9.6. Figure P9.6 shows the graph representation of a decimation-in-time FFT algorithm for $N = 8$. The heavy line shows a path from sample $x[7]$ to DFT sample $X[2]$.

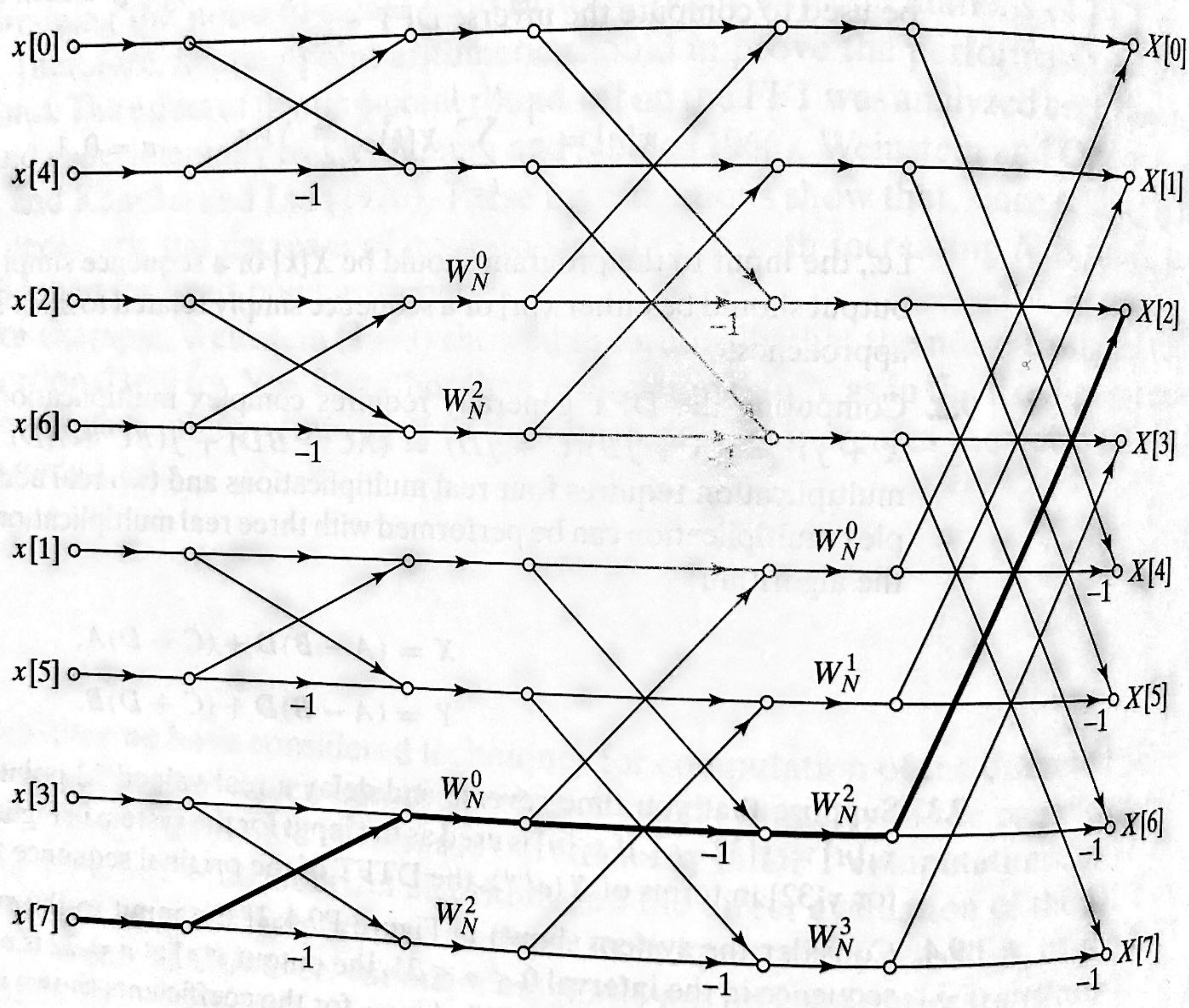

Figure P9.6

(a) What is the "gain" along the path that is emphasized in Figure P9.6?

(b) How many other paths in the flow graph begin at $x[7]$ and end at $X[2]$? Is this true in general? That is, how many paths are there between each input sample and each output sample?

(c) Now consider the DFT sample $X[2]$. By tracing paths in the flow graph of Figure P9.6, show that each input sample contributes the proper amount to the output DFT sample; i.e., verify that

$$X[2] = \sum_{n=0}^{N-1} x[n]e^{-j(2\pi/N)2n}.$$

9.7. Figure P9.7 shows the flow graph for an 8-point decimation-in-time FFT algorithm. Let $x[n]$ be the sequence whose DFT is $X[k]$. In the flow graph, $A[\cdot]$, $B[\cdot]$, $C[\cdot]$, and $D[\cdot]$ represent separate arrays that are indexed consecutively in the same order as the indicated nodes.

(a) Specify how the elements of the sequence $x[n]$ should be placed in the array $A[r]$, $r = 0, 1, \dots, 7$. Also, specify how the elements of the DFT sequence should be extracted from the array $D[r]$, $r = 0, 1, \dots, 7$.

(b) Without determining the values in the intermediate arrays, $B[\cdot]$ and $C[\cdot]$, determine and sketch the array sequence $D[r]$, $r = 0, 1, \dots, 7$, if the input sequence is $x[n] = (-W_N)^n$, $n = 0, 1, \dots, 7$.

(c) Determine and sketch the sequence $C[r]$, $r = 0, 1, \dots, 7$, if the output Fourier transform is $X[k] = 1$, $k = 0, 1, \dots, 7$.

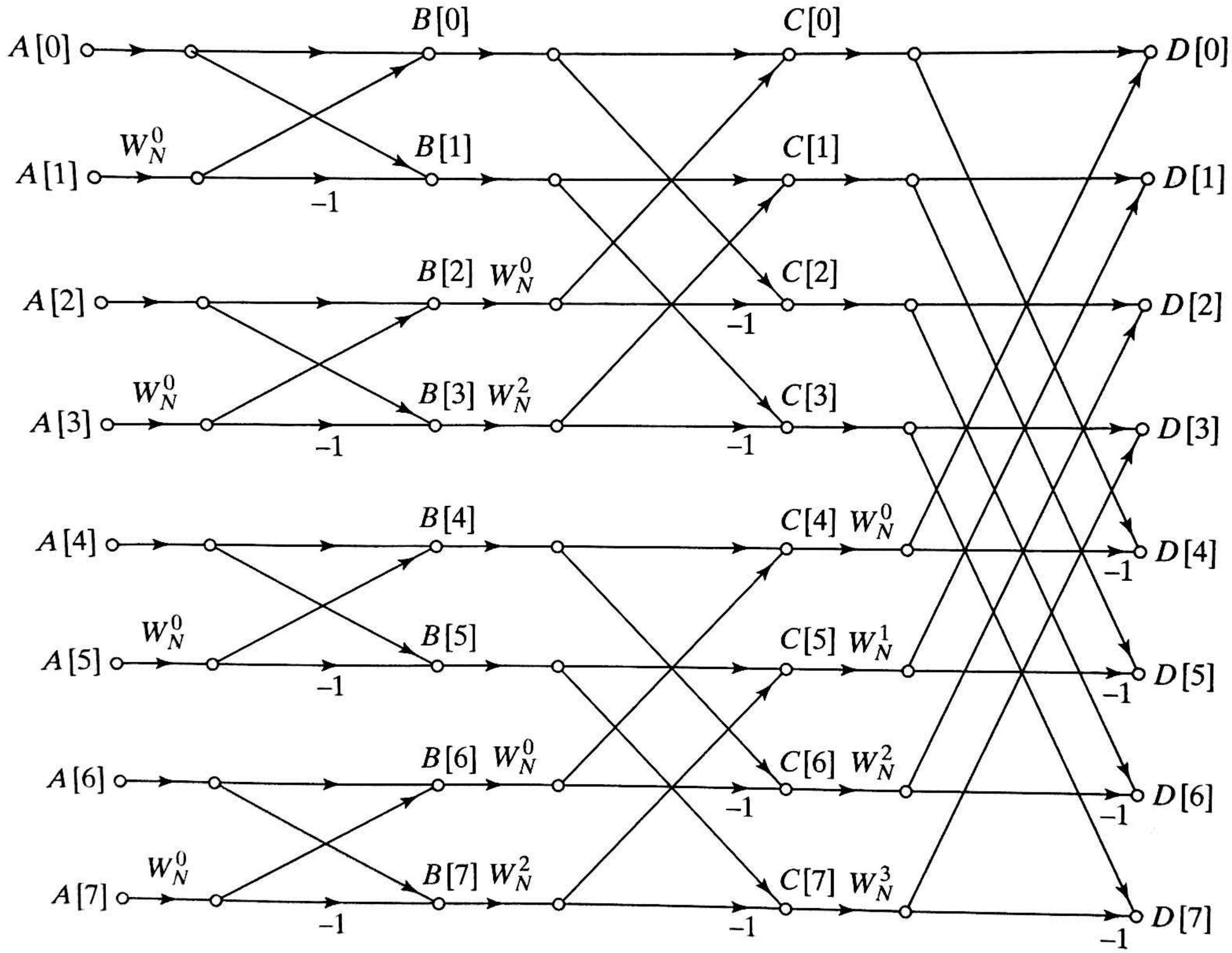

Figure P9.7

9.42. Consider a class of DFT-based algorithms for implementing a causal FIR filter with impulse response $h[n]$ that is zero outside the interval $0 \leq n \leq 63$. The input signal (for the FIR filter) $x[n]$ is segmented into an infinite number of possibly overlapping 128-point blocks $x_i[n]$, for i an integer and $-\infty \leq i \leq \infty$, such that

$$x_i[n] = \begin{cases} x[n], & iL \leq n \leq iL + 127, \\ 0, & \text{otherwise}, \end{cases}$$

where L is a positive integer.

Specify a method for computing

$$y_i[n] = x_i[n] * h[n]$$

for any i. Your answer should be in the form of a block diagram utilizing only the types of modules shown in Figures PP9.42-1 and PP9.42-2. A module may be used more than once or not at all.

The four modules in Figure P9.42-2 either use radix-2 FFTs to compute $X[k]$, the N-point DFT of $x[n]$, or use radix-2 inverse FFTs to compute $x[n]$ from $X[k]$.

Your specification must include the lengths of the FFTs and IFFTs used. For each "shift by n_0" module, you should also specify a value for n_0, the amount by which the input sequence is to be shifted.

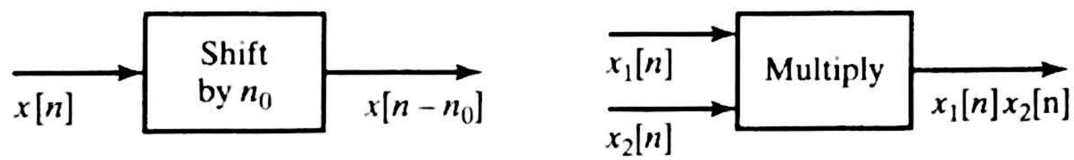

Figure P9.42-1

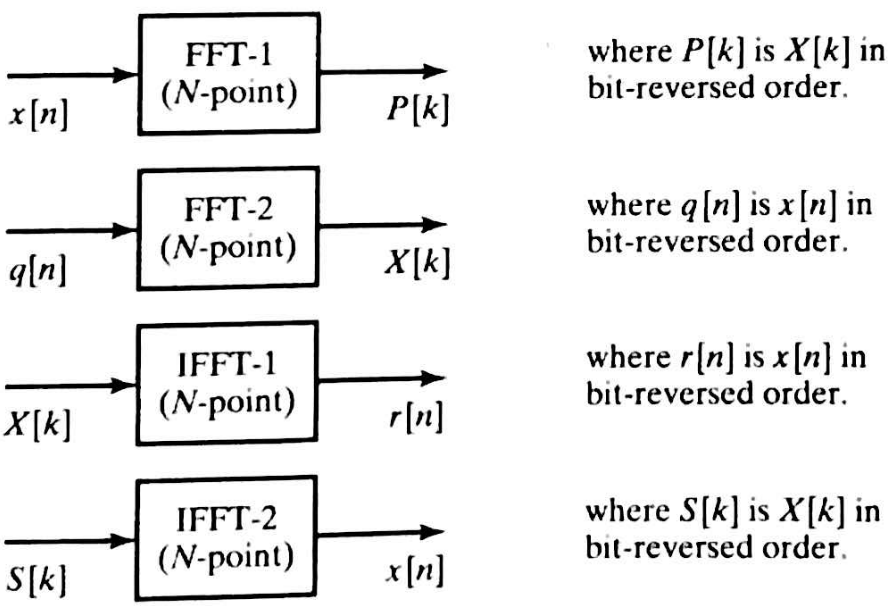

Figure P9.42-2

9.58. In this problem, we will write the FFT as a sequence of matrix operations. Consider the 8-point decimation-in-time FFT algorithm shown in Figure P9.58. Let a and f denote the input and output vectors, respectively. Assume that the input is in bit-reversed order and that the output is in normal order (compare with Figure 9.11). Let b, c, d, and e denote the intermediate vectors shown on the flow graph.

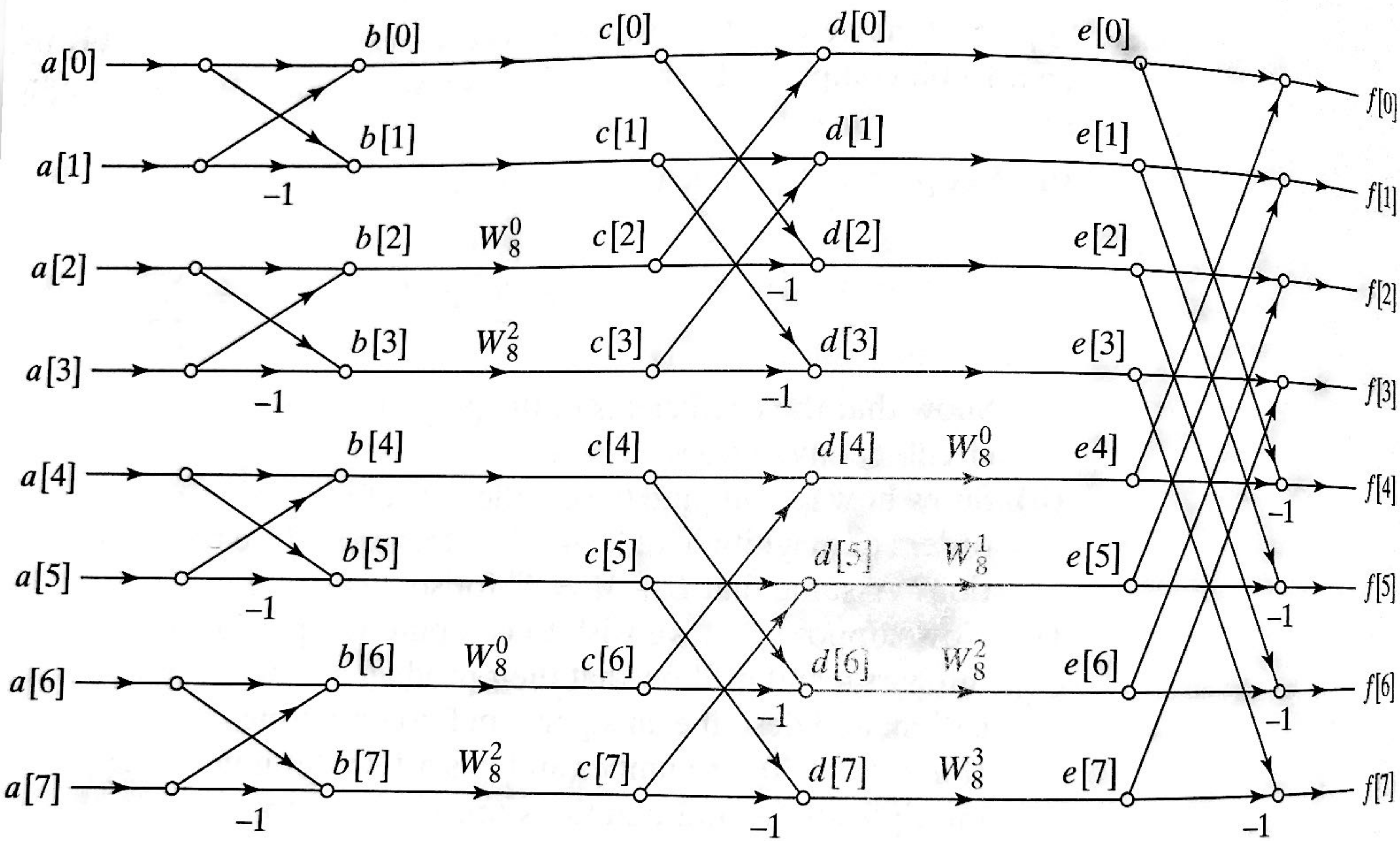

Figure P9.58

(a) Determine the matrices F_1, T_1, F_2, T_2, and F_3 such that

$$b = F_1 a,$$

$$c = T_1 b,$$

$$d = F_2 c,$$

$$e = T_2 d,$$

$$f = F_3 e.$$

(b) The overall FFT, taking input a and yielding output f can be described in matrix notation as $f = Qa$, where

$$Q = F_3 T_2 F_2 T_1 F_1.$$

Let Q^H be the complex (Hermitian) transpose of the matrix Q. Draw the flow graph for the sequence of operations described by Q^H. What does this structure compute?

(c) Determine $(1/N)Q^H Q$.