

Digital Signal Processing (18-491/18-691)  
Spring Semester, 2024

## Selected Transform Tables

TABLE 2.1 SYMMETRY PROPERTIES OF THE FOURIER TRANSFORM

Sequence $x[n]$	Fourier Transform $X(e^{j\omega})$
1. $x^*[n]$	$X^*(e^{-j\omega})$
2. $x^*[-n]$	$X^*(e^{j\omega})$
3. $\mathcal{R}e\{x[n]\}$	$X_e(e^{j\omega})$ (conjugate-symmetric part of $X(e^{j\omega})$ )
4. $j\mathcal{I}m\{x[n]\}$	$X_o(e^{j\omega})$ (conjugate-antisymmetric part of $X(e^{j\omega})$ )
5. $x_e[n]$ (conjugate-symmetric part of $x[n]$ )	$X_R(e^{j\omega}) = \mathcal{R}e\{X(e^{j\omega})\}$
6. $x_o[n]$ (conjugate-antisymmetric part of $x[n]$ )	$jX_I(e^{j\omega}) = j\mathcal{I}m\{X(e^{j\omega})\}$
<i>The following properties apply only when <math>x[n]</math> is real:</i>	
7. Any real $x[n]$	$X(e^{j\omega}) = X^*(e^{-j\omega})$ (Fourier transform is conjugate symmetric)
8. Any real $x[n]$	$X_R(e^{j\omega}) = X_R(e^{-j\omega})$ (real part is even)
9. Any real $x[n]$	$X_I(e^{j\omega}) = -X_I(e^{-j\omega})$ (imaginary part is odd)
10. Any real $x[n]$	$ X(e^{j\omega})  =  X(e^{-j\omega}) $ (magnitude is even)
11. Any real $x[n]$	$\angle X(e^{j\omega}) = -\angle X(e^{-j\omega})$ (phase is odd)
12. $x_e[n]$ (even part of $x[n]$ )	$X_R(e^{j\omega})$
13. $x_o[n]$ (odd part of $x[n]$ )	$jX_I(e^{j\omega})$

**TABLE 2.2** FOURIER TRANSFORM THEOREMS

Sequence	Fourier Transform
$x[n]$	$X(e^{j\omega})$
$y[n]$	$Y(e^{j\omega})$
1. $ax[n] + by[n]$	$aX(e^{j\omega}) + bY(e^{j\omega})$
2. $x[n - n_d]$ ( $n_d$ an integer)	$e^{-j\omega n_d} X(e^{j\omega})$
3. $e^{j\omega_0 n} x[n]$	$X(e^{j(\omega - \omega_0)})$
4. $x[-n]$	$X(e^{-j\omega})$ $X^*(e^{j\omega})$ if $x[n]$ real.
5. $nx[n]$	$j \frac{dX(e^{j\omega})}{d\omega}$
6. $x[n] * y[n]$	$X(e^{j\omega})Y(e^{j\omega})$
7. $x[n]y[n]$	$\frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\theta})Y(e^{j(\omega - \theta)})d\theta$
Parseval's theorem:	
8. $\sum_{n=-\infty}^{\infty}  x[n] ^2 = \frac{1}{2\pi} \int_{-\pi}^{\pi}  X(e^{j\omega}) ^2 d\omega$	
9. $\sum_{n=-\infty}^{\infty} x[n]y^*[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{j\omega})Y^*(e^{j\omega})d\omega$	

**TABLE 2.3** FOURIER TRANSFORM PAIRS

Sequence	Fourier Transform
1. $\delta[n]$	1
2. $\delta[n - n_0]$	$e^{-j\omega n_0}$
3. 1 $(-\infty < n < \infty)$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega + 2\pi k)$
4. $a^n u[n]$ $( a  < 1)$	$\frac{1}{1 - ae^{-j\omega}}$
5. $u[n]$	$\frac{1}{1 - e^{-j\omega}} + \sum_{k=-\infty}^{\infty} \pi\delta(\omega + 2\pi k)$
6. $(n + 1)a^n u[n]$ $( a  < 1)$	$\frac{1}{(1 - ae^{-j\omega})^2}$
7. $\frac{r^n \sin \omega_p (n + 1)}{\sin \omega_p} u[n]$ $( r  < 1)$	$\frac{1}{1 - 2r \cos \omega_p e^{-j\omega} + r^2 e^{-j2\omega}}$
8. $\frac{\sin \omega_c n}{\pi n}$	$X(e^{j\omega}) = \begin{cases} 1, &  \omega  < \omega_c, \\ 0, & \omega_c <  \omega  \leq \pi \end{cases}$
9. $x[n] = \begin{cases} 1, & 0 \leq n \leq M \\ 0, & \text{otherwise} \end{cases}$	$\frac{\sin[\omega(M + 1)/2]}{\sin(\omega/2)} e^{-j\omega M/2}$
10. $e^{j\omega_0 n}$	$\sum_{k=-\infty}^{\infty} 2\pi\delta(\omega - \omega_0 + 2\pi k)$
11. $\cos(\omega_0 n + \phi)$	$\sum_{k=-\infty}^{\infty} [\pi e^{j\phi} \delta(\omega - \omega_0 + 2\pi k) + \pi e^{-j\phi} \delta(\omega + \omega_0 + 2\pi k)]$

**TABLE 3.1** SOME COMMON  $z$ -TRANSFORM PAIRS

Sequence	Transform	ROC
1. $\delta[n]$	1	All $z$
2. $u[n]$	$\frac{1}{1 - z^{-1}}$	$ z  > 1$
3. $-u[-n - 1]$	$\frac{1}{1 - z^{-1}}$	$ z  < 1$
4. $\delta[n - m]$	$z^{-m}$	All $z$ except 0 (if $m > 0$ ) or $\infty$ (if $m < 0$ )
5. $a^n u[n]$	$\frac{1}{1 - az^{-1}}$	$ z  >  a $
6. $-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$	$ z  <  a $
7. $na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  >  a $
8. $-na^n u[-n - 1]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$	$ z  <  a $
9. $\cos(\omega_0 n)u[n]$	$\frac{1 - \cos(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z  > 1$
10. $\sin(\omega_0 n)u[n]$	$\frac{\sin(\omega_0)z^{-1}}{1 - 2\cos(\omega_0)z^{-1} + z^{-2}}$	$ z  > 1$
11. $r^n \cos(\omega_0 n)u[n]$	$\frac{1 - r\cos(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$	$ z  > r$
12. $r^n \sin(\omega_0 n)u[n]$	$\frac{r\sin(\omega_0)z^{-1}}{1 - 2r\cos(\omega_0)z^{-1} + r^2z^{-2}}$	$ z  > r$
13. $\begin{cases} a^n, & 0 \leq n \leq N - 1, \\ 0, & \text{otherwise} \end{cases}$	$\frac{1 - a^N z^{-N}}{1 - az^{-1}}$	$ z  > 0$

**TABLE 3.2** SOME  $z$ -TRANSFORM PROPERTIES

Property Number	Section Reference	Sequence	Transform	ROC
		$x[n]$	$X(z)$	$R_x$
		$x_1[n]$	$X_1(z)$	$R_{x_1}$
		$x_2[n]$	$X_2(z)$	$R_{x_2}$
1	3.4.1	$ax_1[n] + bx_2[n]$	$aX_1(z) + bX_2(z)$	Contains $R_{x_1} \cap R_{x_2}$
2	3.4.2	$x[n - n_0]$	$z^{-n_0}X(z)$	$R_x$ , except for the possible addition or deletion of the origin or $\infty$
3	3.4.3	$z_0^n x[n]$	$X(z/z_0)$	$ z_0 R_x$
4	3.4.4	$nx[n]$	$-z \frac{dX(z)}{dz}$	$R_x$
5	3.4.5	$x^*[n]$	$X^*(z^*)$	$R_x$
6		$\text{Re}\{x[n]\}$	$\frac{1}{2}[X(z) + X^*(z^*)]$	Contains $R_x$
7		$\text{Im}\{x[n]\}$	$\frac{1}{2j}[X(z) - X^*(z^*)]$	Contains $R_x$
8	3.4.6	$x^*[-n]$	$X^*(1/z^*)$	$1/R_x$
9	3.4.7	$x_1[n] * x_2[n]$	$X_1(z)X_2(z)$	Contains $R_{x_1} \cap R_{x_2}$