CHARACTERIZATION OF DT SIGNALS + SYSTEMS

$$
\begin{tikzcd}[column sep=large, row sep=large]
h[n] \arrow[r, shift left, "\text{DTFT}"] \arrow[rr, bend left=40, shift left, "\text{ZT}"] & H(e^{j\omega}) \arrow[l, shift left, "\text{IDTFT}"] \arrow[dl, shift left, "\text{DTFT}"] & H(z) + \text{ROC} \arrow[l, "z = e^{j\omega}"'] \arrow[ll, bend right=40, "\text{(IZT)}"'] \arrow[dll, shift left] \arrow[d, shift left, "\text{ZT}"] \\
\text{DIFF. EQ.} + \text{ICs} \arrow[u] \arrow[ur, shift left] \arrow[urr, shift left] & & \text{POLE-ZERO LOCATIONS} + \text{GAIN} \arrow[u, shift left] \arrow[ul]
\end{tikzcd}
$$

UNIT SAMPLE RESP.

CTFT

$$x(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} X(j\Omega)\, e^{j\Omega t}\, d\Omega$$

$$X(j\Omega) = \int_{-\infty}^{\infty} x(t)\, e^{-j\Omega t}\, dt$$

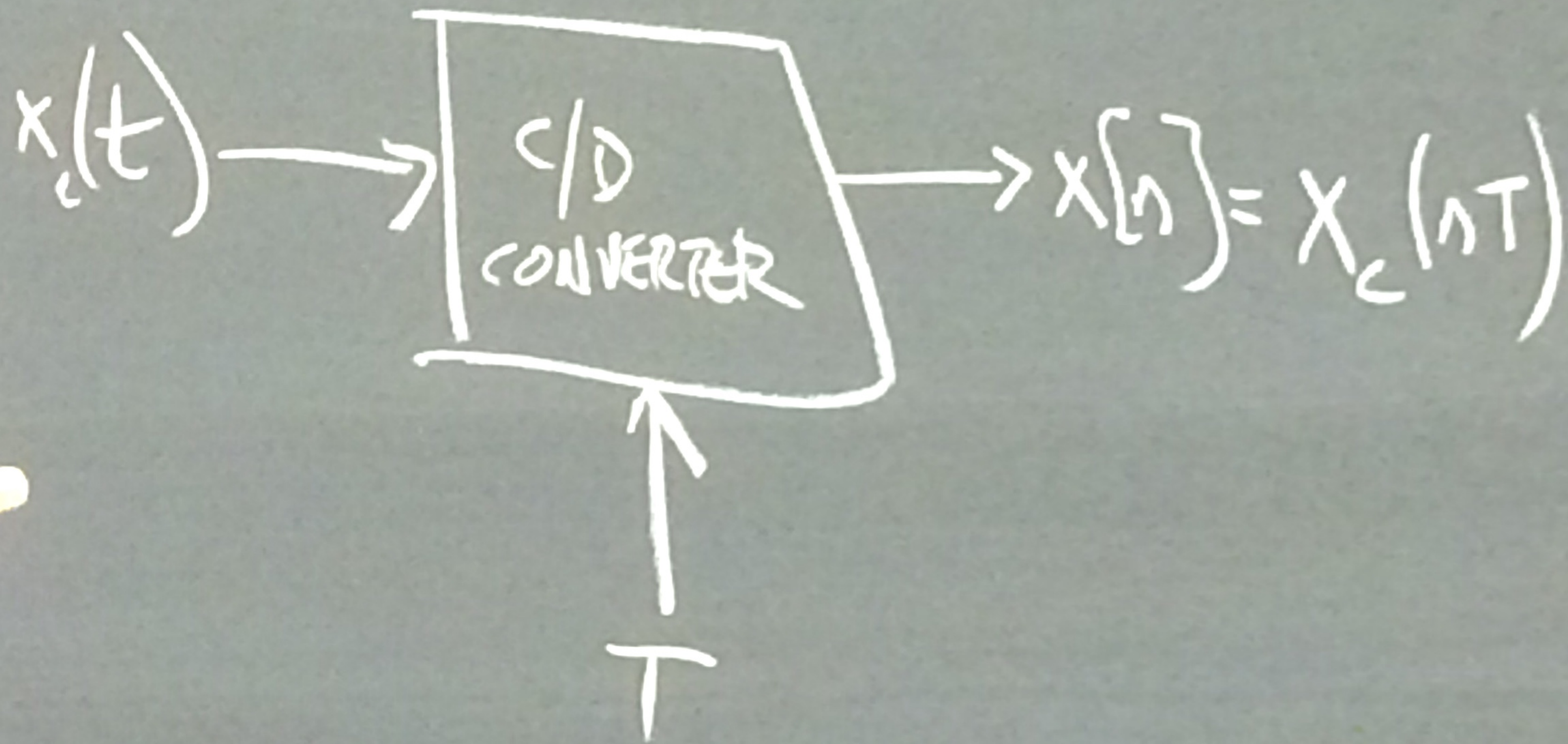
DTFT

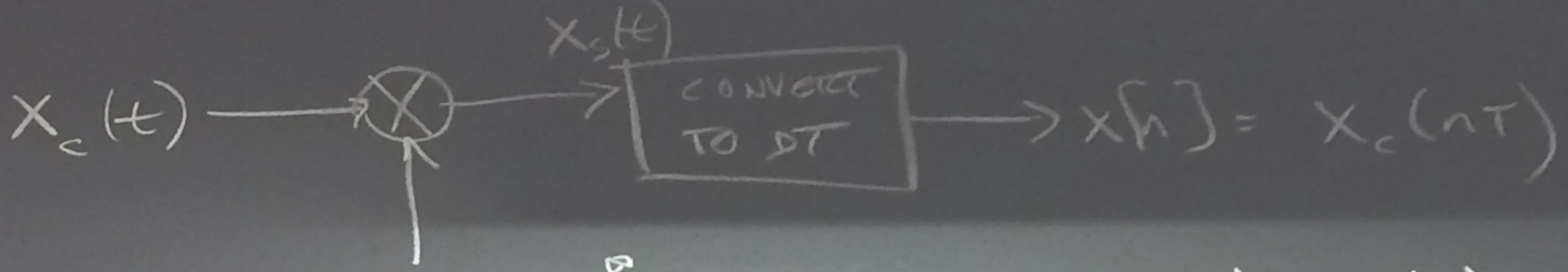
$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j\omega n}$$

SAMPLING CONTINUOUS-TIME SIGNALS (OS4P 4.0-4.5)

IDEAL C/D CONVERSION

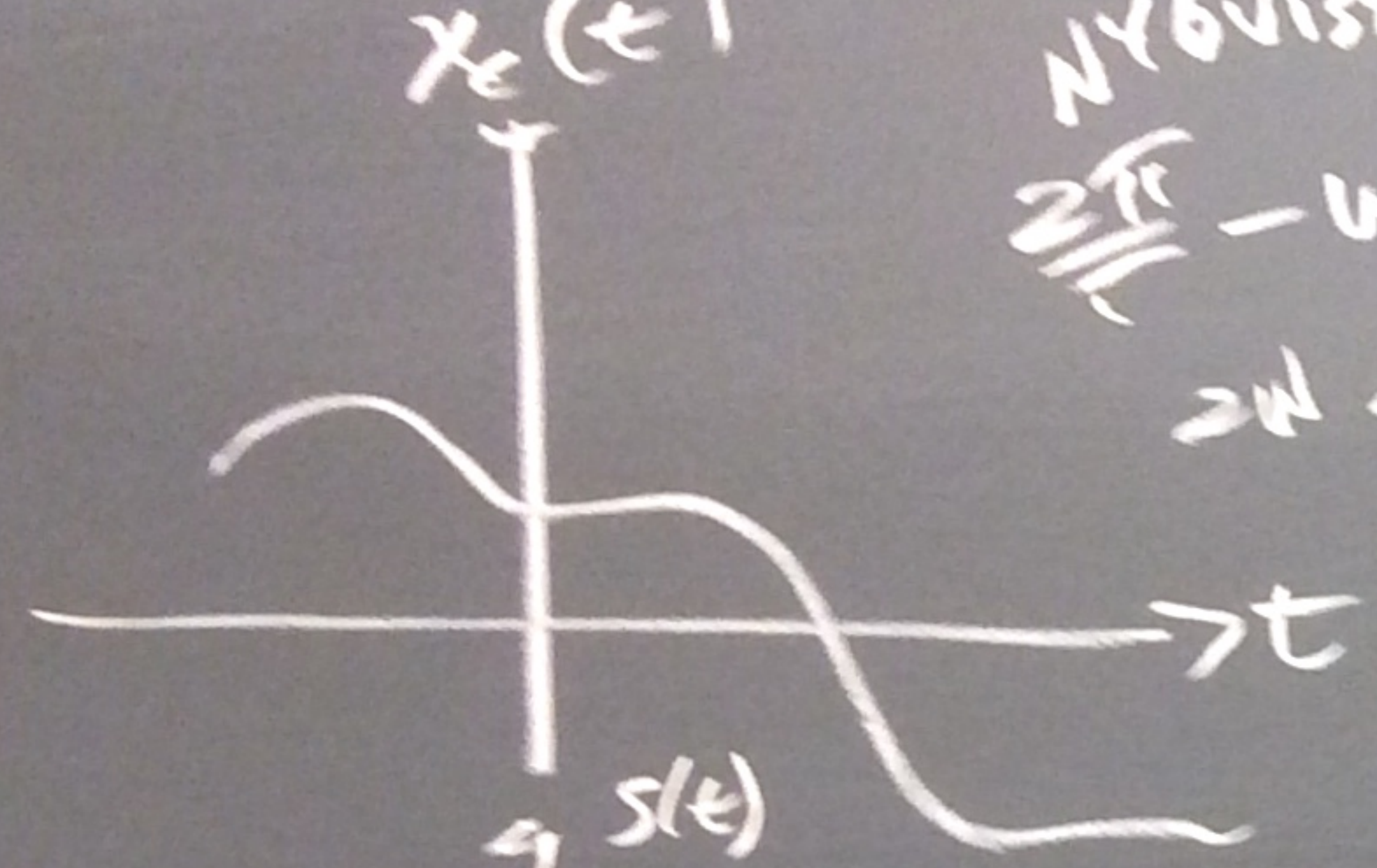




$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t-nT)$$

$$x_s(t) = s(t) \cdot x_c(t)$$

$$X_s(j\Omega) = \frac{1}{2\pi} X_c(j\Omega) * S(j\Omega)$$



NYQUIST CONSTRAINT:

$$\frac{2\pi}{T} - W > W$$

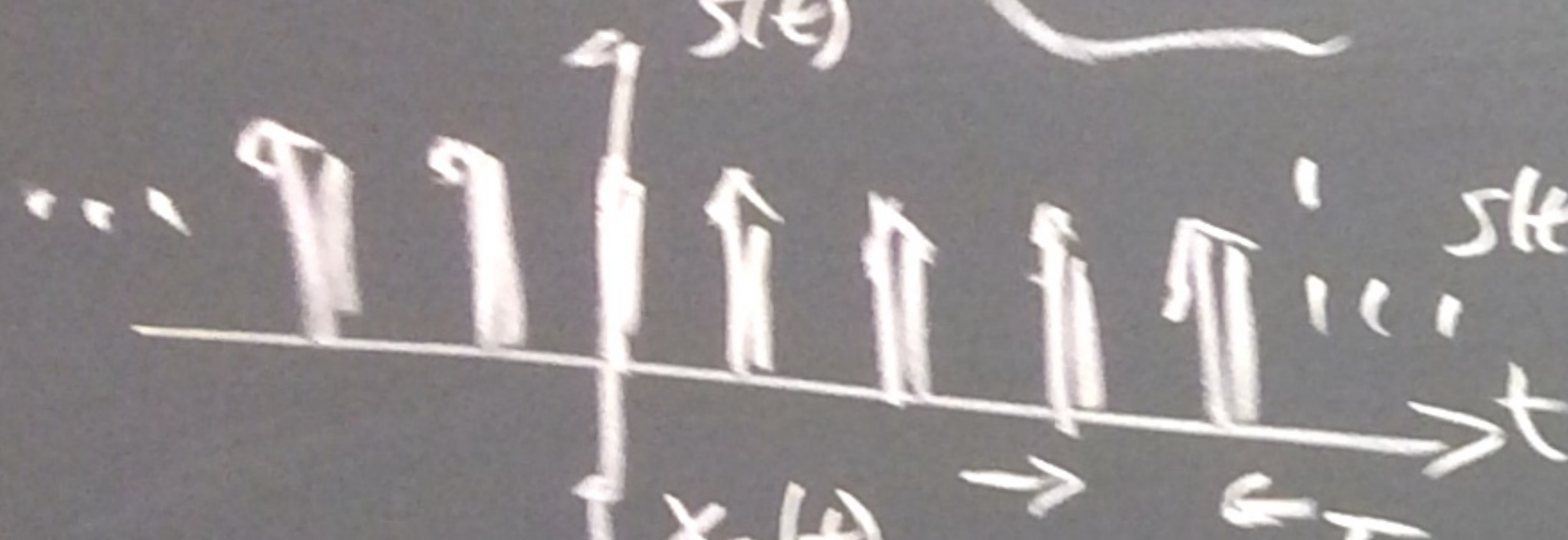
$$2W < \frac{2\pi}{T}$$

$$\boxed{W < \frac{\pi}{T}}$$

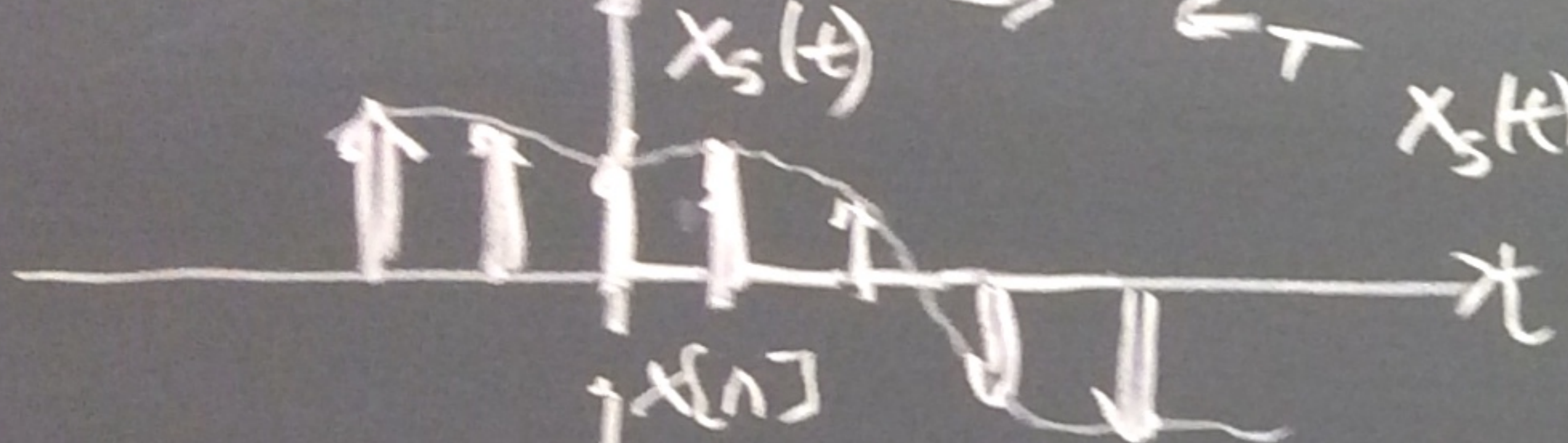
$$X_s(j\Omega) = \sum_{k=-\infty}^{\infty} \frac{1}{T} X_c\left(j\left(\Omega - \frac{2\pi k}{T}\right)\right)$$

$$|X_c(j\Omega)| = 0, \quad |\Omega| > W$$

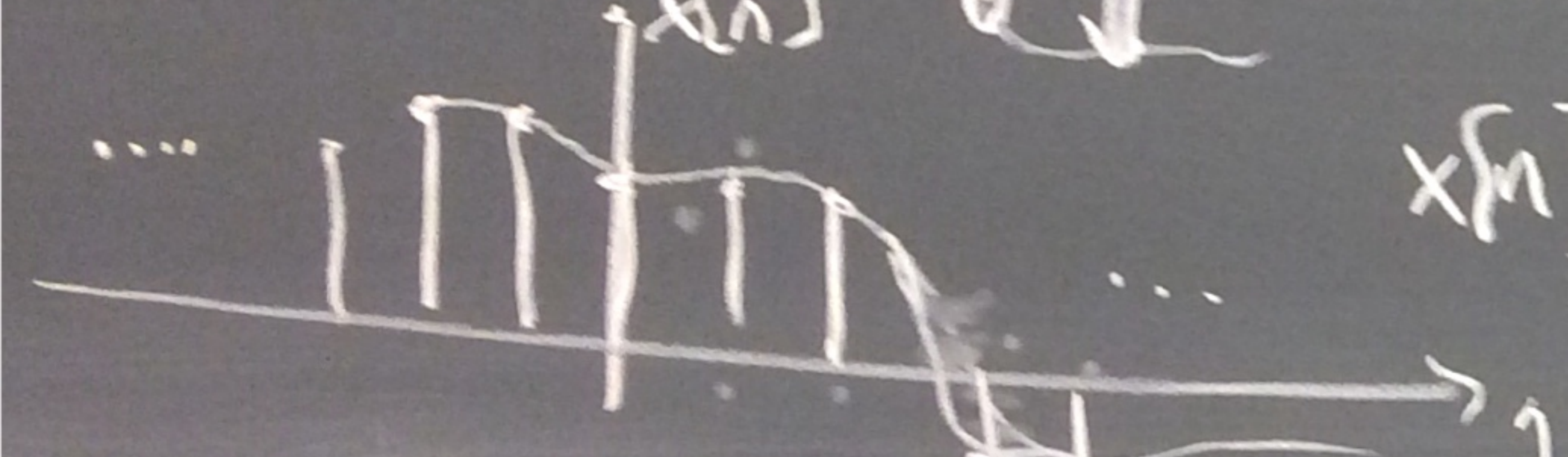
$\Longleftrightarrow$



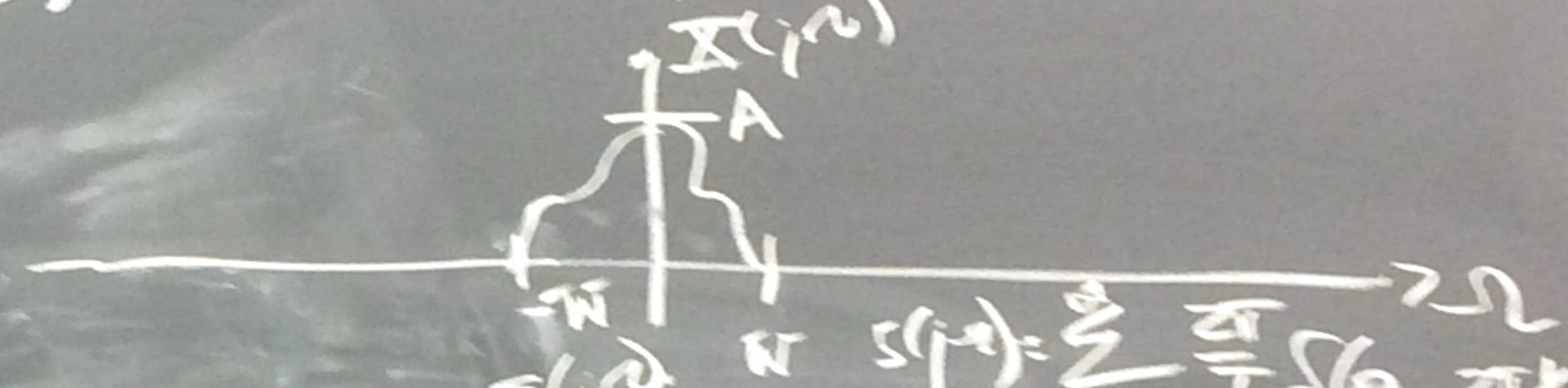
$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$



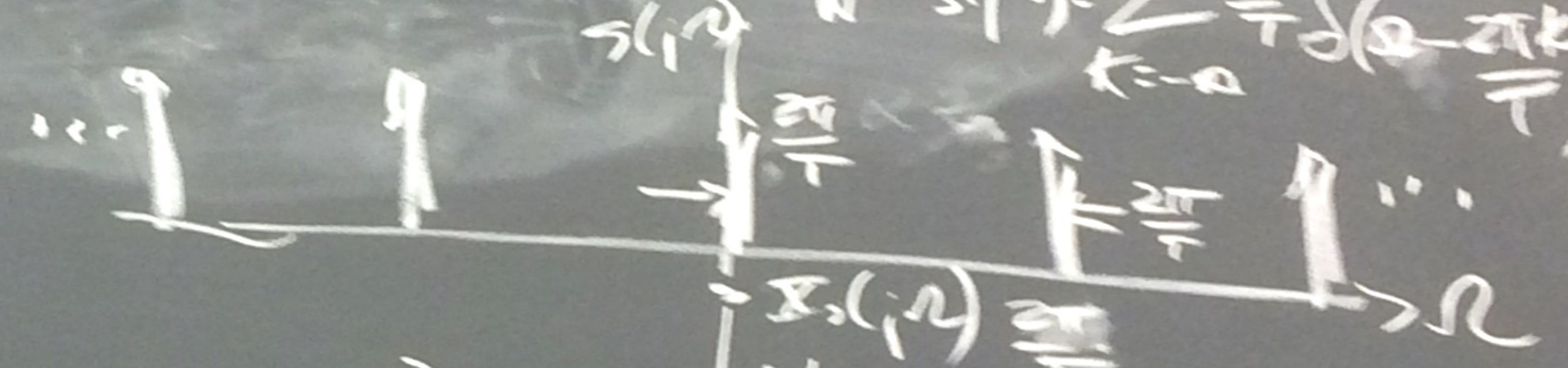
$$x_s(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\,\delta(t - nT)$$

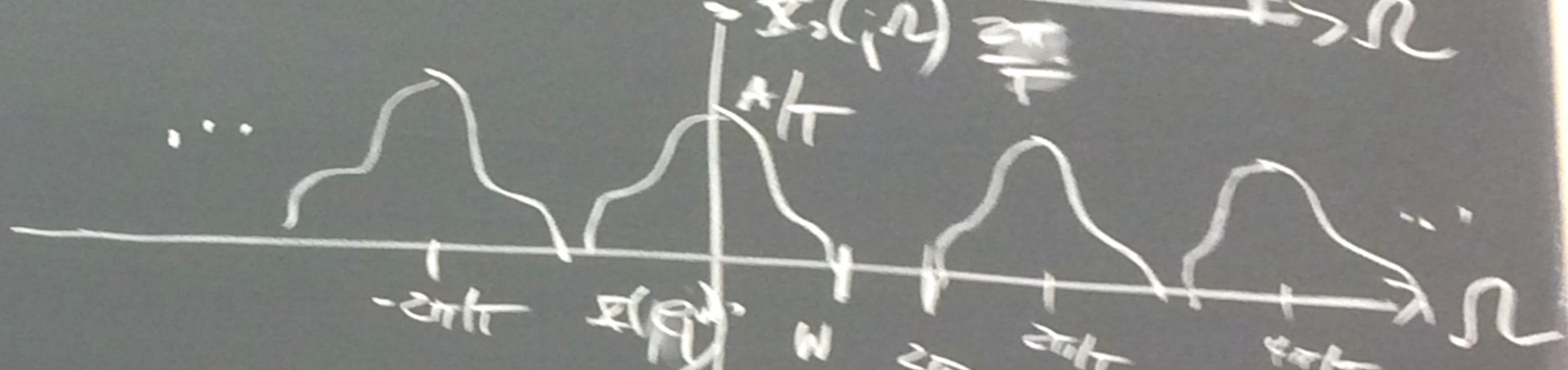


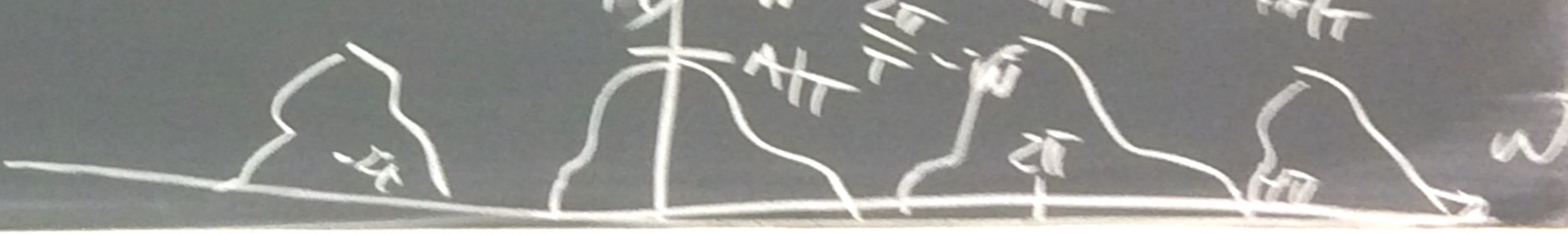
$$x[n] = \sum_{\ell=-\infty}^{\infty} x_c(\ell T)\,\delta[n - \ell]$$



$$S(j\Omega) = \sum_{k=-\infty}^{\infty} \frac{2\pi}{T}\,\delta\left(\Omega - \frac{2\pi k}{T}\right)$$







$$= \sum_{n=-\infty}^{\infty} \int_{-\infty}^{\infty} x_c(t)\,\delta(t-nT)\,e^{-j\Omega t}\,dt \qquad \boxed{\omega = \Omega T}$$

$$X_s(j\Omega) = \sum_{n=-\infty}^{\infty} x_c(nT)\,e^{-j\Omega n T} \qquad X(e^{j\omega}) = X(j\Omega)\Big|_{\Omega = \frac{\omega}{T}}$$

DTFT of $x[n]$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x_c(nT)\,e^{-j\omega n}$$

$$X(e^{j\omega}) = \sum^{\infty} \frac{X_c}{T}\left(j\,\Omega T + 2\pi r\right)$$