Carnegie Mellon DSP Electrical & Computer ENGINEERING

Digital Signal Processing (18-491/18-691)
Spring Semester, 2024

Problem Set 1

Issued: 1/25/24

Due: 2/1/24 at 11:59 pm via Gradescope

Note: You will be submitting your homework using Gradescope, as we will discuss in class. This means that you will write your homework on paper, scan your results, and upload them to the gradescope site.

Reading: This week's short homework covers the first week of classes, during which time we discussed properties of signals and systems, and introduced convolution. This material is covered in Chapter 1 and Sections 2.0-2.4 in Oppenheim, Schafer, Yoder, and Padgett (2009), hereafter to be referred to as OSYP. A handout with analytical examples of solutions to convolution problems is posted on the course website. During the present week we discussed convolution, discrete-time Fourier transforms (DTFTs), and examples and properties of DTFTs. During the next week we will discuss z-transforms (ZTs) following the material in OSYP Secs. 3.0-3.5.

Problem 1.1: For each of the systems with input $x[n]$ and output $y[n]$ described below, determine whether the system is (1) linear, (2) shift invariant, (3) stable, (4) causal, and (5) memoryless. Be sure to state the reasons for your answers.

Note: The failure of a particular property to apply can be asserted by a single counterexample. And if you think that the system is linear and/or shift invariant, an informal argument with a few examples will suffice .... you don't need to develop a formal mathematical proof. In fact, this problem is rather easy!

(a) $$y[n] = \sum_{k=0}^{n} x[k]$$

(b) $$y[n] = x[n-3]$$

(c) $$y[n] = x[3-n]$$

(d) $$y[n] = x[n]\cos(\omega_0 n)$$

(e) $$y[n] = 5x[n] + 9$$

(f) $y[n] = 1/x[n]$

Problem 1.2: Determine which of the following discrete-time signals are periodic. If a signal is periodic, determine its period.

(a) $x[n] = n\cos(n\pi)$

(b) $x[n] = 2\sin(n\pi/7)$

(c) $x[n] = 4e^{j5n}$

(d) $x[n] = e^{j\tan(n\pi/3)}$

Problem 1.3: Problem 2.27 in OSYP

Problem 1.4: Consider an LSI system with impulse response $h[n]$ and input $x[n]$. Determine the output of the system, $y[n]$, for the following pairs of inputs and impulse responses:

(a) $x[n] = (1/3)^n u[n]; \quad h[n] = u[n]$

(b) $x[n] = (1/3)^{n-1} u[n-1]; \quad h[n] = (1/4)^n u[n]$

(c) $x[n] = (1/3)^{n-1} u[n-1]; \quad h[n] = (1/3)^{-n+1} u[-n+1]$

(d) $x[n] = (1/3)^{n-1} u[n-1]; \quad h[n] = \delta[n-4]$

(e) $x[n] = (1/4)^n u[n]; \quad h[n] = \begin{cases} 1, & 0 \le n \le 4 \\ 0, & \text{otherwise} \end{cases}$

(f) $x[n] = u[n]; \quad h[n] = \sin(\omega_0 n) u[n]$

In all of these cases, you should obtain an analytical solution for your results: leave no unevaluated sums or integrals.

MATLAB Problems

Reading: We will be using MATLAB for verifying homework results, and for use as a design tool throughout the course. If you are not already familiar with MATLAB, there are a number of tutorials on the Web that may be found easily by Googling something like `matlab intro` or `matlab tutorial`. (We will download and place on our website any tutorials that are especially highly recommended by the students.)

In working the MATLAB problems, turn in a printout of your results, a copy of the MATLAB code you developed to work the problem, as well as any additional comments you'd like to add.

A note to Python users: Over the years we have had a number of discussions about whether students should be allowed to work the MATLAB problems using Python. As we have discussed in class, the advantages of MATLAB are that it is more stable, better supported, and it has much better documentation than Python. In addition, there are some functions and capabilities that are important for our course that exist in MATLAB but that are not (yet) available in Python. On the other hand, some students prefer Python because it is free, it is better integrated with many standard deep learning packages, and it is a more "legitimate" computer programming language.

This year we are offering students the opportunity to work their MATLAB problems in Python. Because this is only a pilot program, we will be offereing only limited support for students using Python, and as mentioned, some functions do not exist at all in Python, and you will need to use design tools in MATLAB for some of your work on filter design toward the end of the semester. Hence, you will be on your own a bit more if you choose to solve your computer problems in Python rather than in MATLAB. Nevertheless, we will provide support as best as we can.

Problem C1.1: In this problem we will merely use MATLAB to verify the results of your convolution problems.

Among other documentation that you will review, be sure to look over the help files for the routines `conv` and `stem`.

Write four short MATLAB scripts that calculate and display the results of the convolutions of Problem 1.4, parts (a), (c), (e) and (f). Again, you should provide the script and a plot of the results using the `stem` command. Be sure to indicate the key portions of the results, including moments of time at which the nature of the response changes. Show that your computational results in this problem agree with your analytical calculations in the corresponding parts of Problem 1.4.