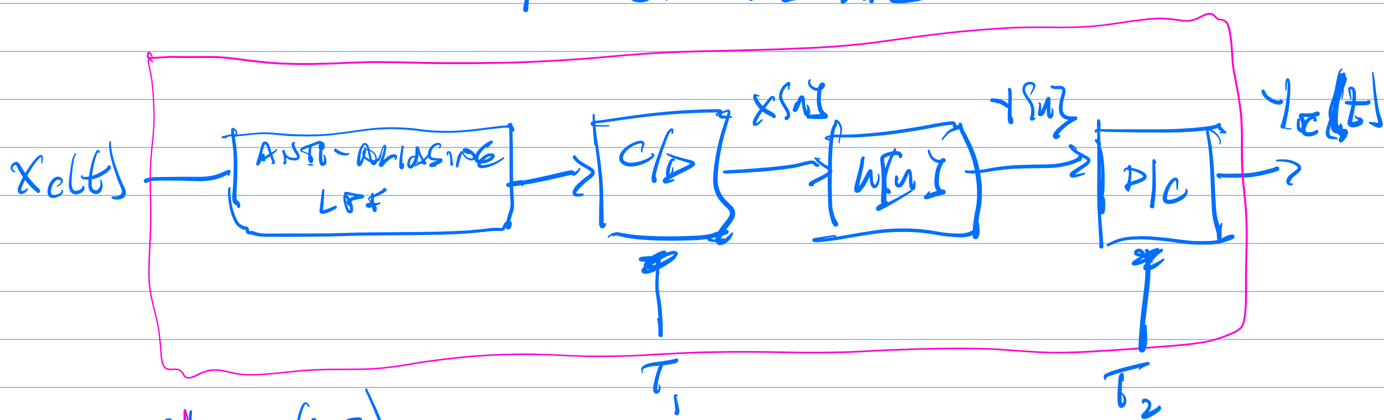
$$y_s(t) = \sum_{n=-\infty}^{\infty} y[n]\,\delta(t-nT)$$

$$y_c(t) = y_s(t) * h_{LP}(t)$$

$$= \sum_{n=-\infty}^{\infty} y[n]\cdot h_{LP}(t-nT)$$



DT PROCESSING OF CT SIGNALS



$H_{eff}(j\Omega)$

IF $T_1 = T_2 = T$ $\quad h[n] \Longleftrightarrow H(e^{j\omega}), \quad \omega = \Omega T$

$$H_{eff}(j\Omega) = \frac{Y_c(j\Omega)}{X_c(j\Omega)} = \begin{cases} H(e^{j\Omega T}), & |\Omega| < \frac{\pi}{T} \\ 0, & \text{ELSE} \end{cases}$$