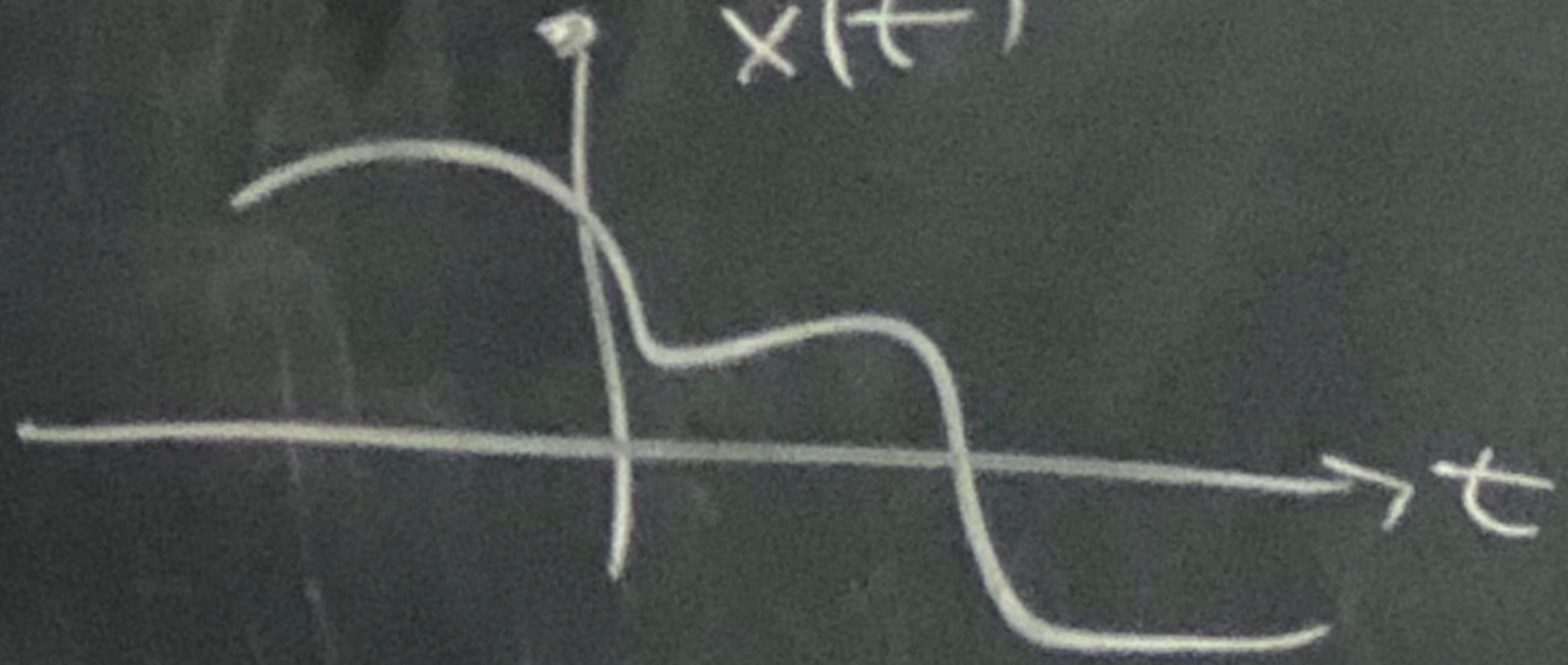
DISCRETE-TIME SIGNALS + SYSTEMS

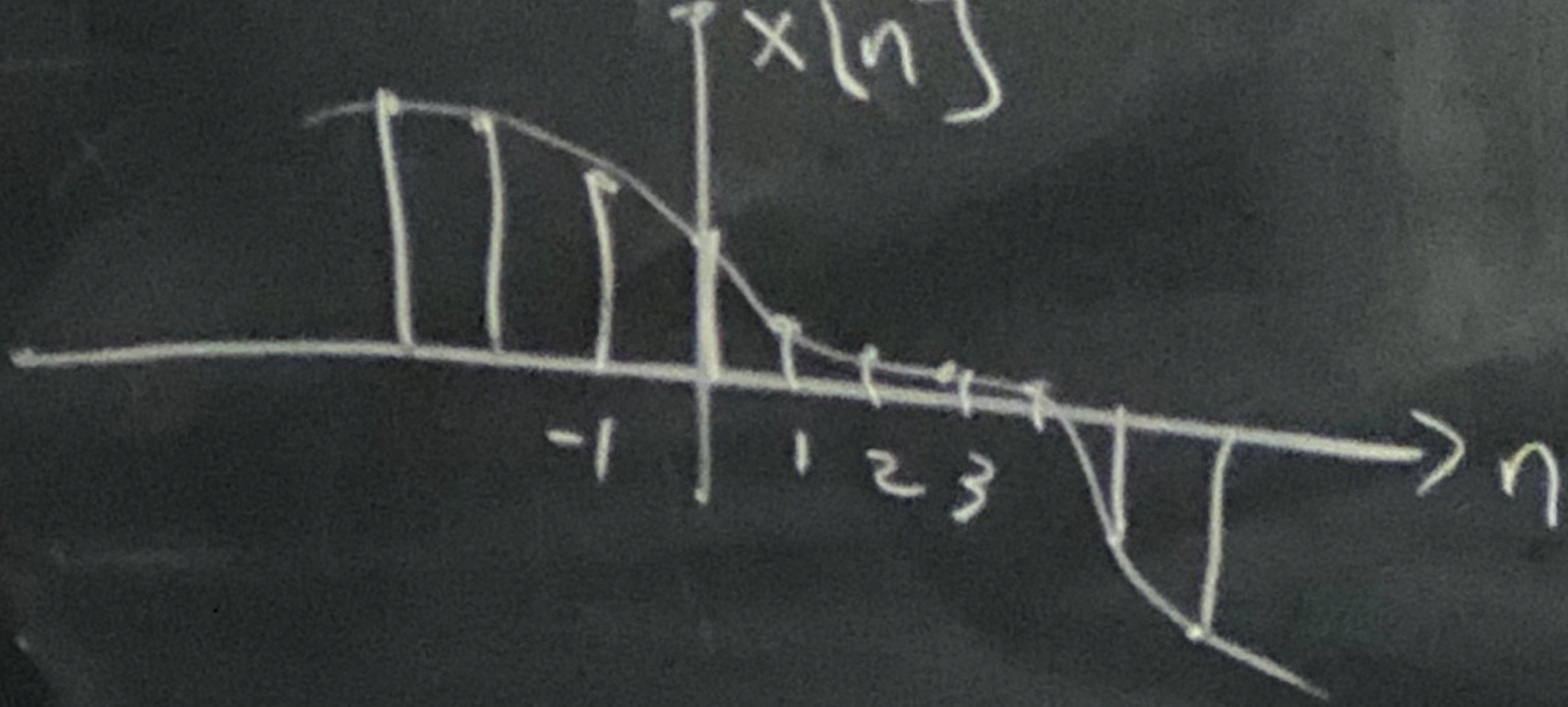
(OSYP 2.0-2.4)

TYPES OF SIGNALS

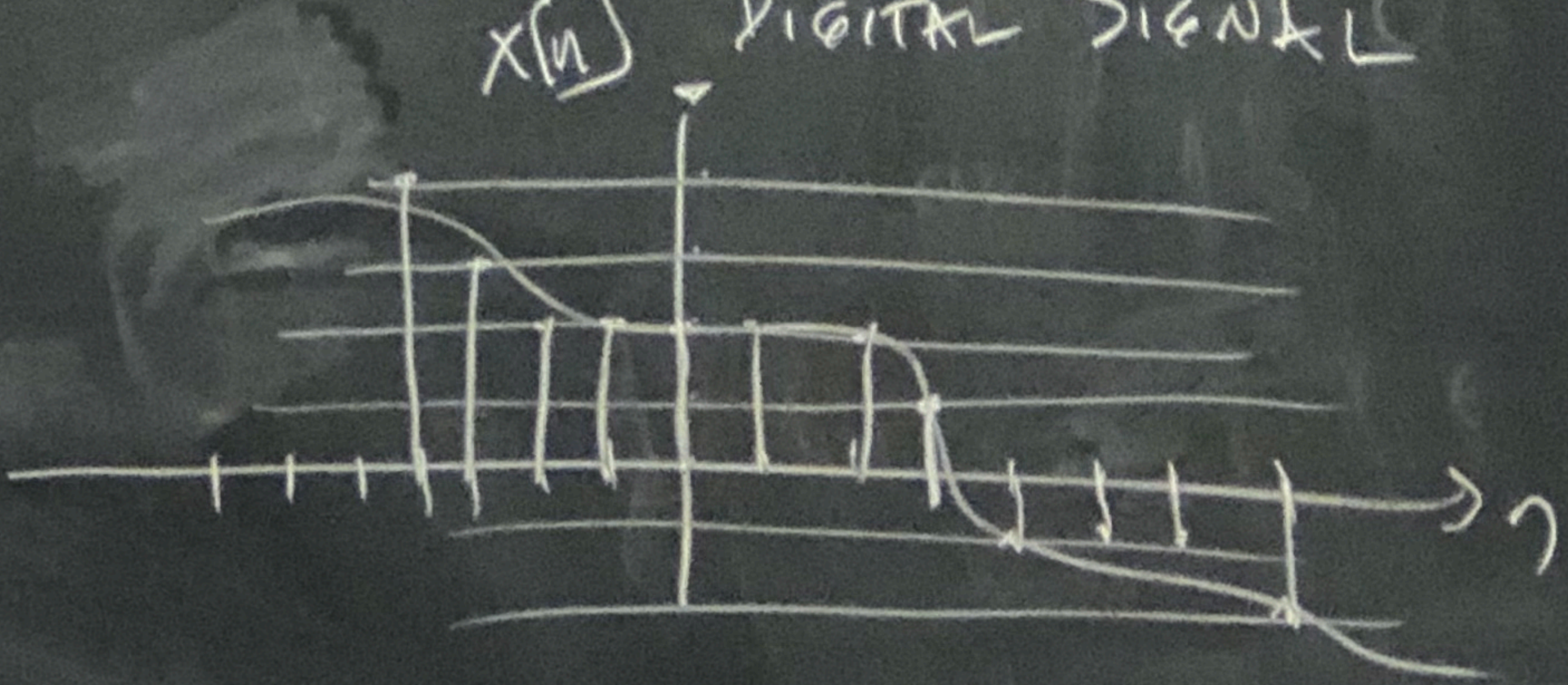
CONTINUOUS-TIME SIGNAL



DISCRETE-TIME SIGNAL



DIGITAL SIGNAL



Transformations of function arguments

$$\frac{3t-4}{9} = 0$$

$$t = \frac{4}{3}$$

$$\frac{3t-4}{9} = 2$$

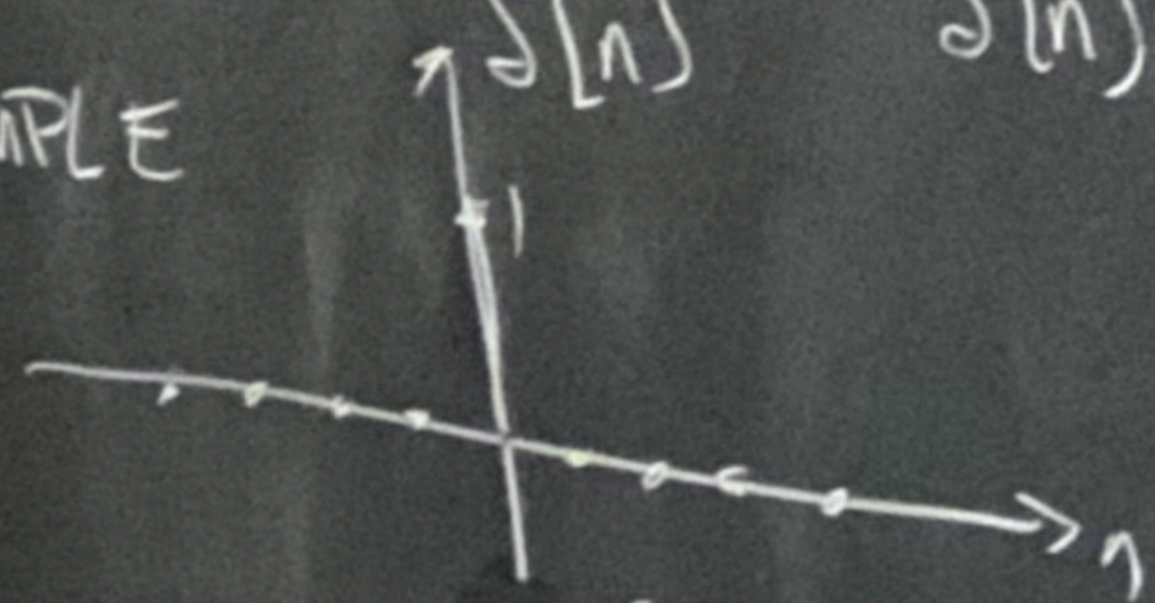
$$3t-4 = 18$$

$$t = \frac{22}{3}$$

SOME IMPORTANT SIGNALS

Σ

UNIT SAMPLE



$$\delta[n] = \begin{cases} 1, & n = 0 \\ 0, & n \neq 0 \end{cases}$$

UNIT STEP

$$u[n] = \begin{cases} 1, & n \geq 0 \\ 0, & n < 0 \end{cases}$$

EXPONENTIAL

$$x[n] = \alpha^n$$

PERIODIC FUNCTIONS

$$x[n] = A\cos(\omega_0 n + \phi)$$

PERIODICIC ?

PERIODIC $\bar{w}$ PERIOD N IF N IS AN INTEGER

PERIODIC $\bar{w}$ SOME OTHER PERIOD IF N IS RATIONAL #

NOT PERIODIC IF N IS IRRATIONAL

$$\cos(\theta) = \frac{e^{j\theta} + e^{-j\theta}}{2}$$

$$\sin(\theta) = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$e^{j\theta} = \cos(\theta) + j\sin(\theta)$$

$x[n]$ IS PERIODIC WITH PERIOD N, IF $x[n] = x[n-N]$ $\forall n$

PROPERTIES OF SYSTEMS

$$x[n] \longrightarrow \boxed{T} \longrightarrow y[n] = T\big[x[n]\big]$$

LINEARITY

IF $x[n] \longrightarrow y[n]$

$a\,x[n] \longrightarrow a\,y[n]$ HOMOGENAITY

IF $x_1[n] \longrightarrow y_1[n]$

AND $x_2[n] \longrightarrow y_2[n]$

$\Rightarrow a x_1[n] + b x_2[n] \longrightarrow a y_1[n] + b x_2[n]$ SUPERPOSITION

} LINEAR

(LSI SYSTEMS)

SHIFT INVARIANCE

IF $x[n] \longrightarrow y[n]$

AND, $x[n-N] \longrightarrow y[n-N]$

FOR $\forall\, n, N$ (INTEGER)

THEN SYSTEM IS SHIFT INVARIAT

L, NOT SI: $y[n] = \sum_{k=0}^{n} x[k]$

LSI: $y[n] = \sum_{k=n-N}^{n+N} x[k]$

L, NOT SI

$y[n] = x[n]\cos(\omega_0 n)$

SI, NOT L

$y[n] = x^2[n]$

CAUSALITY

$x[n] \longrightarrow y[n]$

$y[n]$ DEPENDS ONLY ON
PAST + PRESENT VALUES OF $x[n]$

MEMORYLESSNESS

$y[n]$ DEPENDS ONLY ON
CURRENT VALUE of $x[n]$

STABILITY

IF $|x[n]| < \infty, \forall n$

THEN $|y[n]| < \infty, \forall n$

$$y[n] = \sum_{k=-\infty}^{n} x[k]$$

ORTHOGONAL FUNCTION EXPANSION

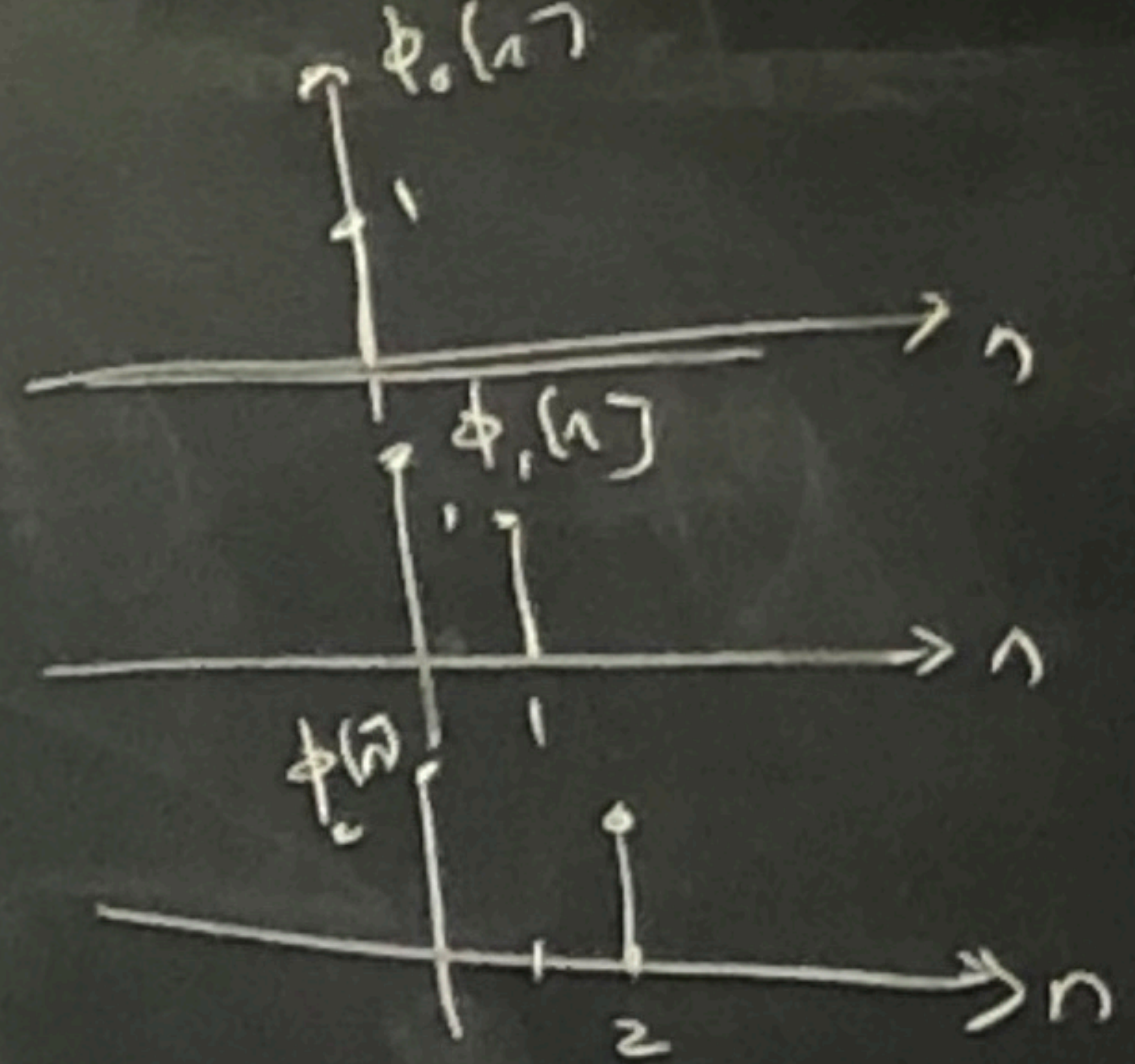
A SET OF FUNCTIONS $\{\phi_k[n]\}$ ARE ORTHONORMAL

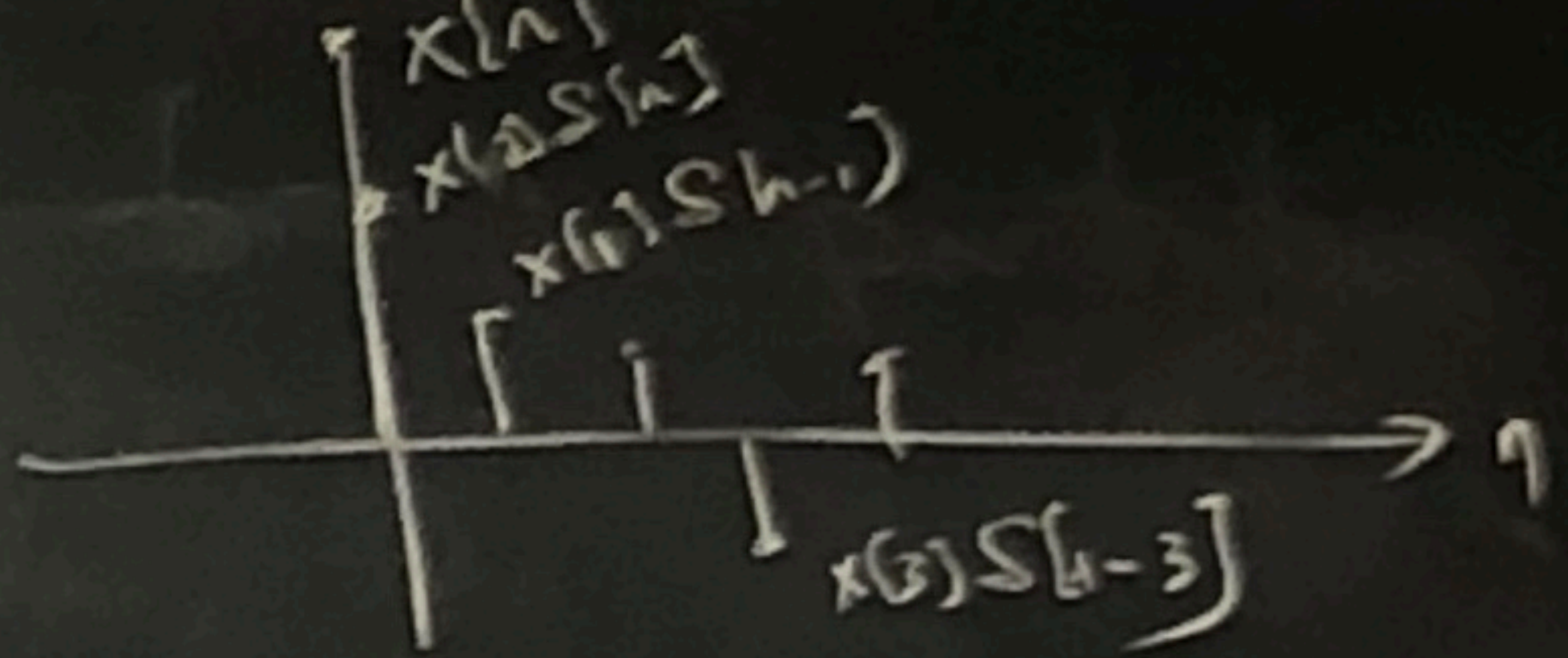
IF $$\sum_{n=-\infty}^{\infty} \phi_k[n]\,\phi_\ell^*[n] = \delta_{k\ell} = \begin{cases}1, & k=\ell\\ 0, & k\neq\ell\end{cases}$$

$$\phi_k[n]=\delta[n-k]$$

IF $$x[n]=\sum_{k=\infty}^{\infty} a_k\,\phi_k[n]$$

$$a_k=\sum_{n=\infty}^{\infty} x[n]\,\phi_k^*[n]$$





$$\phi_k[n]=\delta[n-k]$$

$$a_k=\sum_{n=-\infty}^{\infty} x[n]\cdot\phi_k^*[n] = \sum_{n=-\infty}^{\infty} x[n]\,\delta[n-k] = x[k]$$

$$x[n]=\sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k]$$

RESPONSE OF LSI SYSTEMS

CONSIDER LSI SYSTEM

$x[n] \rightarrow \boxed{} \rightarrow y[n]$

DEFINE $h[n]$ UNIT SAMPLE RESP., OUTPUT FOR $\delta[n]$

$\delta[n] \rightarrow h[n]$ DEF.

$\delta[n-k] \rightarrow h[n-k]$ SI

$x[k]\,\delta[n-k] \rightarrow x[k]\,h[n-k]$ L:H

$$x[n] = \sum_{k=-\infty}^{\infty} x[k]\,\delta[n-k] \rightarrow \sum_{k=-\infty}^{\infty} x[k]\,h[n-k] \quad \text{L:S}$$

CONVOLUTION SUM!

$$y[n] = \sum_{k=-\infty}^{\infty} x[k]\,h[n-k]$$

let $\ell = n-k$

$k = n-\ell$

$$= \sum_{\ell=-\infty}^{\infty} x[n-\ell]\,h[\ell] = \sum_{k=-\infty}^{\infty} x[n-k]\,h[k]$$

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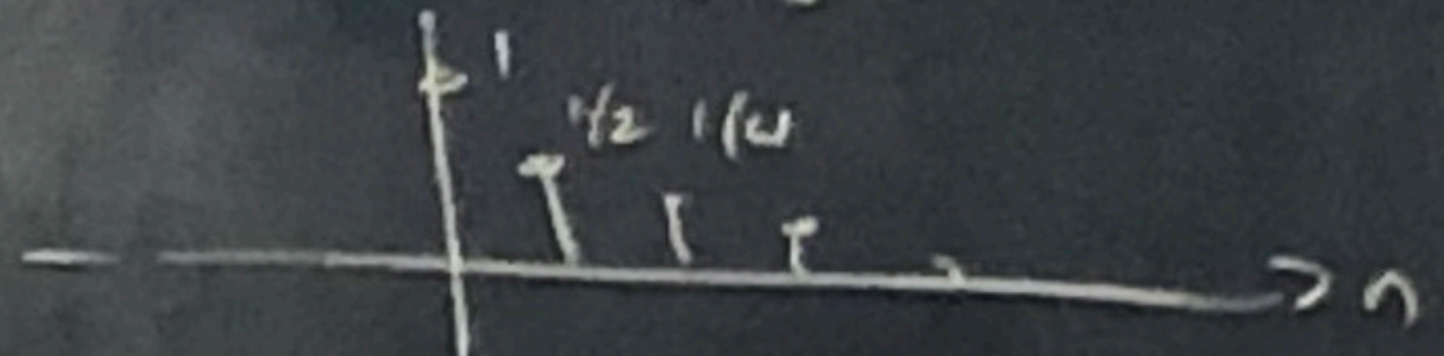
$x[n] = \left(\frac{1}{2}\right)^n u[n]$



$h[n] = u[n]$