Carnegie Mellon — DSP — Electrical & Computer ENGINEERING

Digital Signal Processing (18-491/18-691)
Spring Semester, 2023

QUIZ 2

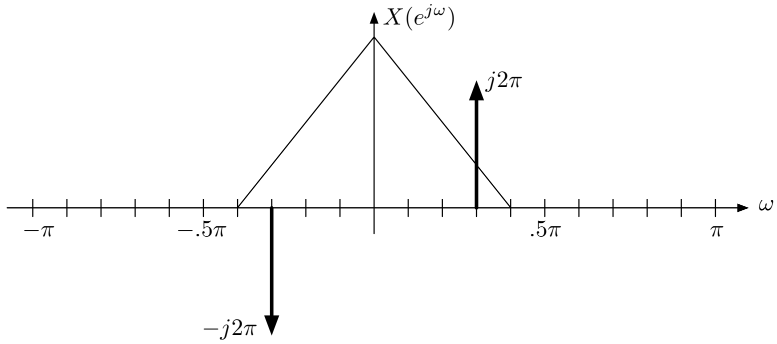
Two hours — **Closed book**
April 5, 2023 — **One sheet of notes**

We list below the ground rules governing the quiz:

- The exam is open book, and you can refer to your own class notes as well, as well as the class notes and text for 18-290 (or a similar pre-requiste signals and systems course taken elsewhere). Reading notes from the screen is also permitted. You are not allowed to search the internet for help in answering the questions. While this is difficult to enforce, the internet is unlikely to be helpful on this exam, and you will be given no credit for insights that appear to come from sources other than the course materials.
- Similarly, you are not allowed to use MATLAB to obtain your solutions, and it will not be helpful in any case. You may use the calculator on your telephone if you wish. Please keep your phones on airplane mode. Texting and phone calls during the exam are strictly prohibited.

Reminder: Be sure to think about ways of answering the questions using reasoning, transform properties, etc., before resorting to brute-force solutions. Please note that the questions are ordered more or less in the sequence that the topics appeared in the course, and they may not be in order of difficulty. In addition, to the greatest extent possible subparts of the questions will be graded independently based on your prior answers to the question.

Question 1: (25%):

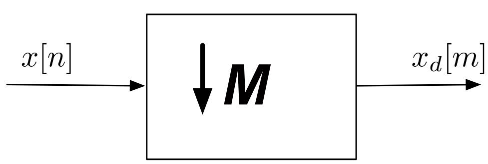


The figure above depicts the spectrum for a signal $[n]$ that contains both periodic and aperiodic components. The DTFT of the signal for $|\omega| \leq \pi$ is

$$X(e^{j\omega}) = j2\pi\ \delta(\omega - 0.3\pi) - j2\pi\delta(\omega + 0.3\pi) + \begin{cases} 1 - \frac{|\omega|}{0.4\pi}, & |\omega| \leq 0.4\pi \\ 0, & \text{otherwise} \end{cases}$$

(a) State whether the time function $x[n]$ is purely even, odd, or neither. You must explain your reasoning to receive full credit.

(b) State whether the time function $x[n]$ is purely real, imaginary, or neither. You must explain your reasoning to receive full credit.



(c) The function $x[n]$ is now downsampled by a factor of M, where $x_d[m] = x[mM]$.

Carefully sketch and dimension $X_d(e^{j\omega'})$, the DTFT of $x_d[m]$, for $M = 4$. Be sure to provide the values of all critical amplitudes, areas, and frequencies.

(d) It is stated that the periodic component of $x_d[n]$ can be written in the form of $A\cos(\omega_0 n + \phi)$. Obtain values for the parameters A, ω_0, and ϕ for $M = 4$.

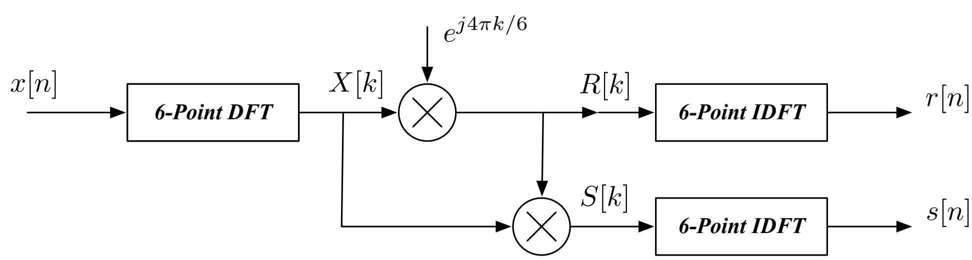
(e) What is the largest integer value of the downsampling ratio M for which the time original function $x[n]$ could be recovered from $x_d[m]$ via upsampling without incurring the effects of aliasing?

Question 2: (25%):

(a) The periodic time function $\tilde{x}[n]$ has period 4 and is described by the following coefficients in time:

- $\tilde{x}[0] = 3$
- $\tilde{x}[1] = 2$
- $\tilde{x}[2] = 1$
- $\tilde{x}[3] = 2$

Sketch and dimension $\tilde{X}(e^{j\omega})$, the DTFT of the periodic function $\tilde{x}[n]$ in terms of its DTFS coefficients $\tilde{X}[k]$. (You do not need to calculate the coefficients $\tilde{X}[k]$ explicitly.)



(b) The function $x[n]$ is finite in duration and defined by

$$x[n] = \begin{cases} \tilde{x}[n], & 0 \leq n \leq 3 \\ 0, & \text{otherwise} \end{cases}$$

The function $x[n]$ undergoes the processing depicted above. Specifically, we observe that:

- $X[k]$ is the 6-point DFT of $x[n]$
- $R[k] = X[k]e^{j4\pi k/6}$
- $S[k] = R[k]X[k]$
- $r[n]$ is the 6-point IDFT of $R[k]$
- $s[n]$ is the 6-point IDFT of $S[k]$

1. Sketch and dimension $r[n]$, the 6-point IDFT of $R[k]$
2. Sketch and dimension $s[n]$, the 6-point IDFT of $S[k]$

Question 3: (25%):

The figure on the last page depicts a mixed-radix 8-point decimation-in-time FFT structure. Although it is hard to see it in this fashion, the right column consists of two overlaid 4-by-4 butterfly structures, one leading to the output points with even indices and one leading to the output points with odd indices. Please detach the last page of this exam, put your name on it, and write your answers directly on the sheet for this question. Turn in this page with your answer book.

(a) The inputs to the structure consist of the samples in time, $x[0]$ through $x[7]$. Mark the diagram to indicate which input line receives which input sample.

(b) The outputs of the structure consist of the DFT coefficients $X[0]$ through $X[7]$. Mark the diagram to indicate which output line corresponds to which DFT coefficient.

(c) The large arrows at the end of the right column and between the two column indicate multiplys by the twiddle factors of the form $W_N^{nk} = e^{-j2\pi nk/N}$. Indicate the values of each of these factors W_N^{nk} that are not equal to 1. If any of the factors W_N^{nk} are equal to ± 1 or $\pm j$ be sure to express them in this form. You will receive maximum credit for the realization that produces a minimum number of factors that are other than ± 1 or $\pm j$.

Again, remember to detach the page with your answer and turn it in with your exam book.

Question 4: (25%):

In our discussions of filter implementation we have avoided discussing filters with multiple poles in the same location. Nevertheless, multiple poles in the same location do not pose any particular problem for filter implementation.

As an example consider an LSI system with a single pole at $z = 1/4$, and a double pole at $z = 1/2$. Assume also that the zeros of the system are at $z = 3/4, -1, -2/3$ and $-1/2$. It can be shown that the transfer function of the filter can be represented as

$$H(z) = \frac{1 + 1.42z^{-1} - 0.125z^{-2} - 0.792z^{-3} - 0.25z^{-4}}{1 - 1.25z^{-1} + .5z^{-2} - 0.062z^{-3}}$$

and (equivalently) as

$$H(z) = \frac{80.3}{1 - \frac{1}{2}z^{-1}} + \frac{-14}{\left(1 - \frac{1}{2}z^{-1}\right)^2} + \frac{-110}{1 - \frac{1}{4}z^{-1}} + 44.7 + 4z^{-1}$$

Note: These expressions are correct within a multiplicative gain constant by which the entire transfer function is multiplied. You are not being asked to determine the value of that constant.

(a) Using signal flow graph notation, draw the **direct form II** implementation of this filter.

(b) Using signal flow graph notation, draw the **direct form II transposed** implementation of this filter.

(c) Using signal flow graph notation, draw the **parallel form** implementation of this filter using second-order sections.

Name ________________________________