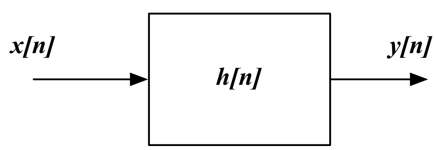
(Sample) 18-491/691 IN-CLASS QUIZ 2

10 minutes | **Closed book**
September 7, 2026 | **One sheet of notes**

Name:______________________________________

Note: This exam is intended to be trivial for students who have worked on Problem Set 2. Full access to OSYP Tables 2.1, 2.2, and 2.3 is provided. Write your answers on the space below and on the reverse side.



A discrete-time signal $x[n]$ is input to an LSI system with unit sample response $h[n]$. The output is $y[n]$.

Question 2.1: (50%)

The input function is

$$x[n] = \left(\frac{1}{2}\right)^{(n-3)} u[n-3]$$

Obtain a closed-form expression for $X(e^{j\omega})$, the DTFT of $x[n]$, either by direct summation or by using examples and properties from the tables provided. Be sure to show your work.

Question 2.2: (50%)

The unit sample response of the LSI filter is

$$h[n] = \frac{\sin(0.8\pi n)}{\pi n}$$

Write an expression for $Y(e^{j\omega})$, the DTFT of the system output $y[n]$, for $-\pi \leq \omega \leq \pi$.

Answers:

Question 2.1:

Direct summation:

$$X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} \left(\frac{1}{2}\right)^{(n-3)} u[n-3]e^{-j\omega n} = \sum_{n=3}^{\infty} \left(\frac{1}{2}\right)^{(n-3)} e^{-j\omega n}$$

Letting $l = n-3; n = l+3$, we obtain

$$X(e^{j\omega}) = \sum_{l=0}^{\infty} \left(\frac{1}{2}\right)^{(l+3-3)} e^{-j\omega(l+3)} = e^{-j3\omega} \sum_{l=0}^{\infty} \left(\frac{1}{2}e^{-j\omega}\right)^{l} = \frac{e^{-j3\omega}}{1-\frac{1}{2}e^{-j\omega}}$$

Solution based on properties:

Direct table lookup will verify that

$$\left(\frac{1}{2}\right)^{n} u[n] \Leftrightarrow \frac{1}{1-\frac{1}{2}e^{-j\omega}}$$

and that the delay by 3 samples corresponds to multiplying the DTFT by $e^{-j3\omega}$.

Question 2.2:

Direct table lookup will show that the DTFT of the unit sample response $h[n]$ that is given is

$$H(e^{j\omega}) = \begin{cases} 1, & |\omega| \leq 0.8\pi \\ 0, & -.8\pi < |\omega| \leq \pi \end{cases}$$

Hence, the answer would be

$$Y(e^{j\omega}) = X(e^{j\omega})H(e^{j\omega}) = \frac{e^{-j3\omega}}{1-\frac{1}{2}e^{-j\omega}} \text{ for } |\omega| \leq 0.8\pi \text{ and}$$

$$Y(e^{j\omega}) = 0 \text{ for } 0.8\pi < |\omega| \leq \pi$$