Carnegie Mellon

DSP

Electrical & Computer ENGINEERING

Digital Signal Processing (18-491/18-691)
Spring Semester, 2023

QUIZ 1

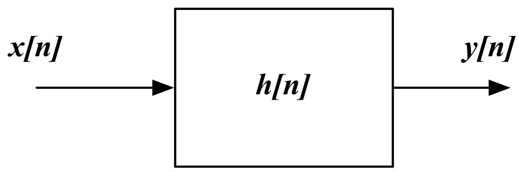
Two hours | **Closed book**
March 1, 2023 | **One sheet of notes**

We list below the ground rules governing the quiz:

- The exam is open book, and you can refer to your own class notes as well, as well as the class notes and text for 18-290 (or a similar pre-requiste signals and systems course taken elsewhere). Reading notes from the screen is also permitted. You are not allowed to search the internet for help in answering the questions. While this is difficult to enforce, the internet is unlikely to be helpful on this exam, and you will be given no credit for insights that appear to come from sources other than the course materials.
- Similarly, you are not allowed to use MATLAB to obtain your solutions, and it will not be helpful in any case. You may use the calculator on your telephone if you wish. Please keep your phones on airplane mode. Texting and phone calls during the exam are strictly prohibited.

Reminder: Be sure to think about ways of answering the questions using reasoning, transform properties, etc., before resorting to brute-force solutions. Please note that the questions are ordered more or less in the sequence that the topics appeared in the course, and they may not be in order of difficulty. In addition, to the greatest extent possible subparts of the questions will be graded independently based on your prior answers to the question.

Question 1: (30%):



An LSI system with unit sample response $h[n]$ has input $x[n]$ and output $y[n]$. We know the following facts about the system and its inputs and outputs:

- The system $h[n]$ is stable.
- The transfer function of the system is

$$H(z) = \frac{z^{-3}}{1 - \frac{1}{2}z^{-1}}$$

- The input function $x[n]$ has finite energy, or

$$\sum_{n=-\infty}^{\infty} |x[n]|^2 < \infty$$

- $Y(z)$, the z-transform of the output function $y[n]$ is

$$Y(z) = \frac{z^{-3}\left(1 + \frac{1}{2}z^{-1}\right)}{\left(1 - \frac{1}{2}z^{-1}\right)(1 + 3z^{-1})} = \frac{z^{-3}\left(1 + \frac{1}{2}z^{-1}\right)}{1 + \frac{5}{2}z^{-1} - \frac{3}{2}z^{-2}}$$

(a) Obtain a closed-form expression for the unit sample response $h[n]$ using any means you wish.

(b) Obtain a closed-form expression for the input function $x[n]$ using any means you wish.

(c) Obtain a closed-form expression for the output function $y[n]$ using any means you wish.

Question 2: (35%):

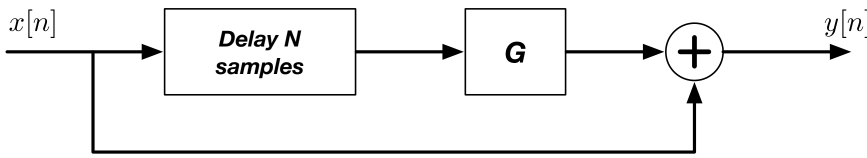


Figure 2.1. Block diagram of a simple system to implement comb filtering.

The diagram above depicts a form of **comb filter**. The comb filter has multiple uses and is part of systems used to implement popular audio processing algorithms such as flanging and chorus.

The comb filter implementation consists of an ideal delay of N samples, an ideal linear amplifier with gain G, and a summer which simply adds its two inputs.

(a) Sketch and dimension the unit sample response of the system depicted in Fig. 2.1.

(b) Write the difference equation that expresses $y[n]$ as a function of $x[n]$.

(c) Using any method, obtain a closed-form expression for $H(e^{J\omega})$, the DTFT of the unit sample response $h[n]$

(d)

1. Using any method obtain a closed-form expression for $H(z)$, the z-transform of $h[n]$.
2. Sketch the locations of the poles and zeros of the system depicted in Fig. 2.1. You may wish to make use of the relationship that the solution to the equation $z^N = G$ is $z = G^{1/N} e^{j2\pi k/N}$ for $0 \leq k \leq N-1$.
3. For what values of G is the system $H(z)$ stable?
4. Now consider the locations of the poles and zeros of the **inverse** system $H_I(z) = \frac{X(z)}{Y(z)}$. For what values of G is the inverse system $H_I(z)$ stable?

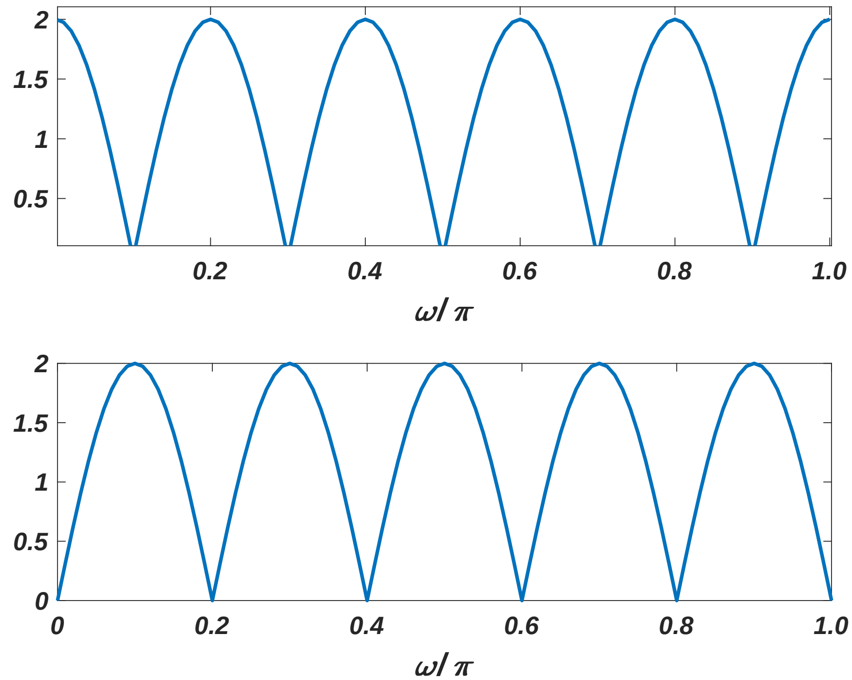


Figure 2.2. Potential comb filter frequency responses.

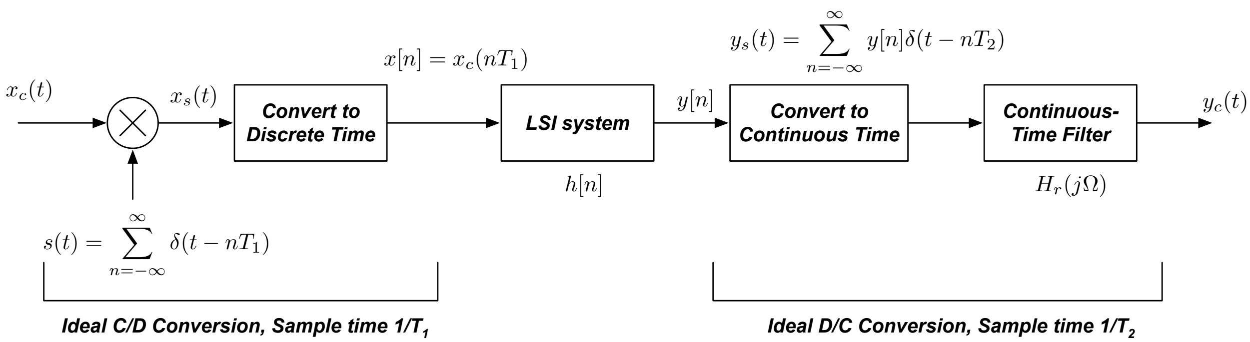
(e) Figure 2.2 depicts two functions, one of which is the magnitude of $H(e^{j\omega})$, the DTFT of the unit sample response $h[n]$. In answering the questions of this section, assume that $G = 1$.

1. Which of the panels (upper or lower) in Fig. 2.2 accurately depicts $|H(e^{j\omega})|$, the magnitude of $H)(e^{j\omega})$? You must explain your reasoning to obtain full credit.

2. From examining Fig. 2.2, what do you believe is the value of the parameter N, the num ber of samples by which the input is delayed in the upper branch? You must explain your reasoning to receive full credit.

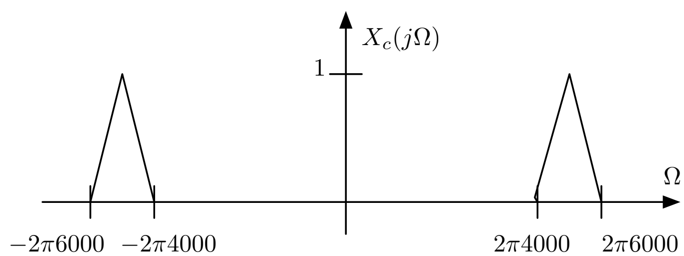
Sketch the locations of the poles and zeros of the system depicted in Fig. 2.1. You may wish to make use of the relationship that the solution to the equation $z^N = 1$ is $z = e^{j2\pi k/N}$ for $0 \le k \le N - 1$.

Question 3: (35%):



In this problem we reconsider some aspects of *bandpass sampling.* The system block diagram above depicts a fairly straightforward sampling system. Note that T_1, the sampling period for C/D conversion, may be different from T_2, the sampling period for D/C conversion.

Consider a bandpass signal with the spectrum depicted below:



Note that the input $x_c(t)$ has a spectrum $X_c(j\Omega)$ that is zero for $|\Omega| < 2\pi 4000$ and for $|\Omega| > 2\pi 6000$, with continuous-time frequencies specified in radians per second. Because $x[n]$ has a bandpass rather than lowpass spectrum, we can use a lower sampling frequency than we would have needed if the function were lowpass in nature with the same maximum frequency.

Let us begin by assuming that the transfer function of the LSI system,

$$H(e^{j\omega}) = \frac{Y(e^{j\omega})}{X(e^{j\omega})} = 1$$

(a) What is the largest value of T_1 in seconds for which $x_c(t)$ can be sampled and reconstructed without distortion, so that $y_c(t) = x_c(t)$?

(b) Sketch and dimension $X_s(j\Omega)$ the *continuous-time* Fourier transform of $x_s(t)$ as depicted in the system block diagram above using the value of T_1 you selected in part(a).

(c) Sketch and dimension $X(e^{j\omega})$ the *discrete-time* Fourier transform of $x[n]$ as depicted in the system block diagram above using the value of T_1 you selected in part(a).

(d) Sketch and dimension $H_r(j\Omega)$, the transfer function of the continuous-time reconstruction filter that would be used to recover the original signal given the value of T_1 that you specified in part (a), assuming that $T_2 = T_1$.

Now assume that the $H(e^{j\omega})$, the transfer function of the LSI system, is unspecified but not necessarily equal to 1.

(e) Express your answers to the questions below in terms of $H(e^{j\omega})$.

1. What is the transfer function of the complete continuous-time system $H_{eff}(j\Omega) = Y_c(j\Omega)/X_c(j\Omega)$ for $T_2 = T_1$?

2. What is the transfer function of the complete continuous-time system $H_{eff}(j\Omega) = Y_c(j\Omega)/X_c(j\Omega)$ for $T_2 = 2T_1$?