Carnegie Mellon DSP Electrical & Computer ENGINEERING

Digital Signal Processing (18-491/18-691)
Spring Semester, 2024

Corrected Solution to Problem 2.2

(a)

$$x[n] = \left(\frac{1}{3}\right)^{n-1} u[n-1]$$

$$x_1[n] = (\frac{1}{3})^n u[n] \iff X_1(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$x[n] = x_1[n-1] \iff X(e^{j\omega}) = X_1(e^{j\omega})e^{-j\omega} = \frac{e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}$$

$$\boxed{X(e^{j\omega}) = \frac{e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}}}$$

$$h[n] = (\frac{1}{3})^{-n+1} u[-n+1]$$

$$h_1[n] = (\frac{1}{3})^n u[n] \iff H_1(e^{j\omega}) = \frac{1}{1-\frac{1}{3}e^{-j\omega}}$$

$$h_2[n] = h_1[n+1] \iff H_2(e^{j\omega}) = H_2(e^{j\omega})e^{j\omega} = \frac{e^{j\omega}}{1-\frac{1}{3}e^{-j\omega}}$$

$$h[n] = h_2[-n] \iff H(e^{j\omega}) = H_2(e^{-j\omega}) = \frac{e^{-j\omega}}{1-\frac{1}{3}e^{j\omega}}$$

$$\boxed{H(e^{j\omega}) = \frac{e^{-j\omega}}{1-\frac{1}{3}e^{j\omega}}}$$

(b)

$$y[n] = \frac{1}{8}(\frac{1}{3})^{-n}u[-n+1] + \frac{81}{3}(\frac{1}{3})^{n}u[n-2]$$

$$Y(e^{j\omega}) = \sum_{n=-\infty}^{\infty}[(\frac{1}{8}(\frac{1}{3})^{-n}u[-n+1] + \frac{81}{3}(\frac{1}{3})^{n}u[n-2]]e^{-j\omega n}$$

$$Y(e^{j\omega}) = \frac{1}{8}\sum_{n_1=-\infty}^{\infty}\frac{1}{3}^{-n_1}u[-n_1+1]e^{-j\omega n_1} + \frac{81}{8}\sum_{n_2=-\infty}^{\infty}(\frac{1}{3})^{n_2}u[n_2-2]e^{-j\omega n_2}$$

$$n_1' = -n_1 + 1 \qquad n_2' = n_2 - 2$$

$$Y(e^{j\omega}) = \frac{1}{8}\sum_{n_1=0}^{\infty}(\frac{1}{3})^{n_1'-1}e^{-j\omega(-n_1'+1)} + \frac{81}{8}\sum_{n_2=0}^{\infty}(\frac{1}{3})^{n_2'+2}e^{-j\omega(n_2'+2)}$$

$$Y(e^{j\omega}) = \frac{3e^{-j\omega}}{8}\sum_{n_1=0}^{\infty}(\frac{1}{3})^{n_1}e^{j\omega n_1} + \frac{81}{8}(\frac{e^{-2j\omega}}{9})\sum_{n_2=0}^{\infty}(\frac{1}{3})^{n_2'}e^{-j\omega n_2}$$

$$Y(e^{j\omega}) = \frac{1}{8}[\frac{3e^{-j\omega}}{1-\frac{1}{3}e^{j\omega}} + \frac{9e^{-2j\omega}}{1-\frac{1}{3}e^{-j\omega}}]$$

$$Y(e^{j\omega}) = \frac{1}{8}\frac{3e^{-j\omega}(1-\frac{1}{3}e^{-j\omega}) + 9e^{-2j\omega}(1-\frac{1}{3}e^{j\omega})}{(1-\frac{1}{3}e^{j\omega})(1-\frac{1}{3}e^{-j\omega})}$$

$$Y(e^{j\omega}) = \frac{1}{8}\frac{3e^{-j\omega} - e^{-2j\omega} + 9e^{-2j\omega} - 3e^{-j\omega}}{(1-\frac{1}{3}e^{j\omega})(1-\frac{1}{3}e^{-j\omega})}$$

$$\boxed{Y(e^{j\omega}) = \frac{e^{-2j\omega}}{(1-\frac{1}{3}e^{j\omega})(1-\frac{1}{3}e^{-j\omega})}}$$

(c)

$$\boxed{X(e^{j\omega})H(e^{j\omega}) = \frac{e^{-j\omega}}{1-\frac{1}{3}e^{-j\omega}} \times \frac{e^{-j\omega}}{1-\frac{1}{3}e^{j\omega}} = \frac{e^{-2j\omega}}{(1-\frac{1}{3}e^{j\omega})(1-\frac{1}{3}e^{-j\omega})} = Y(e^{j\omega})}$$