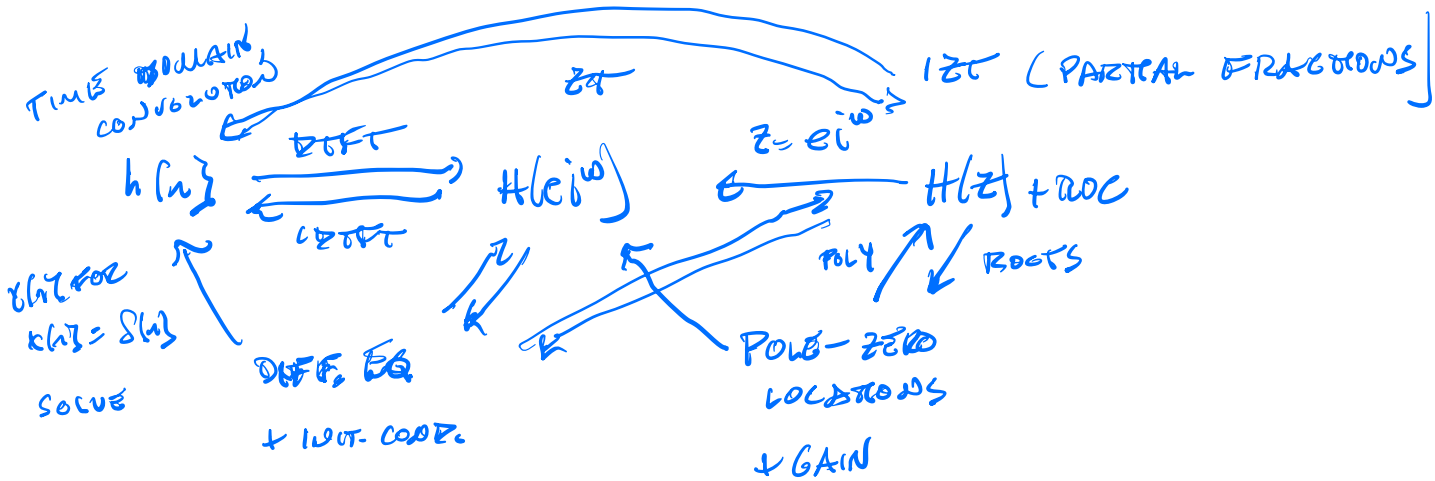
2/7/24 RELATING POLE + ZERO LOCATIONS TO FREQUENCY RESPONSE;

ALL PASS, MINIMUM PHASE, LINEAR PHASE SYSTEMS

[OSUB 5.0-5.3, 5.5-5.7]

COMPLEMENTARY WAYS OF REPRESENTING LSI SYSTEMS



RELATING POLE + ZERO LOCATIONS TO FREQUENCY RESPONSE

Consider

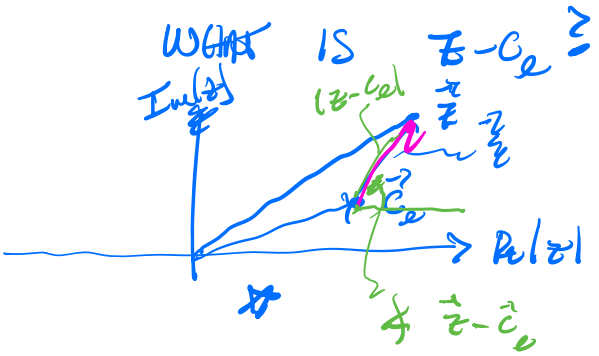
$$H(z) = \frac{\sum_{k=0}^{M} b_k z^{-k}}{1 - \sum_{k=1}^{N} a_k z^{-k}} = \frac{B(z)}{A(z)} = \frac{b_0}{a_0} \cdot \frac{\prod_{k=1}^{M} (1 - c_k z^{-1})}{\prod_{k=1}^{N} (1 - d_k z^{-1})} \quad \star$$

MATLAB: $A(z) = a_0 + \sum_{k=1}^{N} a_k z^{-k}$

$\{c_k\}, \{d_k\}$ ARE ZEROS, POLES

$$H(z) = z^{N-M} \frac{b_0}{a_0} \frac{\prod_{k=1}^{M} (z - c_k)}{\prod_{k=1}^{N} (z - d_k)} \quad \star$$

WHAT IS $z - c_\ell$?



$$\vec{c}_\ell + \vec{\xi} = \vec{z} \quad \Rightarrow \quad \vec{\xi} = \vec{z} - \vec{c}_\ell$$

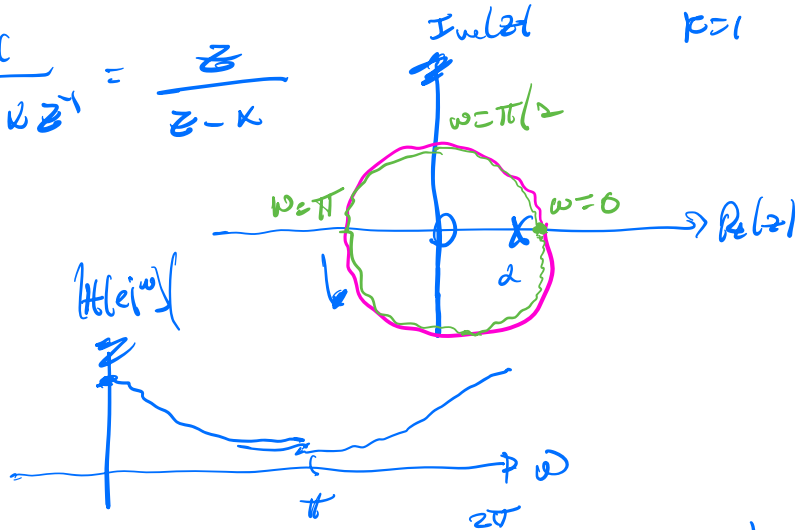
MAGNITUDE & PHASE OF DTFT ...

$$H(z) = z^{N-M} \frac{b_0}{a_0} \frac{\prod_{\ell=1}^{M} (z - c_\ell)}{\prod_{k=1}^{N} (z - d_k)}$$

$$|H(z)| = |z^{N-M}| \left|\frac{b_0}{a_0}\right| \frac{\prod_{\ell=1}^{M} |z - c_\ell|}{\prod_{k=1}^{N} |z - d_k|}$$

$$|H(e^{j\omega})| = |H(z)|_{z=e^{j\omega}} = |e^{j\omega(N-M)}| \left|\frac{b_0}{a_0}\right| \frac{\prod_{\ell=1}^{M} |e^{j\omega} - c_\ell|}{\prod_{k=1}^{N} |e^{j\omega} - d_k|}$$

$$H(z) = \frac{C}{1 - \alpha z^{-1}} = \frac{z}{z - \alpha}$$

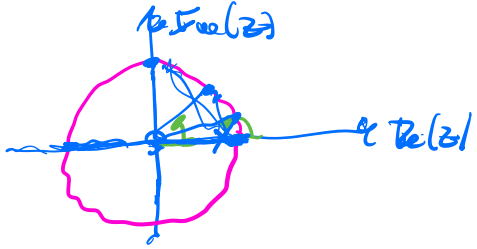


$$\angle H(z) = \angle z^{N-M} + \angle\left(\frac{b_0}{a_0}\right) + \sum_{\ell=1}^{M} \angle(z - c_\ell) - \sum_{k=1}^{N} \angle(z - d_k)$$

$$\angle H(e^{j\omega}) = \underbrace{\angle e^{j\omega(N-M)}}_{\omega(N-M)} + \angle\left(\frac{b_0}{a_0}\right) + \sum_{\ell=1}^{M} \angle(e^{j\omega} - c_\ell) - \sum_{k=1}^{N} \angle(e^{j\omega} - d_k)$$

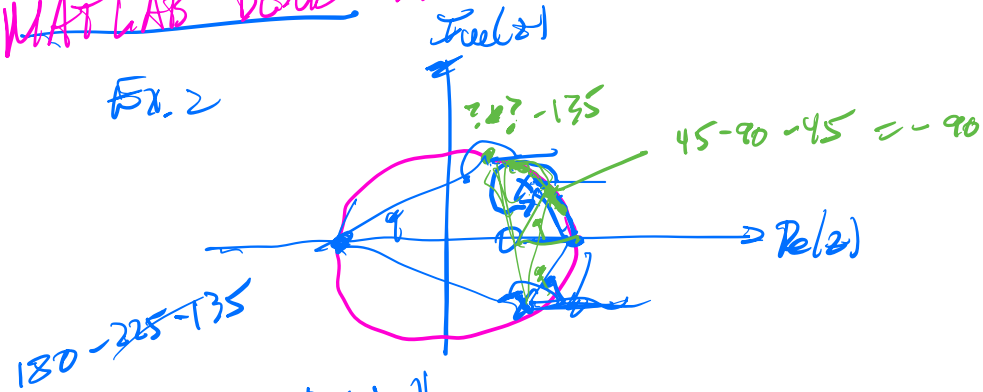
$= H(z)|_{z=e^{j\omega}}$; $\angle\left(\frac{b_0}{a_0}\right) \in \{0, \pi\}$

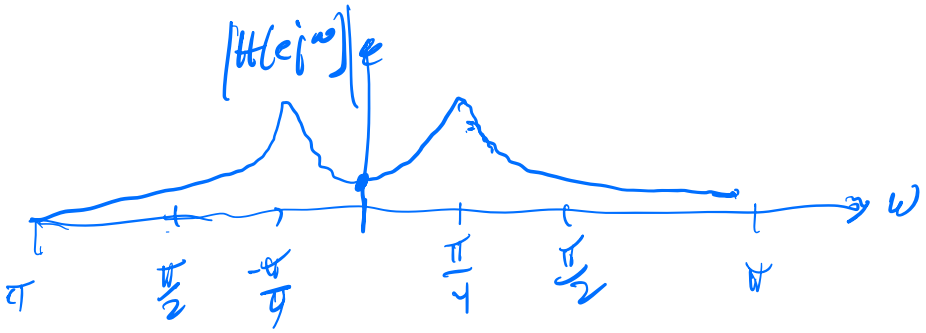
$H(z) = \frac{z}{z-a}$

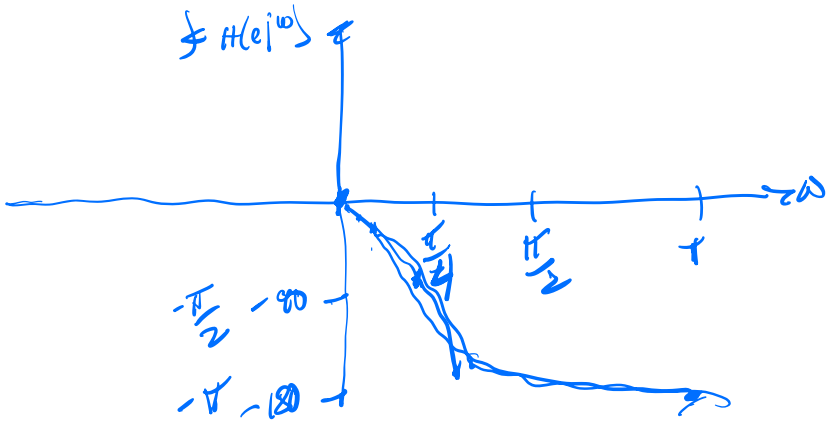


MATLAB DEMO SHOWN HERE

Ex. 2





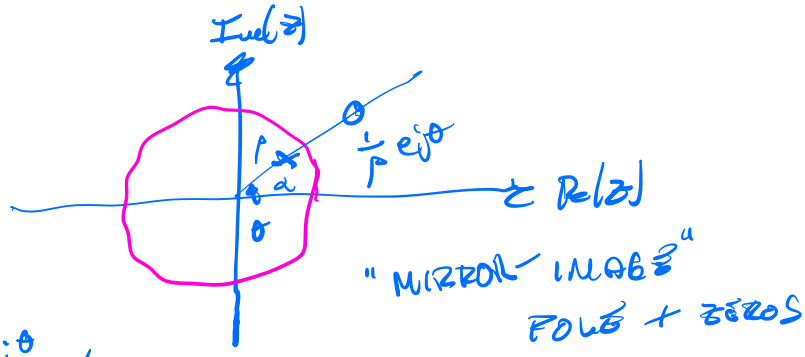


ALL PASS SYSTEMS

CONSIDER $H(z) = \dfrac{z^{-1} - \alpha^*}{1 - \alpha z^{-1}} = \dfrac{1 - z\alpha^*}{z - \alpha}$

Let $\alpha = \rho e^{j\theta}$

$\alpha^* = \rho e^{-j\theta}$



"MIRROR IMAGE" POLE + ZEROS

POLE @ $\rho e^{j\theta} = \alpha$

ZERO $(1 - z\alpha^*) = 0$; $z = \dfrac{1}{\alpha^*} = \dfrac{1}{\rho e^{-j\theta}} = \dfrac{1}{\rho} e^{j\theta}$

THIS DISCUSSION WILL BE COMPLETED FRIDAY, COVERING ALL PASS, MINIMUM PHASE, + LINEAR PHASE SYSTEMS