Question 4: (25%): SOLUTIONS TO ORIGINAL PROBLEM WITH TYPOS

In our discussions of filter implementation we have avoided discussing filters with multiple poles in the same location. Nevertheless, multiple poles in the same location do not pose any particular problem for filter implementation.

As an example consider an LSI system with a single pole at $z = 1/4$, and a double pole at $z = 1/2$. Assume also that the zeros of the system are at $z = 3/4, -1, -2/3$ and $-1/2$. It can be shown that the transfer function of the filter can be represented as

$$H(z) = \frac{1 + 1.42z^{-1} - 0.125z^{-2} - 0.792z^{-3} - 0.25z^{-4}}{1 - 1.25z^{-1} + .5z^{-2} - 0.0625z^{-3}}$$

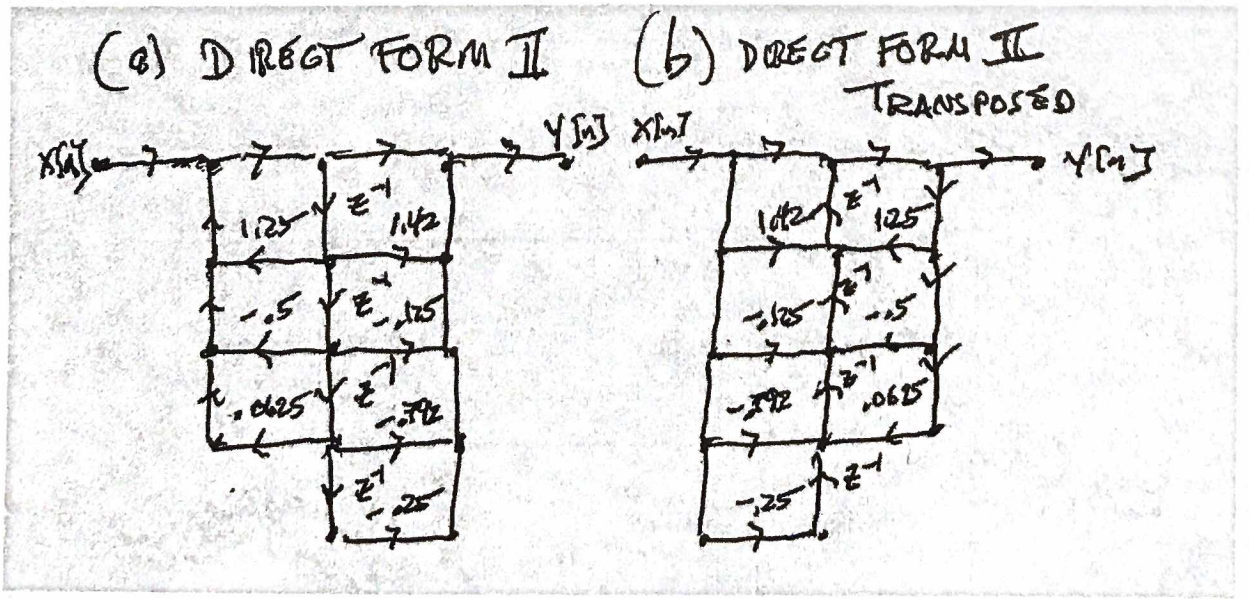
and (equivalently) as

$$H(z) = \frac{80.3}{1 - \frac{1}{2}z^{-1}} + \frac{-14}{\left(1 - \frac{1}{2}z^{-1}\right)^2} + \frac{-110}{1 - \frac{1}{4}z^{-1}} + 44.7 + 4z^{-1}$$

Note: These expressions are correct within a multiplicative gain constant by which the entire transfer function is multiplied. You are not being asked to determine the value of that constant.

(a) Using signal flow graph notation, draw the **direct form II** implementation of this filter.

(b) Using signal flow graph notation, draw the **direct form II transposed** implementation of this filter.



(c) Using signal flow graph notation, draw the **parallel form** implementation of this filter using second-order sections.

$$H(z) = \frac{80.3}{1-\frac{1}{2}z^{-1}} + \frac{-14}{\left(1-\frac{1}{2}z^{-1}\right)^2} + \frac{-110}{1-\frac{1}{4}z^{-1}} + 44.7 + 4z^{-1}$$

Let's begin by combining terms to get second-order sections in the standard form:

$$\frac{80.3}{1-\frac{1}{2}z^{-1}} + \frac{-110}{1-\frac{1}{4}z^{-1}} = \frac{80.3 - \frac{80.3}{4}z^{-1} - 110 + 55z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} = \frac{-29.7 + 34.9z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}}$$

$$\frac{-14}{\left(1-\frac{1}{2}z^{-1}\right)^2} = \frac{-14}{1-z^{-1}+\frac{1}{4}z^{-2}}$$

Hence

$$H(z) = \frac{-29.7 + 34.9z^{-1}}{1-\frac{3}{4}z^{-1}+\frac{1}{8}z^{-2}} + \frac{-14}{1-z^{-1}+\frac{1}{4}z^{-2}} + 44.7 + 4z^{-1}$$

This leads directly to the parallel form: