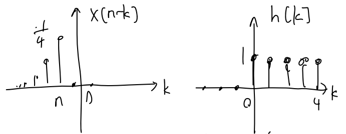
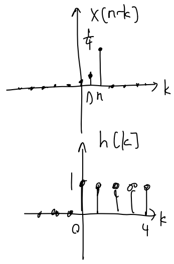
(d) By sifting properties, $y[n] = x[n-4] = (\frac{1}{3})^{n-5} u[n-5]$

(e)

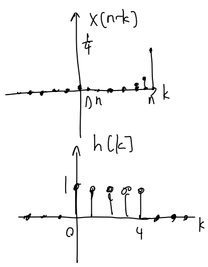


Case 1: $n<0$, no overlap. $y(n) = 0$



Case 2: $0 \le n \le 4$

$$y(n) = \sum_{k=0}^{n} (\tfrac{1}{4})^{n-k}$$
$$= \frac{1-(\frac{1}{4})^{n+1}}{1-\frac{1}{4}}$$
$$= \frac{4}{3}\left[1-(\tfrac{1}{4})^{n+1}\right]$$

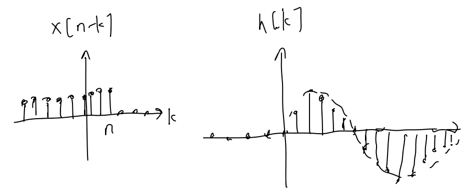


Case 3: $n > 4$

$$y(n) = \sum_{k=0}^{4} (\tfrac{1}{4})^{n-k}$$
$$= (\tfrac{1}{4})^n \sum_{k=0}^{4} (\tfrac{1}{4})^{-k} \quad \text{let } r = -k$$
$$= (\tfrac{1}{4})^n \sum_{r=0}^{-4} (\tfrac{1}{4})^r$$
$$= (\tfrac{1}{4})^n \cdot \frac{1-(\frac{1}{4})^{-3}}{1-\frac{1}{4}}$$
$$= (\tfrac{1}{4})^n \cdot 4(1-64)$$
$$= 63\,(\tfrac{1}{4})^{n-1}$$

(f) $y[n] = \sum_{k=-\infty}^{\infty} \sin(\omega_0 k) u[k] u(n-k)$

$$= \sum_{k=-\infty}^{\infty} \frac{1}{2j}\left(e^{j\omega_0 k} - e^{-j\omega_0 k}\right) u(k) u(n-k)$$



When $n \le 0$, $y[n] = 0$

When $n > 0$, $y[n] = \frac{1}{2j} \sum_{k=0}^{n} \left(e^{j\omega_0 k} - e^{-j\omega_0 k}\right)$

$$= \frac{1}{2j} \cdot \left[\frac{1-e^{j\omega_0(n+1)}}{1-e^{j\omega_0}} - \frac{1-e^{-j\omega_0(n+1)}}{1-e^{-j\omega_0}}\right]$$

$$= \frac{1}{2j}\left[\frac{e^{j\omega_0(n+1)/2}\left(e^{-j\omega_0(n+1)/2} - e^{j\omega_0(n+1)/2}\right)}{e^{j\omega_0/2}\left(e^{-j\omega_0/2} - e^{j\omega_0/2}\right)} - \frac{e^{-j\omega_0(n+1)/2}\left(e^{j\omega_0(n+1)/2} - e^{-j\omega_0(n+1)/2}\right)}{e^{-j\omega_0/2}\left(e^{j\omega_0/2} - e^{-j\omega_0/2}\right)}\right]$$

$$= \frac{1}{2j}\left[\frac{e^{j\omega_0(n+1)/2}\left(e^{-j\omega_0(n+1)/2} - e^{j\omega_0(n+1)/2}\right)}{e^{j\omega_0/2}\left(e^{-j\omega_0/2} - e^{j\omega_0/2}\right)} - \frac{-e^{-j\omega_0(n+1)/2}\left(e^{-j\omega_0(n+1)/2} - e^{j\omega_0(n+1)/2}\right)}{-e^{-j\omega_0/2}\left(e^{-j\omega_0/2} - e^{j\omega_0/2}\right)}\right]$$

$$= \left[\frac{e^{-j\omega_0(n+1)/2} - e^{j\omega_0(n+1)/2}}{e^{-j\omega_0/2} - e^{j\omega_0/2}}\right] \cdot \frac{1}{2j}\left[\frac{e^{j\omega_0(n+1)/2}}{e^{j\omega_0/2}} - \frac{e^{-j\omega_0(n+1)/2}}{e^{-j\omega_0/2}}\right]$$

$$= \frac{\sin[\omega_0(n+1)/2]}{\sin(\omega_0/2)} \cdot \frac{1}{2j}\left(e^{j\omega_0 n/2} - e^{-j\omega_0 n/2}\right)$$

$$= \frac{\sin[\omega_0(n+1)/2]}{\sin(\omega_0/2)} \sin(\omega_0 n/2)$$