7.31. Suppose that we have used the Parks–McClellan algorithm to design a causal FIR linear-phase lowpass filter. The system function of this system is denoted $H(z)$. The *length* of the impulse response is 25 samples, i.e., $h[n] = 0$ for $n < 0$ and for $n > 24$, and $h[0] \neq 0$. The desired response and weighting function used were

$$H_d(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq 0.3\pi \\ 0 & 0.4\pi \leq |\omega| \leq \pi \end{cases} \qquad W(e^{j\omega}) = \begin{cases} 1 & |\omega| \leq 0.3\pi \\ 2 & 0.4\pi \leq |\omega| \leq \pi. \end{cases}$$

In each case below, determine whether the statement is true or false or that insufficient information is given. Justify your conclusions.

(a) $h[n+12] = h[12-n]$ or $h[n+12] = -h[12-n]$ for $-\infty < n < \infty$.

(b) The system has a stable and causal inverse.

(c) We know that $H(-1) = 0$.

(d) The maximum weighted approximation error is the same in all approximation bands.

(e) If z_0 is a zero of $H(z)$, then $1/z_0$ is a pole of $H(z)$.

(f) The system can be implemented by a network (flow graph) that has no feedback paths.

(g) The group delay is equal to 24 for $0 < \omega < \pi$.

(h) If the coefficients of the system function are quantized to 10 bits each, the system is still optimum in the Chebyshev sense for the original desired response and weighting function.

(i) If the coefficients of the system function are quantized to 10 bits each, the system is still guaranteed to be a linear-phase filter.

(j) If the coefficients of the system function are quantized to 10 bits each, the system may become unstable.

7.36. The graphs in Figure P7.36 depict four frequency-response magnitude plots of linear-phase FIR filters, labelled $|A_e^i(e^{j\omega})|$, $i = 1, 2, 3, 4$. One or more of these plots may belong to equiripple linear-phase FIR filters designed by the Parks–McClellan algorithm. The maximum approximation errors in the passband and the stopband, as well as the desired cutoff frequencies of those bands, are also shown in the plots. Please note that the approximation error and filter length specifications may have been chosen differently to ensure that the cutoff frequencies are the same in each design.

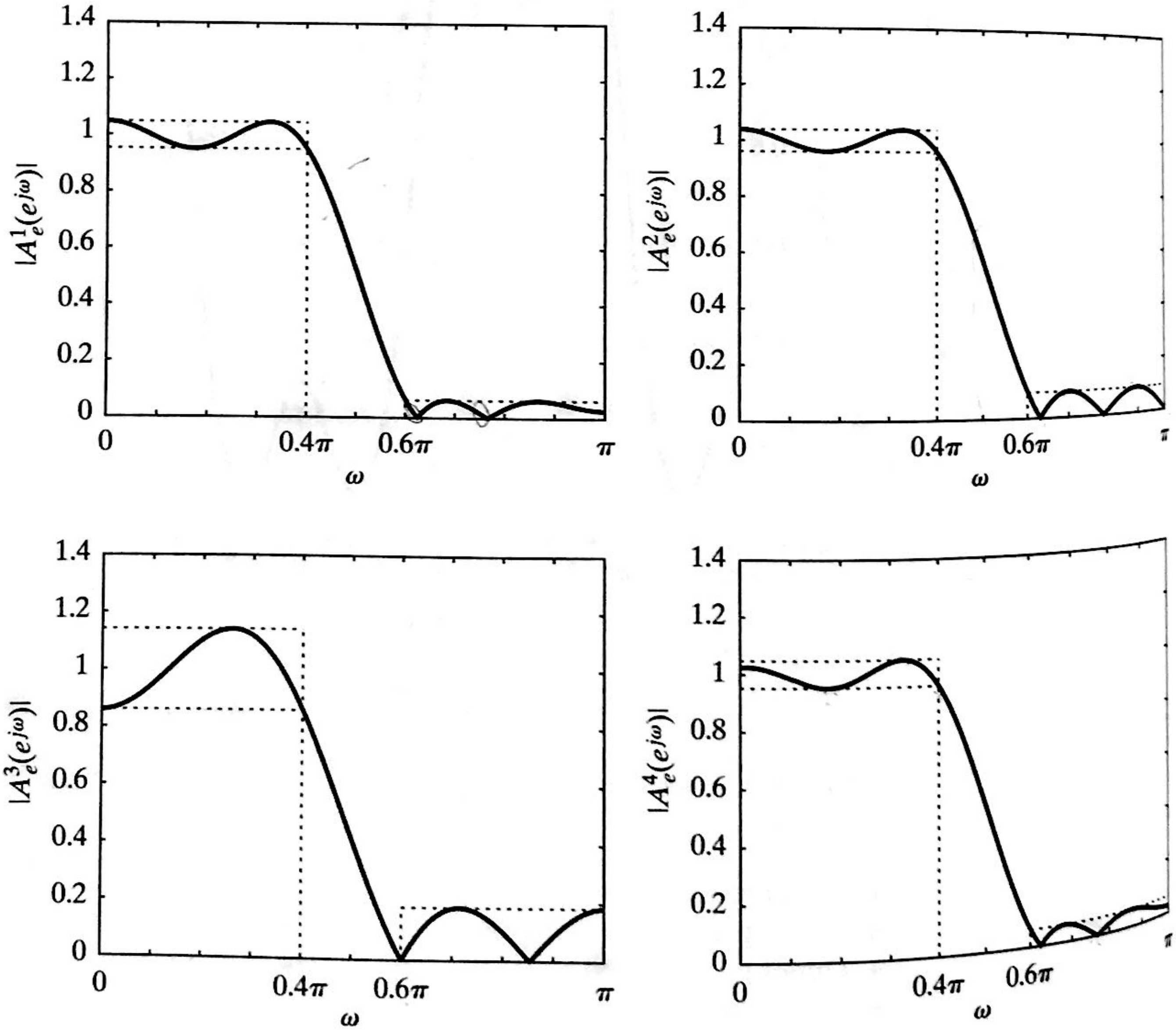

Figure P7.36

(a) What type(s) (I, II, III, IV) of linear-phase FIR filters can $|A_e^i(e^{j\omega})|$ correspond to, for $i = 1, 2, 3, 4$? Please note that there may be more than one linear-phase FIR filter type corresponding to each $|A_e^i(e^{j\omega})|$. If you feel this is the case, list all possible choices.

(b) How many alternations does each $|A_e^i(e^{j\omega})|$ exhibit, for $i = 1, 2, 3, 4$?

(c) For each i, $i = 1, 2, 3, 4$, can $|A_e^i(e^{j\omega})|$ belong to an output of the Parks–McClellan algorithm?

(d) If you claimed that a given $|A_e^i(e^{j\omega})|$ could correspond to an output of the Parks–McClellan algorithm, and that it could be type I, what is the length of the impulse response of $|A_e^i(e^{j\omega})|$?

7.42. As discussed in Chapter 12, an *ideal discrete-time Hilbert transformer* is a system that introduces −90 degrees ($-\pi/2$ radians) of phase shift for $0 < \omega < \pi$ and +90 degrees ($+\pi/2$ radians) of phase shift for $-\pi < \omega < 0$. The magnitude of the frequency response is constant (unity) for $0 < \omega < \pi$ and for $-\pi < \omega < 0$. Such systems are also called *ideal 90-degree phase shifters*.

(a) Give an equation for the ideal desired frequency response $H_d(e^{j\omega})$ of an ideal discrete-time Hilbert transformer that also includes constant (nonzero) group delay. Plot the phase response of this system for $-\pi < \omega < \pi$.

(b) What type(s) of FIR linear-phase systems (I, II, III, or IV) can be used to approximate the ideal Hilbert transformer in part (a)?

(c) Suppose that we wish to use the window method to design a linear-phase approximation to the ideal Hilbert transformer. Use $H_d(e^{j\omega})$ given in part (a) to determine the ideal impulse response $h_d[n]$ if the FIR system is to be such that $h[n] = 0$ for $n < 0$ and $n > M$.

(d) What is the delay of the system if $M = 21$? Sketch the magnitude of the frequency response of the FIR approximation for this case, assuming a rectangular window.

(e) What is the delay of the system if $M = 20$? Sketch the magnitude of the frequency response of the FIR approximation for this case, assuming a rectangular window.

7.63. Consider the design of a type I bandpass linear-phase FIR filter using the Parks–McClellan algorithm. The impulse response length is $M+1 = 2L+1$. Recall that for type I systems, the frequency response is of the form $H(e^{j\omega}) = A_e(e^{j\omega})e^{-j\omega M/2}$, and the Parks–McClellan algorithm finds the function $A_e(e^{j\omega})$ that minimizes the maximum value of the error function

$$E(\omega) = W(\omega)[H_d(e^{j\omega}) - A_e(e^{j\omega})], \qquad \omega \in F,$$

where F is a closed subset of the interval $0 \leq \omega \leq \pi$, $W(\omega)$ is a weighting function, and $H_d(e^{j\omega})$ defines the desired frequency response in the approximation intervals F. The tolerance scheme for a bandpass filter is shown in Figure P7.63.

(a) Give the equation for the desired response $H_d(e^{j\omega})$ for the tolerance scheme in Figure P7.63.

(b) Give the equation for the weighting function $W(\omega)$ for the tolerance scheme in Figure P7.63.

(c) What is the *minimum* number of alternations of the error function for the optimum filter?

(d) What is the *maximum* number of alternations of the error function for the optimum filter?

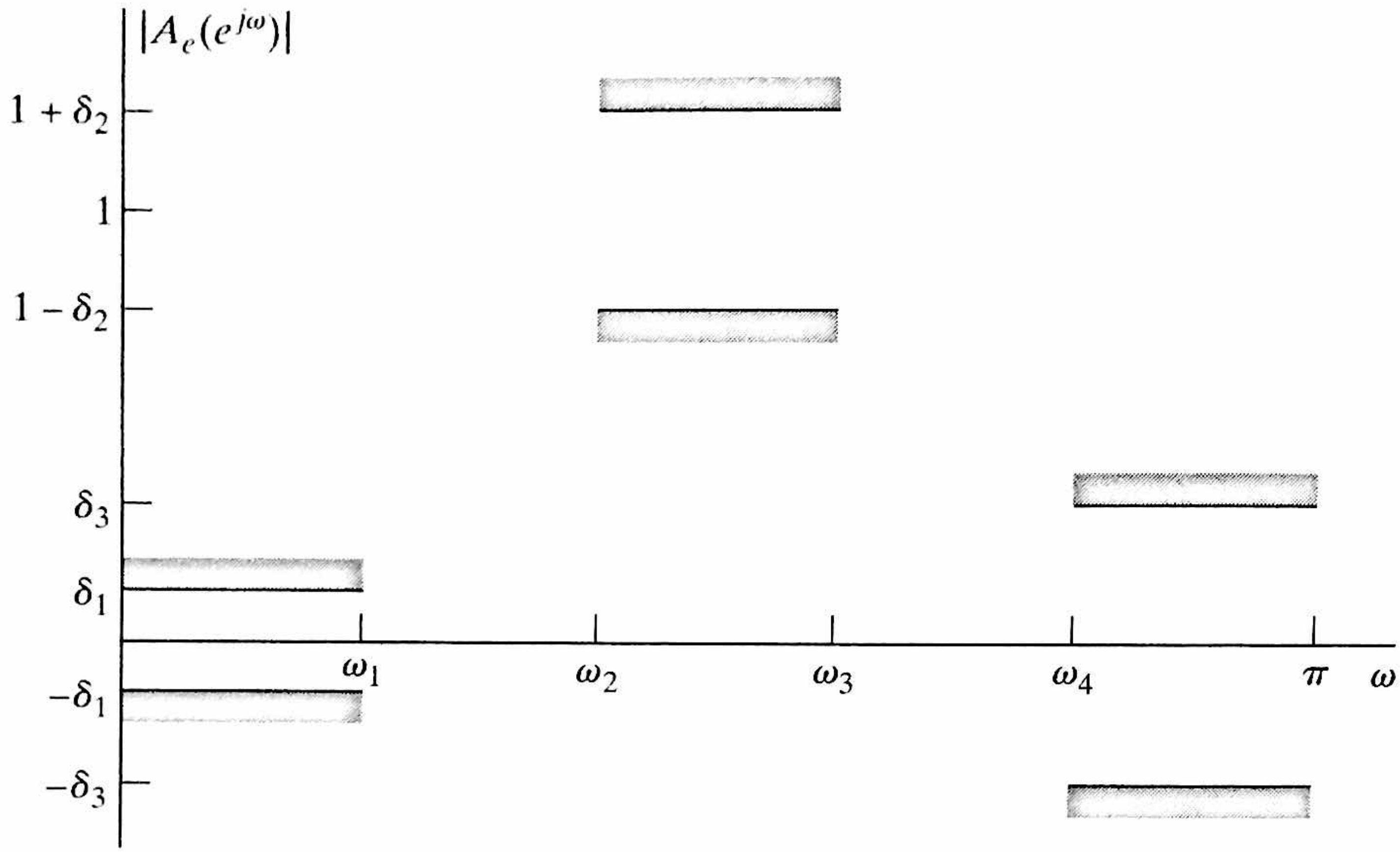

Figure P7.63

(e) Sketch a "typical" weighted error function $E(\omega)$ that could be the error function for an optimum bandpass filter if $M = 14$. Assume the *maximum* number of alternations.

(f) Now suppose that M, ω_1, ω_2, ω_3, the weighting function, and the desired function are kept the same, but ω_4 is *increased*, so that the transition band $(\omega_4 - \omega_3)$ is increased.