Carnegie Mellon DSP Electrical & Computer ENGINEERING

Digital Signal Processing (18-491/18-691)
Spring Semester, 2024

QUIZ 1

Two hours
February 28, 2024

Open book
One sheet of notes

We list below the ground rules governing the quiz:

- The quiz is in open book format. You may refer to any edition of Oppenheim and Schafer (including OSYP of course), as well as your own notes from class. Nothing else. (Specifically, you are not allowed to look at the solutions to the problem sets and prior quizzes.) You may use a .pdf copy of the text on your laptop or tablet, but the device must be on airplane mode.
- Please quit all web browsers. If it looks like you may be consulting the web, your exam will be confiscated immediately and you will be disqualified from the exam. But don't worry, the web will not be of any help on this exam.
- You can use your phone as a calculator but you may not use MATLAB in any form to work problems on the exam. (You will probably not have much need for a calculator, if any.) Please keep all phones and calculators in airplane mode.

Reminder: Be sure to think about ways of answering the questions using reasoning, transform properties, etc., before resorting to brute-force solutions. Please note that the questions are ordered more or less in the sequence that the topics appeared in the course, and they may not be in order of difficulty. In addition, to the greatest extent possible subparts of the questions will be graded independently based on your prior answers to the question.

Question 1: (35%):

A causal LSI system is described by the difference equation

$$y[n] = \frac{3}{2}y[n-1] + y[n-2] + x[n] - \frac{1}{2}x[n-1]$$

(a) It is asserted that this system is unstable. How can we know this from the information given?

(b) Using any method you wish, obtain a closed-form analytical expression for the system's unit sample response $h[n]$. (This means that you may not use iterative techniques such as direct iteration of the difference equation or long division of the transfer function to obtain your result.)

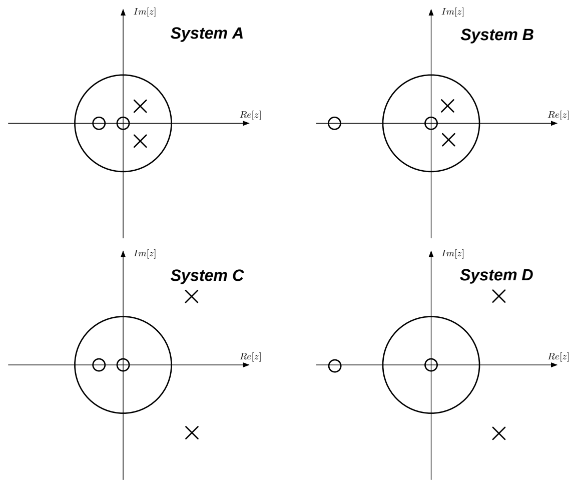
(c) What are the initial conditions of the difference equation that are needed to constrain the system if the input arrives at $n = 0$?

(d) Using any method you wish, obtain a closed-form expression for the output of the system for each of the following inputs:

1. $x_1[n] = (1/3)^n u[n]$
2. $x_2[n] = \cos(2\pi 500 n)$ for all n

(e) Let $H(z) = Y(z)/X(z)$ be the transfer function of the system. Obtain the transfer function $H_2(z)$ for the simplest possible system that is stable and for which the magnitude of $H_2(e^{j\omega}) = H(z)|_{z=e^{j\omega}}$ is the same as the magnitude of $H(e^{j\omega})$.

Question 2: (30%):



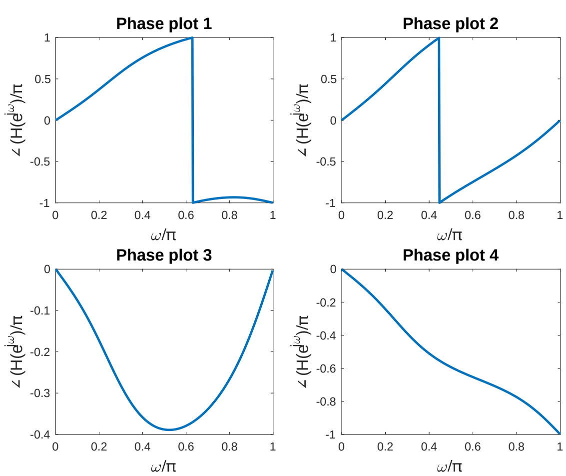
The figure above depict the pole-zero plots of four stable systems. In each case, the poles and zeros have magnitudes that are either 0, 0.5, or 2, and phase angles that are either $\pm\pi/4$ or π radians. It can easily be shown that the magnitude of the frequency response of all four systems is the same.

(a) Which of the systems (if any) are causal? Explain your reasoning.

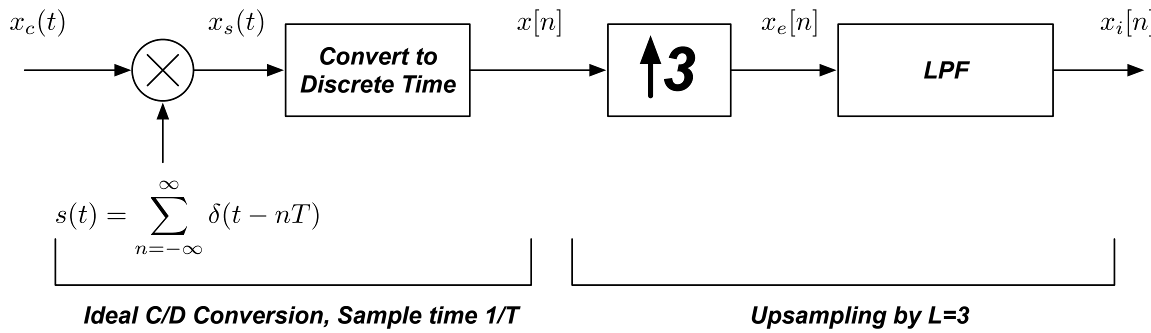
(b) Which of the systems (if any) are minimum phase? Explain your reasoning.

(c) Which of the systems (if any) are linear phase? Explain your reasoning.

(d) The figure below depicts the phase response of the four systems. Note that the axes are normalized so that the maximum frequencies and phases are 1. Identify which system corresponds to each phase response. You must explain your reasoning to receive full credit.



Question 3 (35%):



The block diagram above depicts a simple multi-rate sampling system. The input to the system is the continuous-time signal $x_c(t) = \sin((2\pi)(1.2)(10^4)t)$. The input $x_c(t)$ is multiplied by the continuous-time sampling signal

$$s(t) = \sum_{n=-\infty}^{\infty} \delta(t - nT)$$

where $1/T = (2)(10^4)$. The intermediate function $x_s(t)$ is a train of continuous-time delta functions

$$x_s(t) = \sum_{n=-\infty}^{\infty} x_c(nT)\delta(t - nT)$$

The function $x[n]$ is a discrete-time train of delta functions:

$$x[n] = \sum_{r=-\infty}^{\infty} x_c(rT)\delta(n - r)$$

The signal $x_{[}n] = x[n/3]$ for $n/3$ integer and zero otherwise. The signal $x_e[n]$ is passed through an ideal filter with gain 3 and cutoff frequency $\omega_c = \pi/3$.

(a) Sketch and dimension $X(j\Omega)$, the CTFT of $x_c(t)$.

(b) Sketch and dimension $X_s(j\Omega)$, the CTFT of $x_s(t)$.

(c) Sketch and dimension $X(e^{j\omega})$, the DTFT of $x[n]$.

(d) Sketch and dimension $X_e(e^{j\omega})$, the DTFT of $_e[n]$.

(e) Sketch and dimension $X_i(e^{j\omega})$, the DTFT of $x_i[n]$.

(f) It is said that $x_i[n]$ can be expressed as $x[_i[n] = A\cos(\omega_0 n + \phi)$. Obtain numerical values for the parameters A, ω_0, and ϕ.