PROVING THE INVERSE RELATIONSHIP

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\pi}^{\pi} \underbrace{\sum_{\ell=-\infty}^{\infty} x[\ell] e^{-j\omega \ell}}_{X(e^{j\omega})} e^{j\omega n}\, d\omega$$

$$= \sum_{\ell=-\infty}^{\infty} x[\ell] \underbrace{\frac{1}{2\pi}\int_{-\pi}^{\pi} e^{j\omega(n-\ell)}\, d\omega}_{\begin{cases} 1 & n=\ell \\ 0 & n\neq \ell \end{cases}} = x[n]$$

DTFT

IF $\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$

$$x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{j\omega}) e^{j\omega n}\, d\omega$$

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega n}$$

EXAMPLES + PROPERTIES OF DTFTs

(OS4P 2.6-2.9)

DTFT of $\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$

$$= \sum_{n=-\infty}^{\infty}\sum_{k=-\infty}^{\infty} x[k]\,h[n-k]\,e^{-j\omega n}$$

$$= \sum_{k=-\infty}^{\infty} x[k] \cdot \underbrace{\sum_{n=-\infty}^{\infty} h[n-k]\,e^{-j\omega n}}_{H(e^{j\omega})\,e^{-j\omega k}} = H(e^{j\omega}) \cdot \underbrace{\sum_{k=-\infty}^{\infty} x[k]\,e^{-j\omega k}}_{X(e^{j\omega})} = X(e^{j\omega}) \cdot H(e^{j\omega})$$

$x[-n] \longleftrightarrow \sum_{n=-\infty}$

let $\ell = -n$

$n = -\ell$

$$\sum_{n=-\infty}^{\infty} x[n-N] = \sum_{n=-\infty}^{\infty} x[n-N]\, e^{-j\omega n}$$

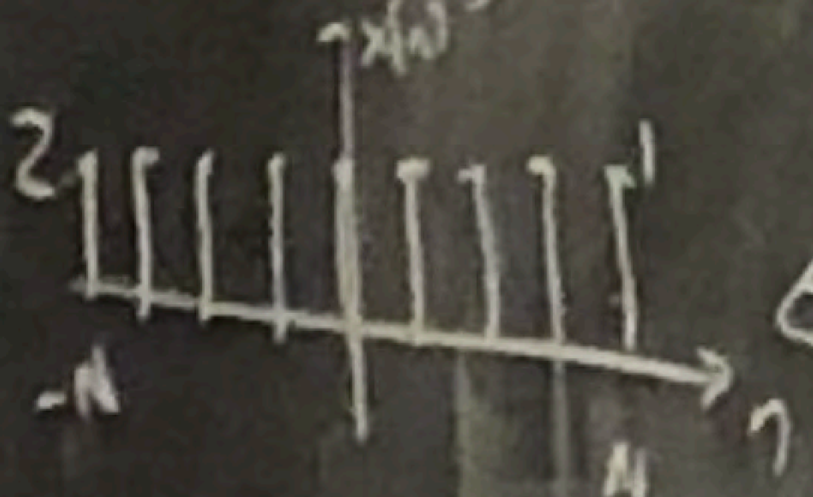
Let $l = n - N$; $n = l + N$

$$= \sum_{l=-\infty}^{\infty} x[l]\, e^{-j\omega(l+N)} = e^{-j\omega N} \underbrace{\sum_{l=-\infty}^{\infty} x[l]\, e^{-j\omega l}}_{X(e^{j\omega})}$$

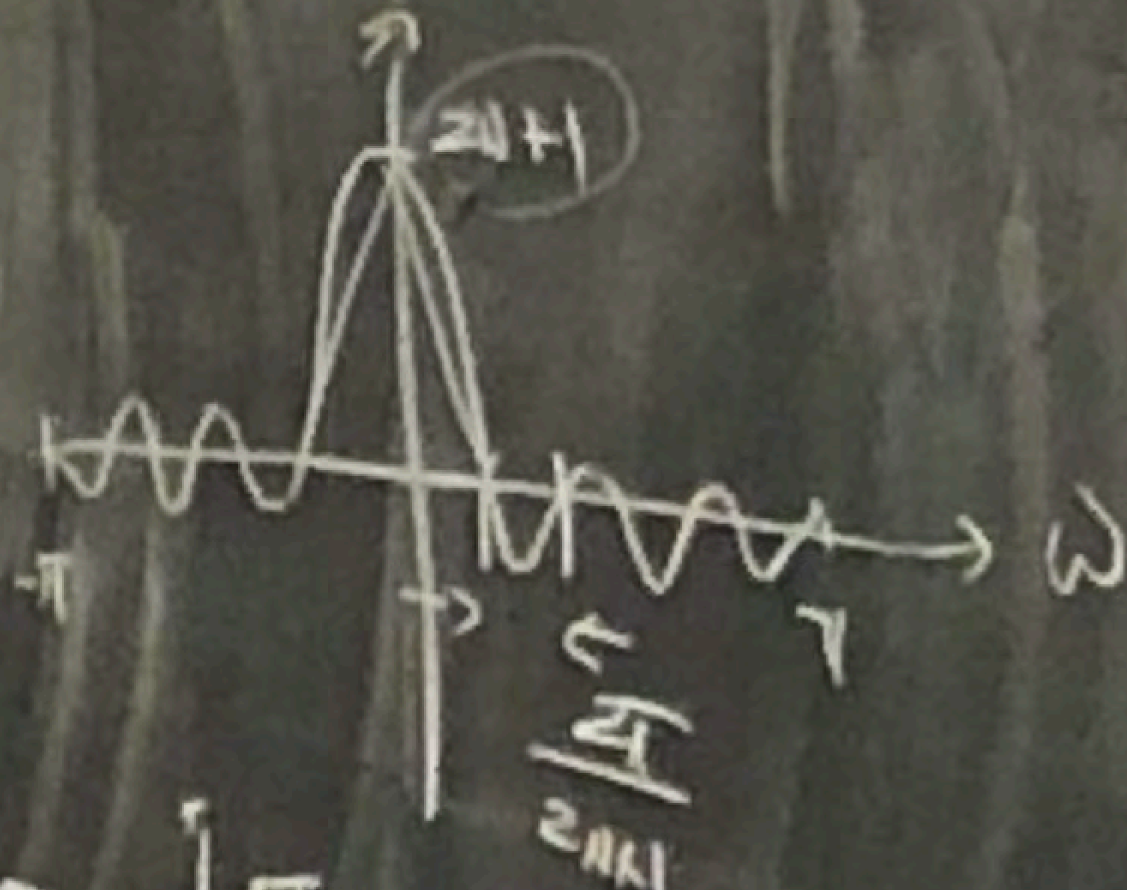
DTFT EXAMPLES

$x[n] \Longleftrightarrow X(e^{j\omega})$

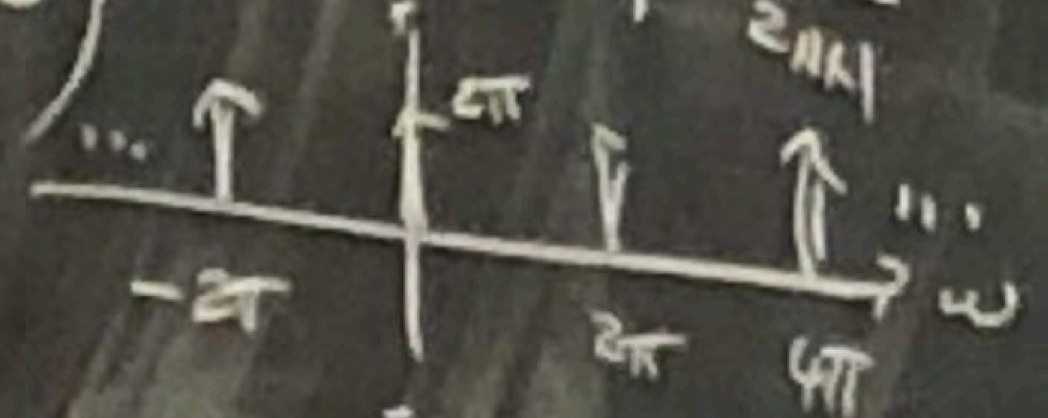
1. $\alpha^n u[n],\ |\alpha|<1 \Longleftrightarrow \dfrac{1}{1-\alpha e^{-j\omega}}$



$\Longleftrightarrow \dfrac{\sin\left(\frac{(2N+1)\omega}{2}\right)}{\sin\left(\frac{\omega}{2}\right)}$

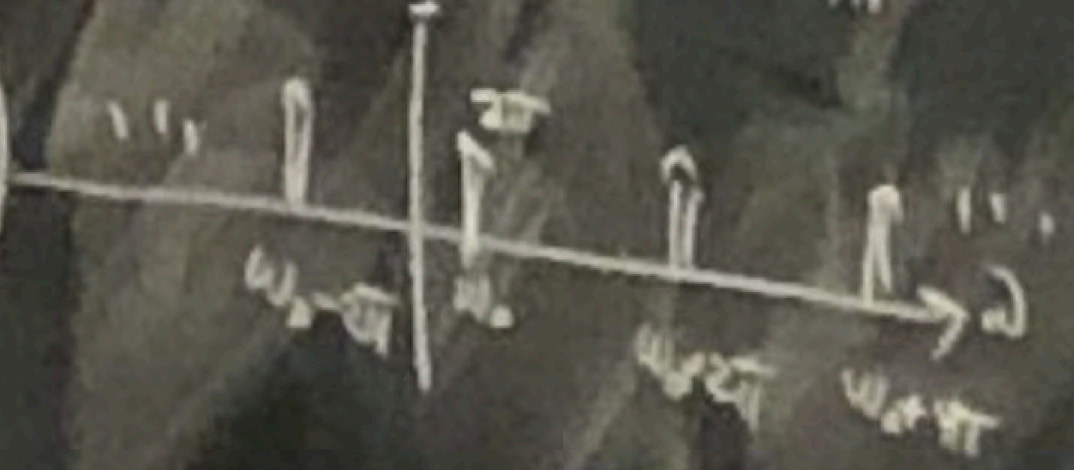


$1 \Longleftrightarrow 2\pi \sum_{r=-\infty}^{\infty} \delta(\omega-2\pi r)$



$\delta[n] \Longleftrightarrow 1$

$1\cdot e^{j\omega_0 n} \Longleftrightarrow 2\pi \sum_{r=-\infty}^{\infty} \delta(\omega-\omega_0-2\pi r)$



DTFT PROPERTIES

$x[n] \Longleftrightarrow X(e^{j\omega})$

LINEARITY $a x_1[n] + b x_2[n] \Longleftrightarrow a X_1(e^{j\omega}) + b X_2(e^{j\omega})$

SHIFT $x[n-N] \Longleftrightarrow e^{-j\omega N} X(e^{j\omega})$

MULT BY COMP. EXP.

$x[n] e^{+j\omega_0 n} \Longleftrightarrow X(\omega-\omega_0)$

CONVOLUTION IF $x[n] \Longleftrightarrow X(e^{j\omega})$

$h[n] \Longleftrightarrow H(e^{j\omega})$

$\sum_{k=-\infty}^{\infty} x[k]\, h[n-k] \Longleftrightarrow X(e^{j\omega})\cdot H(e^{j\omega})$

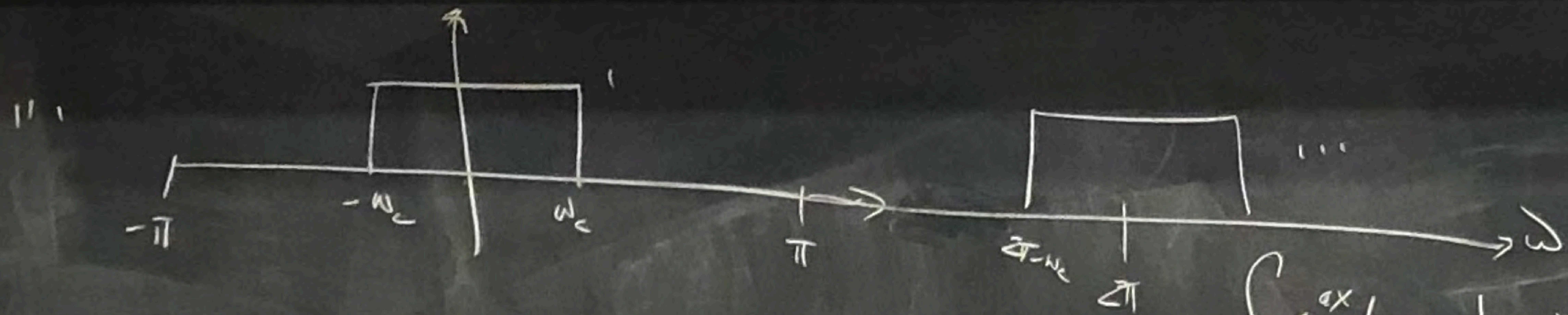
MULT.

$x_1[n]\cdot x_2[n] \Longleftrightarrow \frac{1}{2\pi}\int_{-\pi}^{\pi} X_1(e^{j\theta})\cdot X_2(e^{j(\omega-\theta)})\, d\theta$

$\frac{1}{2\pi} X_1(e^{j\omega}) \circledast X_2(e^{j\omega})$ CIRCULAR CONVOLUTION

$x[n]$
$-\frac{j}{2}\frac{\sin\left((\omega-\omega_0)\frac{N}{2}\right)}{\sin\left(\frac{\omega-\omega_0}{2}\right)}e^{-j(\omega-\omega_0)\left(\frac{N-1}{2}\right)}$
$+\frac{j}{2}\frac{\sin\left((\omega+\omega_0)\frac{N}{2}\right)}{\sin\left(\frac{\omega+\omega_0}{2}\right)}e^{-j(\omega+\omega_0)\left(\frac{N-1}{2}\right)}$
$x[n] = \sin(\omega_0 n)\left[u[n] - u[n-N]\right]$
$N = k\frac{2\pi}{\omega}$
$\sin(\omega_0 n)$
$N-1$
$\frac{\sin\left(\frac{\omega N}{2}\right)}{\sin\left(\frac{\omega}{2}\right)} e^{-j\omega\frac{(N-1)}{2}}$
ω

IDEAL LPF



$$\int e^{ax}\,dx = \frac{1}{a} e^{x}$$

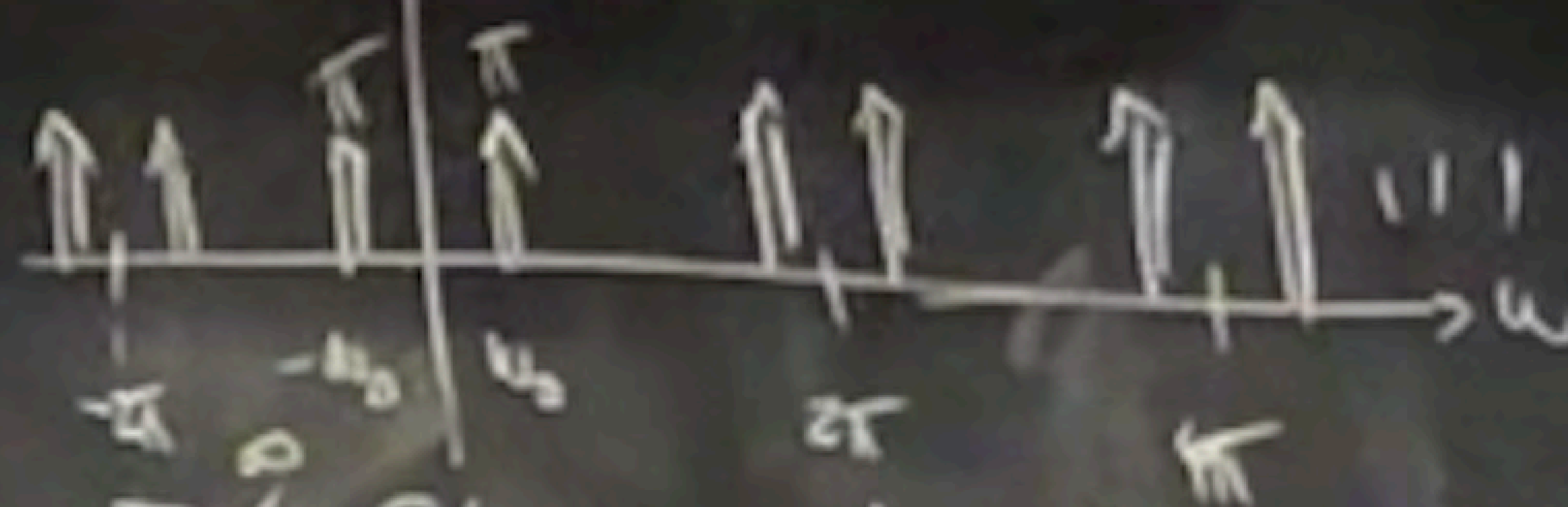
$$h[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} H(e^{j\omega})\, e^{j\omega n}\, d\omega$$

$$= \frac{1}{2\pi}\int_{-\omega_c}^{\omega_c} 1 \; e^{j\omega n}\, d\omega = \frac{1}{2\pi}\,\frac{1}{jn}\left[e^{j\omega n}\right]_{\omega=-\omega_c}^{\omega_c}$$

$$= \frac{1}{2\pi j n}\underbrace{\left[e^{j\omega_c n} - e^{-j\omega_c n}\right]}_{2j\,\sin(\omega_c n)} = \frac{\cancel{2j}\,\sin(\omega_c n)}{\cancel{2}\pi \cancel{j} n}$$

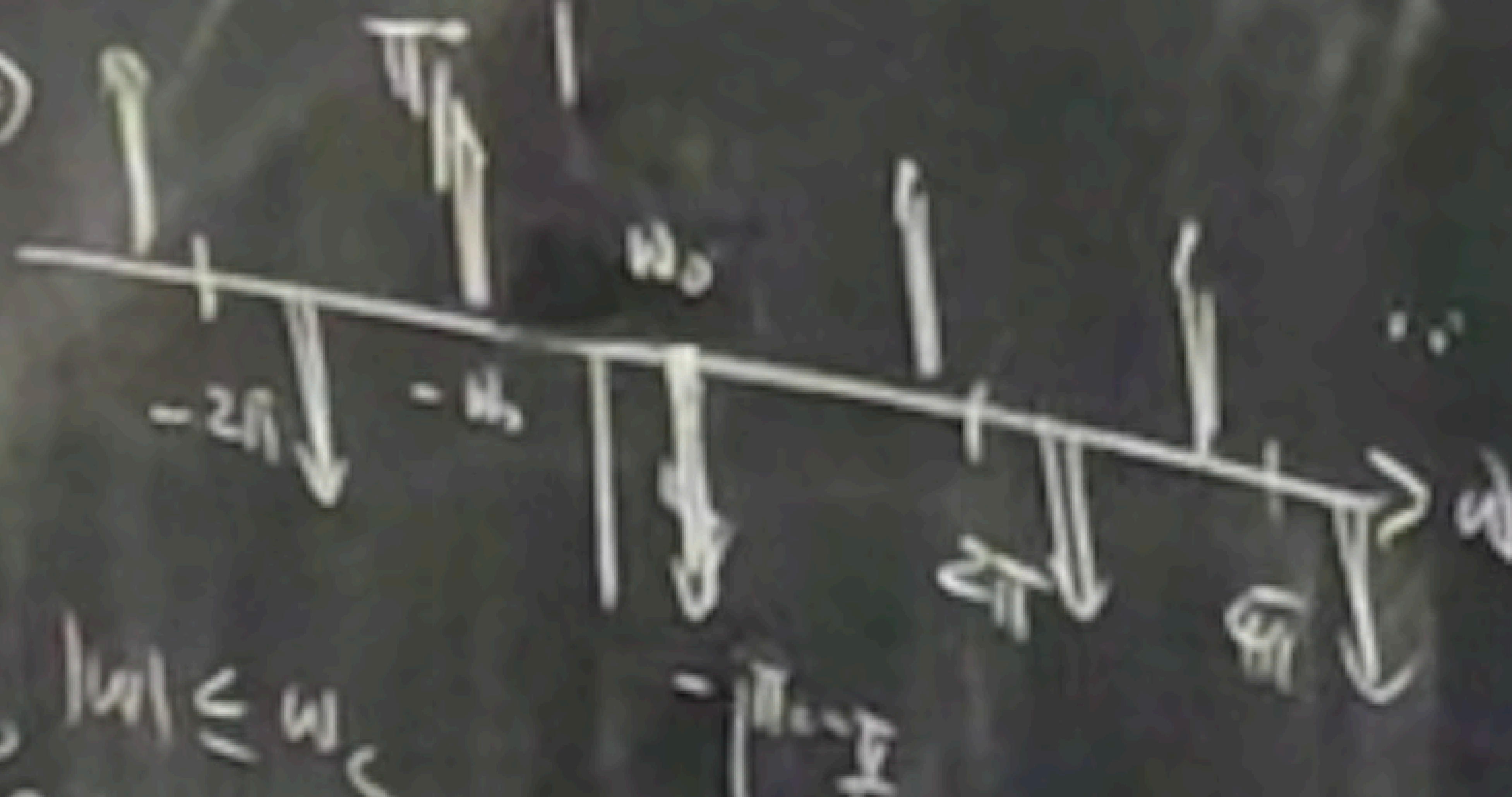
$$= \frac{\sin(\omega_c n)}{\pi n}$$

$$\cos(\omega_0 n) = \frac{e^{j\omega_0 n} + e^{-j\omega_0 n}}{2} \Longleftrightarrow$$



$$\pi \sum_{r=-\infty}^{\infty} \delta(\omega - \omega_0 - 2\pi r) + \delta(\omega + \omega_0 - 2\pi r)$$

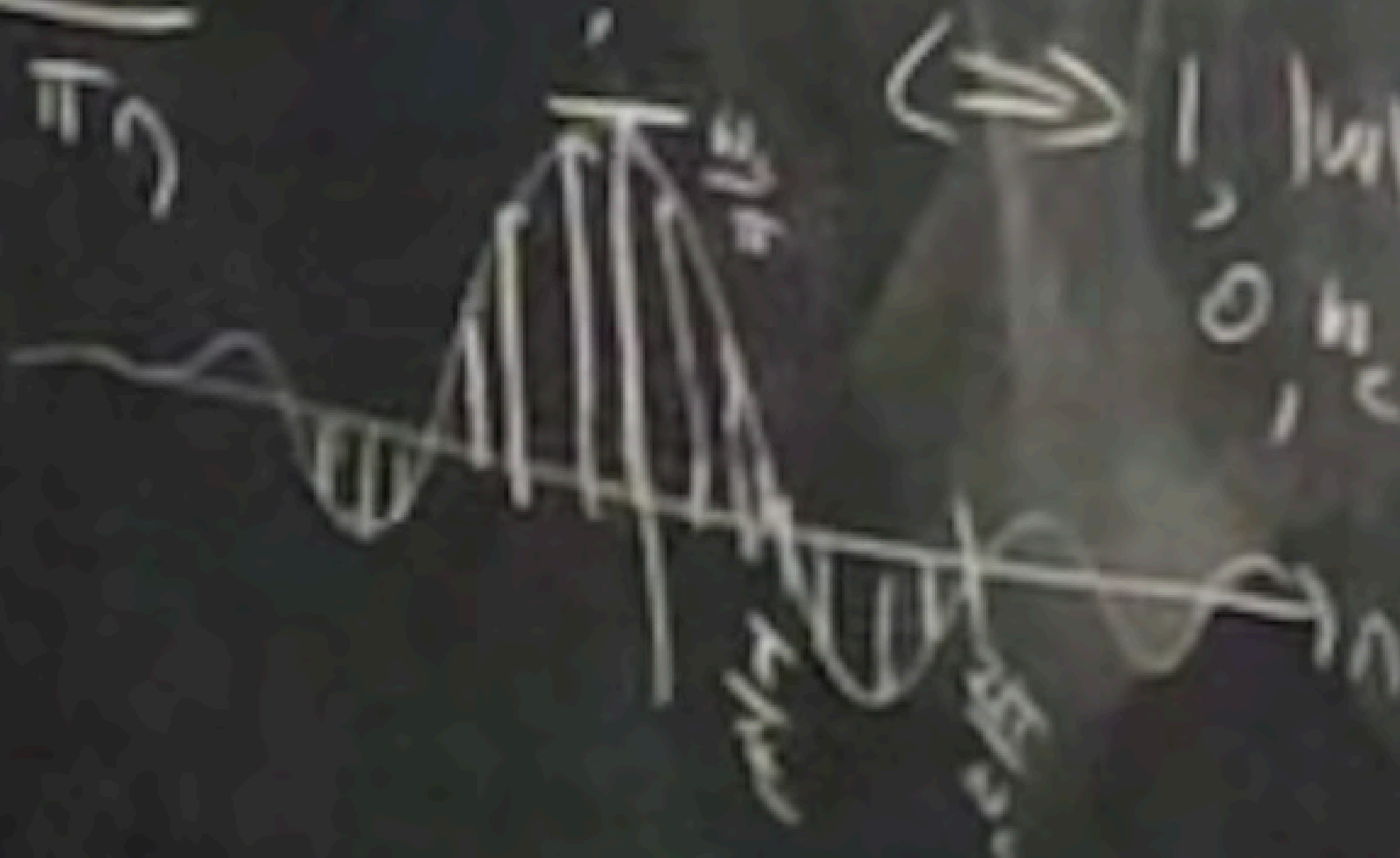
$$\sin(\omega_0 n) = \frac{e^{j\omega_0 n} - e^{-j\omega_0 n}}{2j} \Longleftrightarrow$$

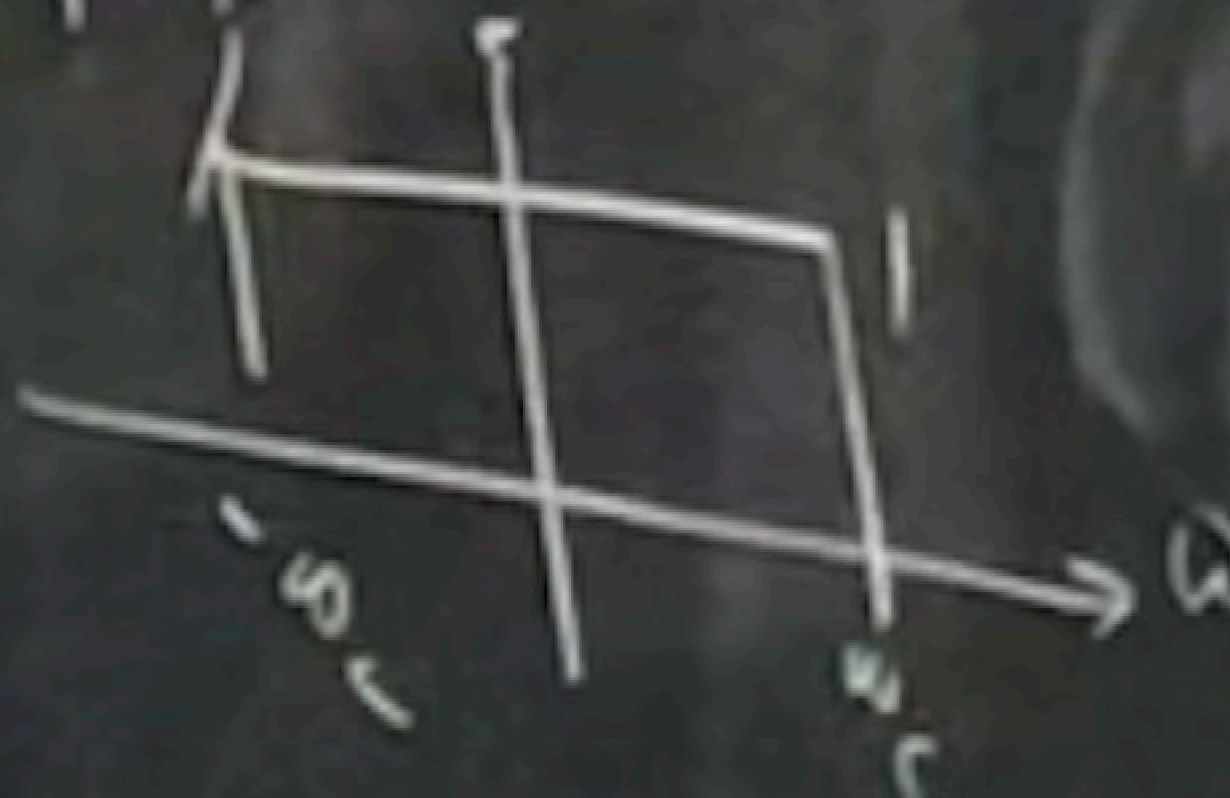


$\omega_c n = k\pi$

$n = k\pi / \omega_c$

$$\frac{\sin(\omega_c n)}{\pi n} \Longleftrightarrow \begin{cases} 1, & |\omega| \le \omega_c \\ 0, & \omega_c < |\omega| \le \pi \end{cases}$$





EVEN $x[n] = x[-n] \iff X(e^{j\omega}) = X(e^{-j\omega})$

ODD $x[n] = -x[-n] \iff X(e^{j\omega}) = -X(e^{-j\omega})$

REAL $x[n] = x^*[n] \iff X(e^{j\omega}) = X^*(e^{-j\omega})$ HERMITIAN SYMMETRY

LET $X(e^{j\omega}) = X_R(e^{j\omega}) + j\, X_I(e^{j\omega})$

$X(e^{j\omega}) = X^*(e^{-j\omega}) =$

$X_R(e^{j\omega}) = X_R(e^{-j\omega})$

$X_I(e^{j\omega}) = -X_I(e^{-j\omega})$

$\mathrm{Re}\left[X(e^{j\omega})\right]$ IS EVEN

$\mathrm{Im}\left[X(e^{j\omega})\right]$ IS ODD

$|X(e^{j\omega})|$ EVEN

$\angle X(e^{j\omega})$ ODD

$$C(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x^*[n]\, e^{-j\omega n}$$

$$C^*(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]\, e^{+j\omega n} = \sum_{n=-\infty}^{\infty} x[n]\, e^{-j(-\omega)n} = X(e^{-j\omega})$$

$$\underline{C(e^{j\omega}) = X^*(e^{-j\omega})}$$

$x[n]$ REAL, EVEN $\iff X(e^{j\omega})$ REAL, EVEN

$x[n]$ REAL, ODD $\iff X(e^{j\omega})$ IMAG, ODD