Digital Signal Processing (18-491/18-691)
Spring Semester, 2025

QUIZ 2

Two hours | **Closed book**
April 9, 2025 | **Two sheets of notes**

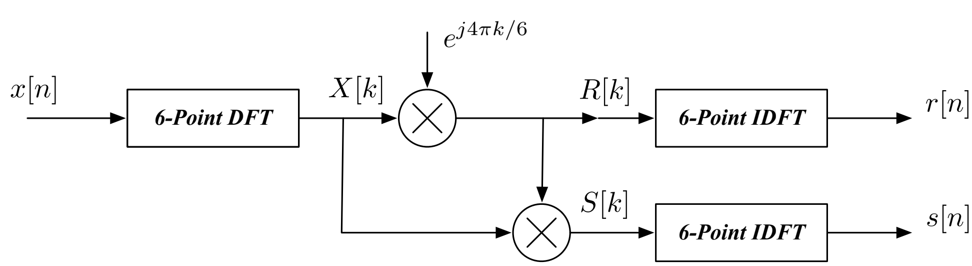
Reminder: Be sure to think about ways of answering the questions using reasoning, transform properties, etc., before resorting to brute-force solutions. Please note that the questions are ordered more or less in the sequence that the topics appeared in the course, and they may not be in order of difficulty. In addition, to the greatest extent possible, subparts of the questions will be graded independently based on your prior answers to the question.

Question 1: (25%):

(a) The periodic time function $\tilde{x}[n]$ has period 4 and is described by the following coefficients in time:

- $\tilde{x}[0] = 3$
- $\tilde{x}[1] = 2$
- $\tilde{x}[2] = 1$
- $\tilde{x}[3] = 2$

Sketch and dimension $\tilde{X}(e^{j\omega})$, the DTFT of the periodic function $\tilde{x}[n]$ in terms of its DTFS coefficients $\tilde{X}[k]$. (You do not need to calculate the coefficients $\tilde{X}[k]$ explicitly.)



(b) The function $x[n]$ is finite in duration and defined by

$$x[n] = \begin{cases} \tilde{x}[n], & 0 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

The function $x[n]$ undergoes the processing depicted above. Specifically, we observe that:

- $X[k]$ is the 6-point DFT of $x[n]$
- $R[k] = X[k]e^{j4\pi k/6}$
- $S[k] = R[k]X[k]$
- $r[n]$ is the 6-point IDFT of $R[k]$
- $s[n]$ is the 6-point IDFT of $S[k]$

1. Sketch and dimension $r[n]$, the 6-point IDFT of $R[k]$
2. Sketch and dimension $s[n]$, the 6-point IDFT of $S[k]$

Question 2: (30%):

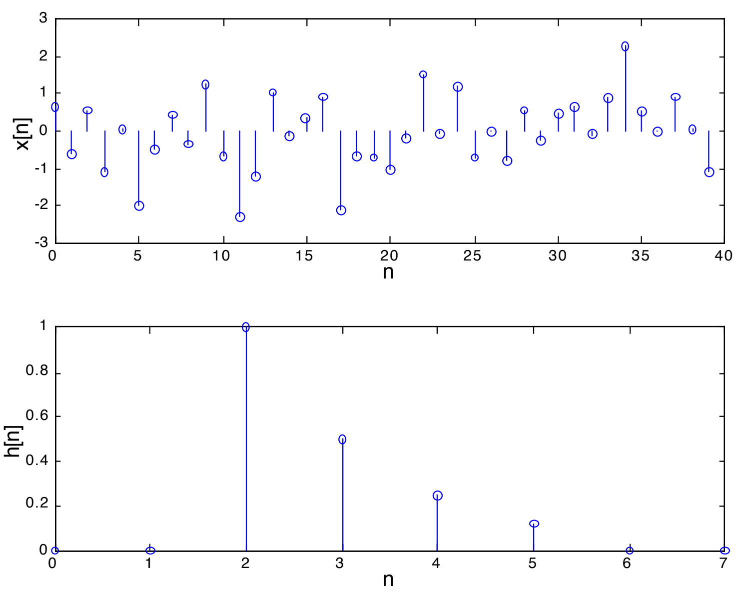


Figure 2. Input and unit sample response of a discrete-time filter.

Figure 2 above shows the input $x[n]$ and unit sample response $h[n]$ of a simple discrete-time system. Note that $x[n]$ is nonzero only for $0 \leq n \leq 39$. Although $h[n]$ is defined for $0 \leq n \leq 7$, this function is nonzero only for $2 \leq n \leq 5$. The specific numerical values of $x[n]$ and $h[n]$ are unimportant for the purposes of this question.

(a) Consider first $y[n]$, the result of the *linear* convolution of $x[n]$ and $h[n]$. For exactly which values of n will $y[n]$ be nonzero?

(b) Now suppose that we compute $X[k]$ and $H[k]$, the 40-point DFTs of $x[n]$ and $h[n]$. (The 40-point DFT of $h[n]$ is obtained by appending an appropriate number of zeros to the original time function.) Let $y_2[n]$ be the inverse 40-point DFT of the product of $X[k]$ and $H[k]$. For which values of n will $y_2[n]$ be equal to $y[n]$?

(c) We would now like to obtain the linear convolution of $x[n]$ and $h[n]$ using an implementation of the overlap-add method. Our implementation includes the following steps:

- Segment $x[n]$ into two or more abutting sections of length 20.
- Extract the first 6 points of $h[n]$ (which include all of the nonzero points of the function).

As usual with the overlap-add method, sequences of output samples are generated by obtaining the circular convolution of $h[n]$ with the abutting sections of $x[n]$, and then adding the results obtained from these convolutions together.

1. What is the minimum length of the DFT needed in order to obtain the circular convolution of $h[n]$ with the sections of $x[n]$ without having the results corrupted by temporal aliasing? Assume that zeros are appended to both $h[n]$ and the abutting sections of $x[n]$ before computing their DFTs.
2. How many sections of $x[n]$ will be needed to obtain a complete result that is equal to $y[n]$?
3. How are the output samples combined? Be specific, stating the manipulations of inputs and outputs in terms of the values of the time index n.

Question 3: (20%):

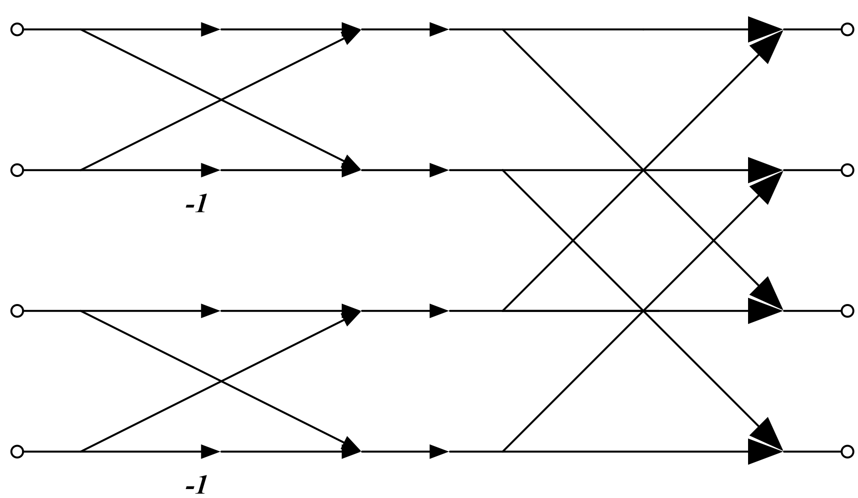


Figure 3. Signal flowgraph for a 4-point DFT.

The figure above depicts an odd implementation of a 4-point decimation-in-time FFT structure. Please copy this simple figure onto your answer pages.

(a) The inputs to the structure consist of the samples in time, $x[0]$ through $x[3]$. Mark your figure to indicate which input line receives which input sample.

(b) The outputs of the structure consist of the DFT coefficients $X[0]$ through $X[3]$. Mark your figure to indicate which output line corresponds to which DFT coefficient.

(c) Indicate the values of each of the multiplicative factors W_N^{nk} that are anything other than 1 by marking the value of the multiplicative factors by the appropriate arrows. Note that because this is a 4-point DFT, all twiddle factors will be equal to ± 1 or $\pm j$.

Question 4: (25%):

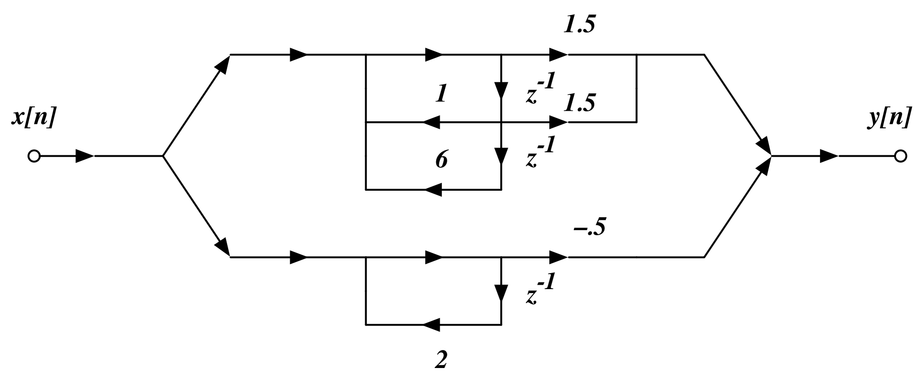


Figure 4.1. IIR filter structure.

(a) The signal flow graph above depicts a simple structure for a causal IIR filter.

1. What is the difference equation that specifies this filter? (You should be able to obtain this by hand calculations without much difficulty.)
2. Sketch the transposed form of the direct-form II implementation of this filter.
3. Is the filter stable? Why or why not?
4. Obtain the locations of the filter's poles and zeros.
5. Sketch the magnitude of the filter's transfer function $H(e^{j\omega})$.

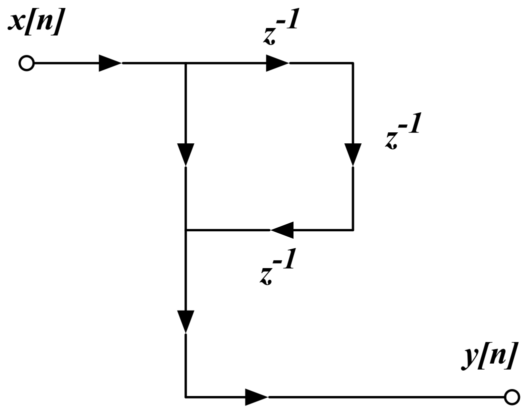


Figure 4.2. FIR filter structure.

(b) The signal flow graph above depicts the implementation of an FIR filter.

1. Write the difference equation that specifies this system.
2. Sketch the implementation of this system in the direct form. Use real coefficients only to obtain maximum credit.