9.6. Figure P9.6 shows the graph representation of a decimation-in-time FFT algorithm for $N = 8$. The heavy line shows a path from sample $x[7]$ to DFT sample $X[2]$.

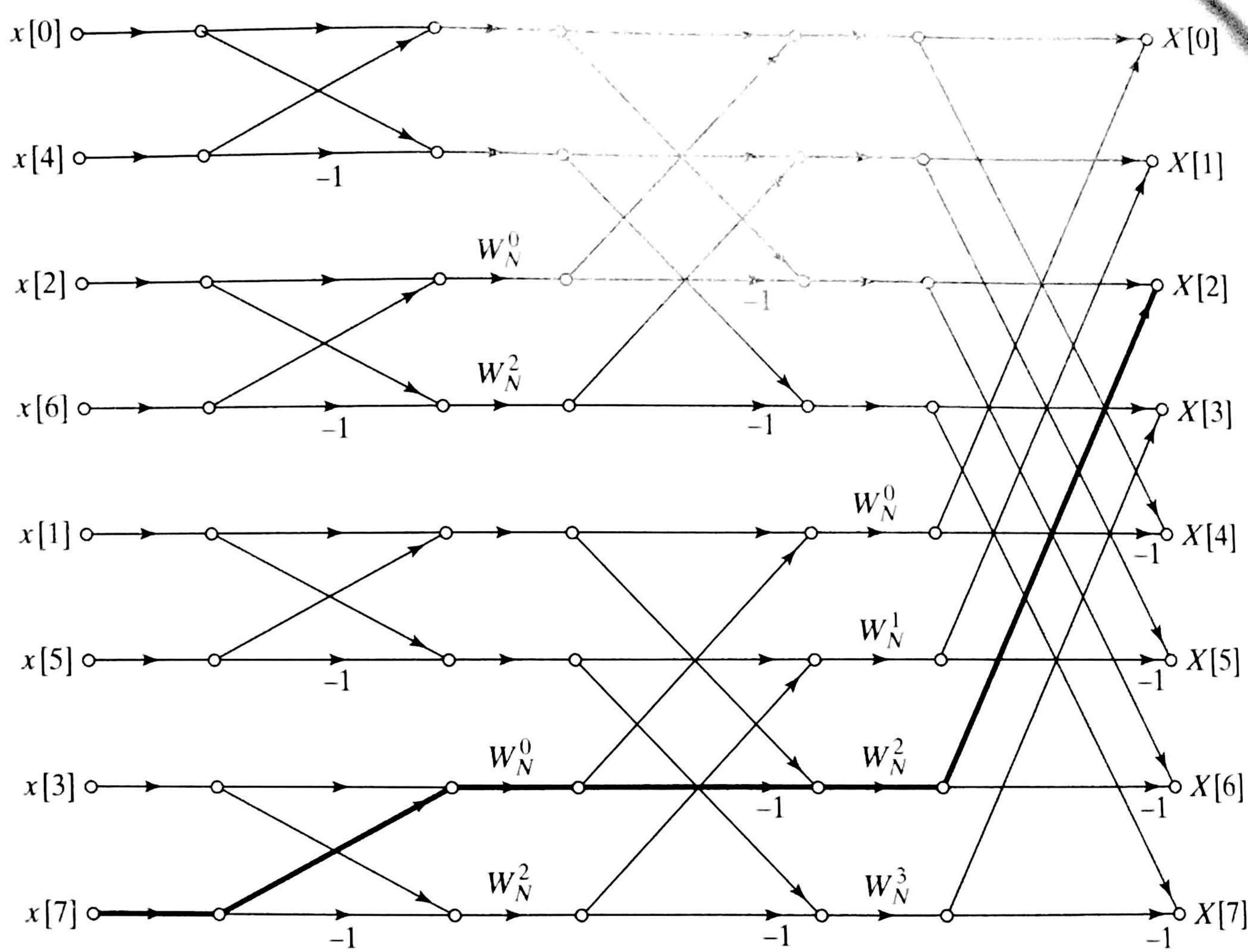

Figure P9.6

(a) What is the "gain" along the path that is emphasized in Figure P9.6?

(b) How many other paths in the flow graph begin at $x[7]$ and end at $X[2]$? Is this true in general? That is, how many paths are there between each input sample and each output sample?

(c) Now consider the DFT sample $X[2]$. By tracing paths in the flow graph of Figure P9.6, show that each input sample contributes the proper amount to the output DFT sample; i.e., verify that

$$X[2] = \sum_{n=0}^{N-1} x[n]e^{-j(2\pi/N)2n}.$$

9.7. Figure P9.7 shows the flow graph for an 8-point decimation-in-time FFT algorithm. Let $x[n]$ be the sequence whose DFT is $X[k]$. In the flow graph, $A[\cdot]$, $B[\cdot]$, $C[\cdot]$, and $D[\cdot]$ represent separate arrays that are indexed consecutively in the same order as the indicated nodes.

(a) Specify how the elements of the sequence $x[n]$ should be placed in the array $A[r]$, $r = 0, 1, \ldots, 7$. Also, specify how the elements of the DFT sequence should be extracted from the array $D[r]$, $r = 0, 1, \ldots, 7$.

(b) Without determining the values in the intermediate arrays, $B[\cdot]$ and $C[\cdot]$, determine and sketch the array sequence $D[r]$, $r = 0, 1, \ldots, 7$, if the input sequence is $x[n] = (-W_N)^n$, $n = 0, 1, \ldots, 7$.

(c) Determine and sketch the sequence $C[r]$, $r = 0, 1, \ldots, 7$, if the output Fourier transform is $X[k] = 1$, $k = 0, 1, \ldots, 7$.

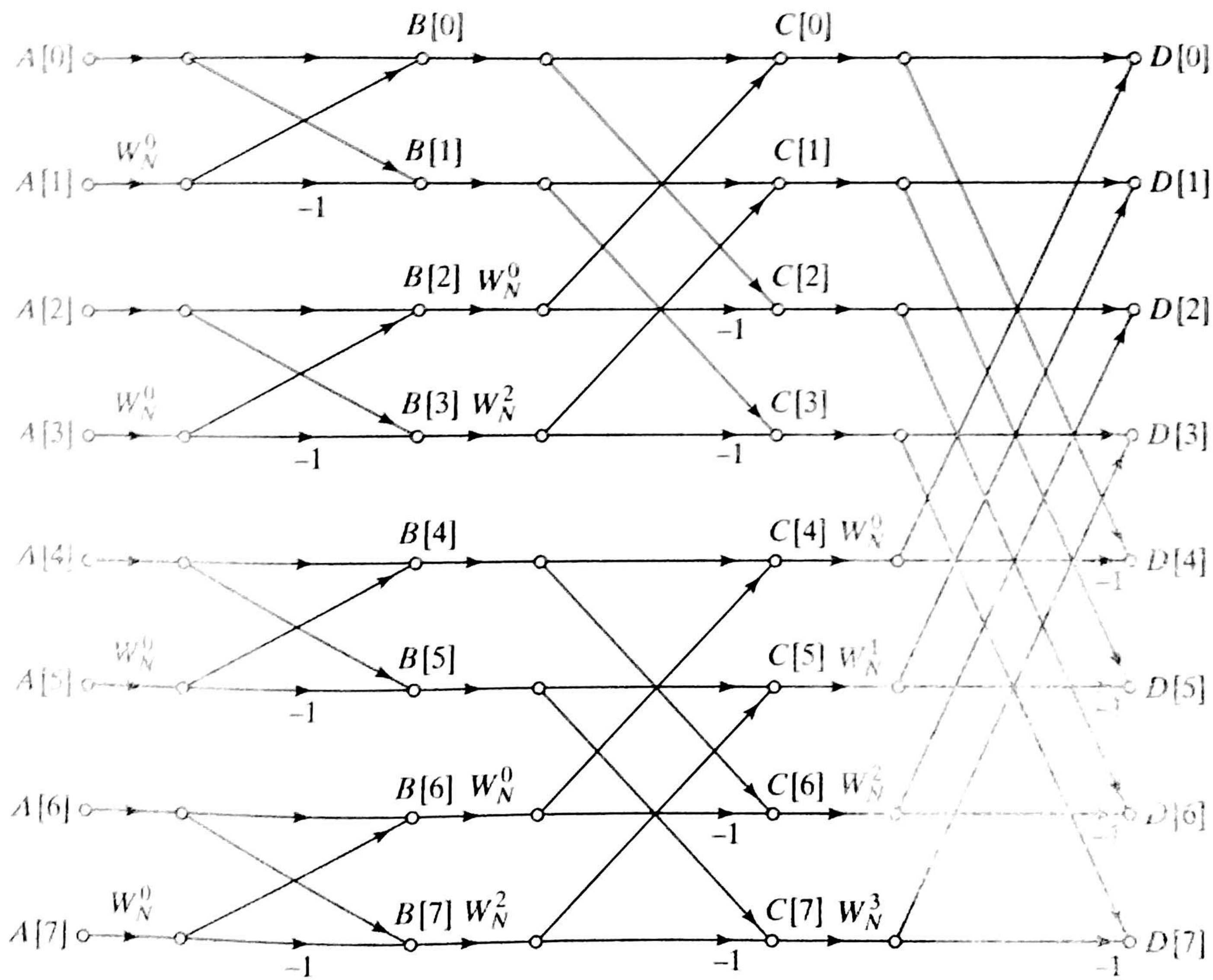

Figure P9.7

9.45. A modified FFT algorithm called the *split-radix* FFT, or SRFFT, was proposed by Duhamel and Hollman (1984) and Duhamel (1986). The flow graph for the split-radix algorithm is similar to the radix-2 flow graph, but it requires fewer real multiplications. In this problem, we illustrate the principles of the SRFFT for computing the DFT $X[k]$ of a sequence $x[n]$ of length N.

(a) Show that the even-indexed terms of $X[k]$ can be expressed as the $N/2$-point DFT

$$X[2k] = \sum_{n=0}^{(N/2)-1} (x[n] + x[n + N/2])W_N^{2kn}$$

for $k = 0, 1, \ldots, (N/2) - 1$.

(b) Show that the odd-indexed terms of the DFT $X[k]$ can be expressed as the $N/4$-point DFTs

$$X[4k+1] = \sum_{n=0}^{(N/4)-1} \{(x[n] - x[n + N/2]) - j(x[n + N/4] - x[n + 3N/4])\}W_N^{n} W_N^{4kn}$$

for $k = 0, 1, \ldots, (N/4) - 1$, and

$$X[4k+3] = \sum_{n=0}^{(N/4)-1} \{(x[n] - x[n + N/2]) + j(x[n + N/4] - x[n + 3N/4])\}W_N^{3n} W_N^{4kn}$$

for $k = 0, 1, \ldots, (N/4) - 1$.

(c) The flow graph in Figure P9.45 represents the preceding decomposition of the DFT for a 16-point transform. Redraw this flow graph, labeling each branch with the appropriate multiplier coefficient.

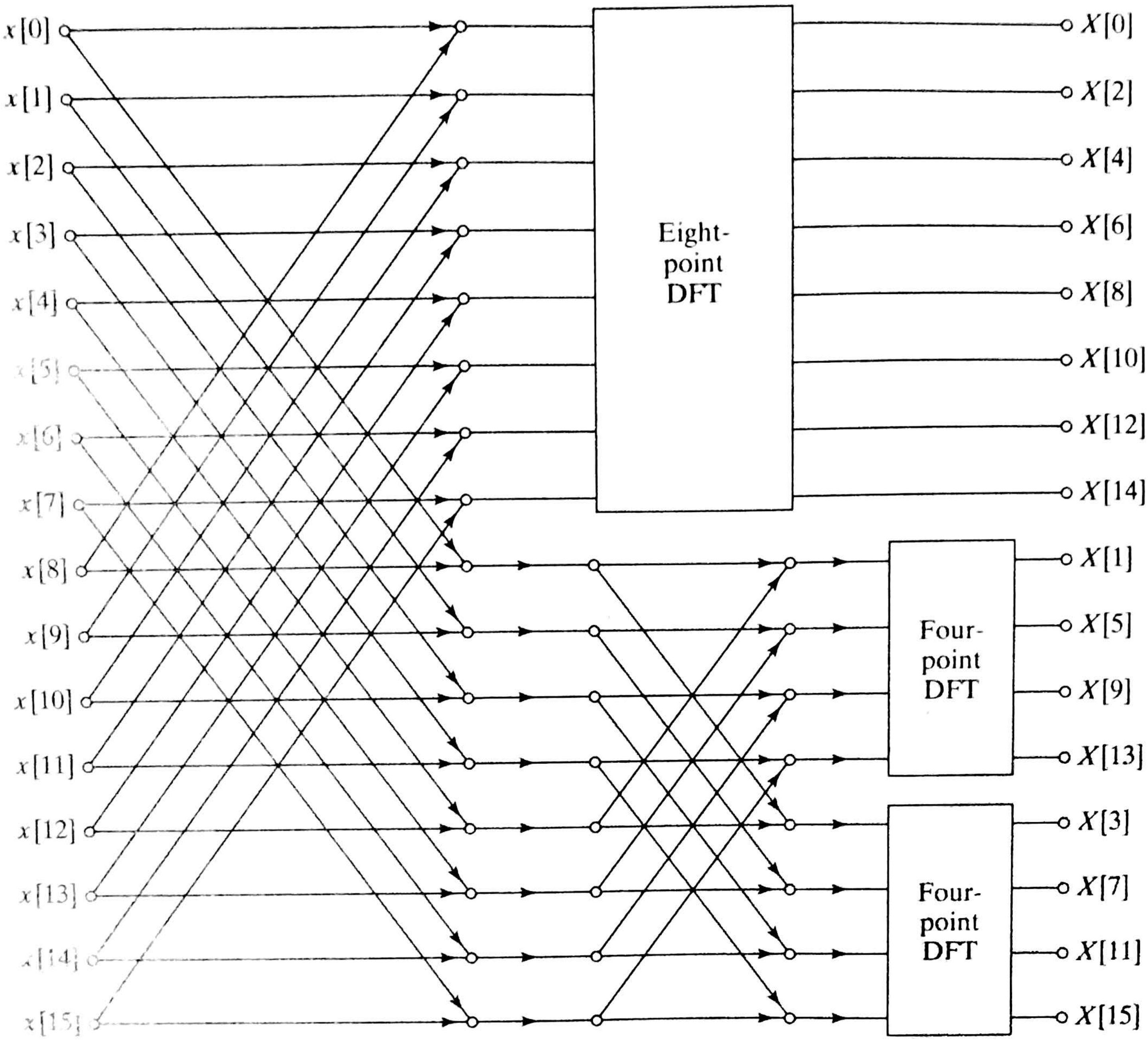

Figure P9.45

(d) Determine the number of real multiplications required to implement the 16-point transform when the SRFFT principle is applied to compute the other DFTs in Figure P9.45. Compare this number with the number of real multiplications required to implement a 16-point radix-2 decimation-in-frequency algorithm. In both cases, assume that multiplications by W_N^0 are not done.

9.58. In this problem, we will write the FFT as a sequence of matrix operations. Consider the 8-point decimation-in-time FFT algorithm shown in Figure P9.58. Let **a** and **f** denote the input and output vectors, respectively. Assume that the input is in bit-reversed order and that the output is in normal order (compare with Figure 9.11). Let **b**, **c**, **d**, and **e** denote the intermediate vectors shown on the flow graph.

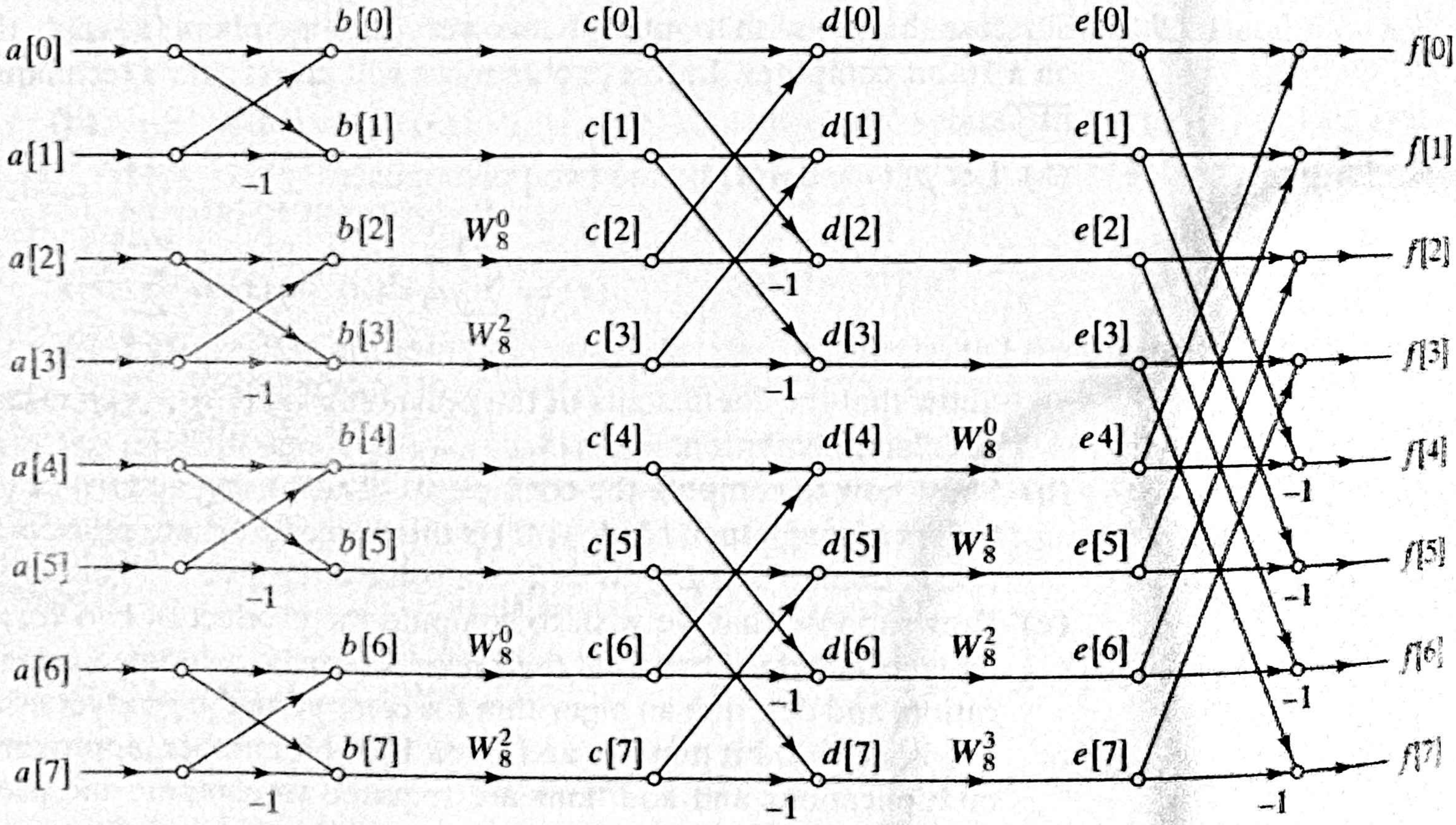

Figure P9.58

(a) Determine the matrices F_1, T_1, F_2, T_2, and F_3 such that

$$b = F_1 a,$$

$$c = T_1 b,$$

$$d = F_2 c,$$

$$e = T_2 d,$$

$$f = F_3 e.$$

(b) The overall FFT, taking input a and yielding output f can be described in matrix notation as $f = Qa$, where

$$Q = F_3 T_2 F_2 T_1 F_1.$$

Let Q^H be the complex (Hermitian) transpose of the matrix Q. Draw the flow graph for the sequence of operations described by Q^H. What does this structure compute?

(c) Determine $(1/N)Q^H Q$.