Carnegie Mellon DSP Electrical & Computer ENGINEERING

Digital Signal Processing (18-491/18-691)
Spring Semester, 2025

Problem Set 8

Issued: 3/20/25

Due: 3/28/25 at 0100 via Gradescope

Reminder: Quiz 2 will take place in class on April 9. More details to follow next week.

As with Quiz 1, Quiz 2 will be administered as a **closed-book** exam. Each student will be provided with transform tables from OSYP, and each student may bring two pages of paper with notes (US letter or A4 size), written on both sides.

Reading: This problem set covers IIR and FIR filter implementation. The relevant sections of OSYP are Secs. 6.0 to 6.5. We are also likely to return to Sec. 6.6 in our discussion of linear prediction at the end of the semester. Next week we will begin a discussion of digital filter design, following Secs. 7.0 to 7.4.

Problem 8.1: An IIR discrete-time digital filter has poles at $1/2$, $-2/3$, $e^{j\pi/4}$, and $e^{-j\pi/4}$ and zeros at -1, $-3/2$, $e^{j\pi/2}$, and $e^{-j\pi/2}$. The magnitude of the transfer function of the filter at $\omega = 0$ or $z = 1$ equals 1.

Sketch the signal flowgraph diagram for the following implementations of this filter.

(a) direct form II

(b) the cascade form

(c) the parallel form

(d) the transposed version of the parallel form

Note: You are encouraged to use MATLAB for calculations anywhere you wish on this problem set. The commands `roots`, `poly` and `residuez` are likely to be especially useful.

Problem 8.2: Problem 6.1 in OSYP. Because the answers are in the back of the book, you must show your work to receive full credit for this problem. Please note that the structure in Fig. P6.1(b) is called a *coupled-form oscillator*. It degrades more gracefully when coefficients are quantized than the conventional direct-form does for second-order systems.

Problem 8.3:

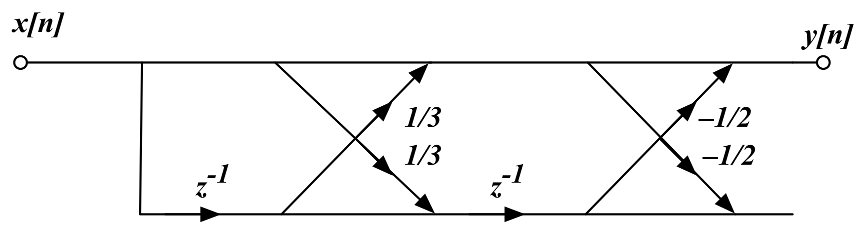

Figure 8.3. Lattice filter structure.

Figure 8.3 above depicts an implementation of a structure known as a lattice filter. (There are also other types of lattice filters.) If time permits we will discuss lattice filtering in terms of linear prediction at the end of the course. If not, lattice structures will be treated in detail in 18-792. (They are also discussed superficially in OSYP Sec. 6.6.) You already have all the information you need to answer the questions below about lattice filters, even though we have not studied them formally yet. (In other words, you do not need to refer to OSYP Sec. 6.6 or elsewhere to answer these questions.)

(a) Is this filter IIR or FIR? Why? If it FIR, what is the length of the unit sample response?

(b) Is the filter stable or unstable?

(c) Draw the signal-flow diagram of the equivalent filter in direct form. This is easily accomplished by tracing the various signal flow for each given possible amount of delay.

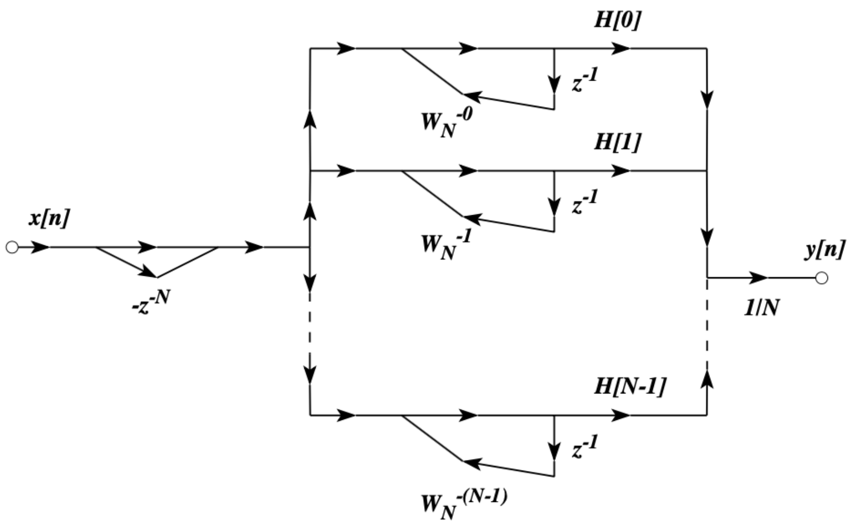

Figure 8.4a. Frequency-sampling FIR filter implementation.

Problem 8.4: (an old quiz problem)

We discussed in class the *frequency-sampling implementation* of FIR discrete-time filters. As you may recall, this form was obtained by expressing $H(z)$ as

$$H(z) = (1 - z^{-N})\frac{1}{N}\sum_{k=0}^{N-1}\frac{H[k]}{1 - W_N^{-k}z^{-1}} \tag{1}$$

which implies the filter structure shown in Fig. 8.4a.

If N is odd, Eq. (1) can be written as

$$H(z) = (1 - z^{-N})\frac{1}{N}\frac{H[0]}{1 - z^{-1}} + (1 - z^{-N})\frac{1}{N}\sum_{k=1}^{(N-1)/2}\left[\frac{H[k]}{1 - W_N^{-k}z^{-1}} + \frac{H[N-k]}{1 - W_N^{-(N-k)}z^{-1}}\right] \tag{2}$$

(a) Obtain expressions for $H[N-k]$ in terms of $H[k]$, and $W_N^{-(N-k)}$ in terms of W_N^{-k}, that are valid for $h[n]$ real.

(b) Let us now assume that $h[n]$ actually is real, as it inevitably is in this course. By applying the symmetry properties you developed in part (a) to Eq. (2), it can be shown that it is possible to realize Eq. (2) as a discrete-time filter structure of the form shown in Fig. 8.4b on the next page, where all of the coefficients a_{1k}, a_{2k}, b_{0k}, and b_{1k} are real. Obtain expressions for the coefficients a_{1k}, a_{2k}, b_{0k}, and b_{1k} in terms of $H[k]$, assuming that $h[n]$ is real.

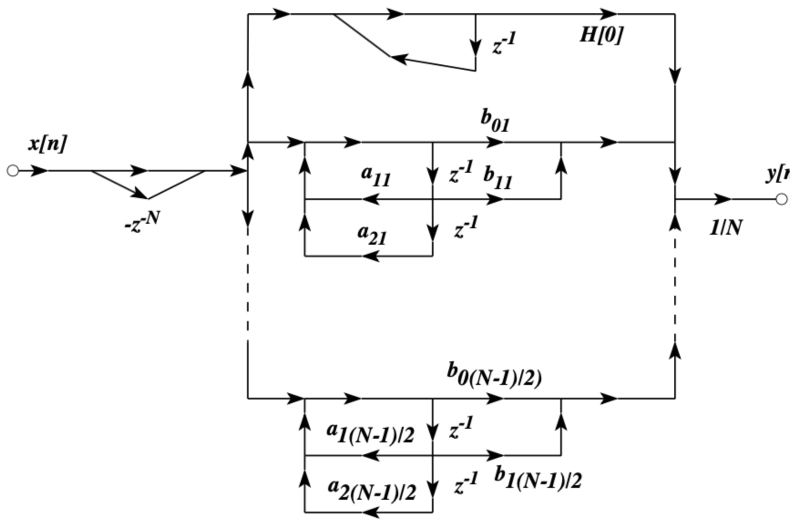

Figure 8.4b. Frequency-sampling FIR filter implementation with real coefficients.

Problem 8.5: A linear-phase FIR filter has the following unit sample response:

- $h[0] = 1$
- $h[1] = 2$
- $h[2] = -3$
- $h[3] = -3$
- $h[4] = 2$
- $h[5] = 1$

Sketch the signal flowgraph diagram for the following implementations of this filter:

(a) the direct form

(b) the cascade form

(c) the linear-phase form

(d) the frequency-sampling form (Use the method discussed in Problem 8.4 to obtain real coefficients)

MATLAB Problems

No MATLAB problems this week!