ECE 18-316
Fall 1998
EXAM # 1
9/16/98

READ THE ENTIRE EXAM BEFORE YOU START WRITING.
YOU CAN TEAR OFF THIS PAGE FOR REFERENCE

Some useful info and formulas:

$$\mathrm{B} = \mu_0 (\mathrm{M} + \mathrm{H})$$

$$\mu_0 = 4\pi 10^{-7} H/m$$

$$\oint \bar{H} \cdot d\bar{s} = \mathrm{N}i$$

$$dB = \frac{\mu_0 i dl \sin\theta}{4\pi r^2}$$

$$E_D = -\frac{1}{2}\mu_0 \bar{M} \cdot \bar{H}_D$$

$$\bar{H}_D = -\bar{N} \cdot \bar{M}$$

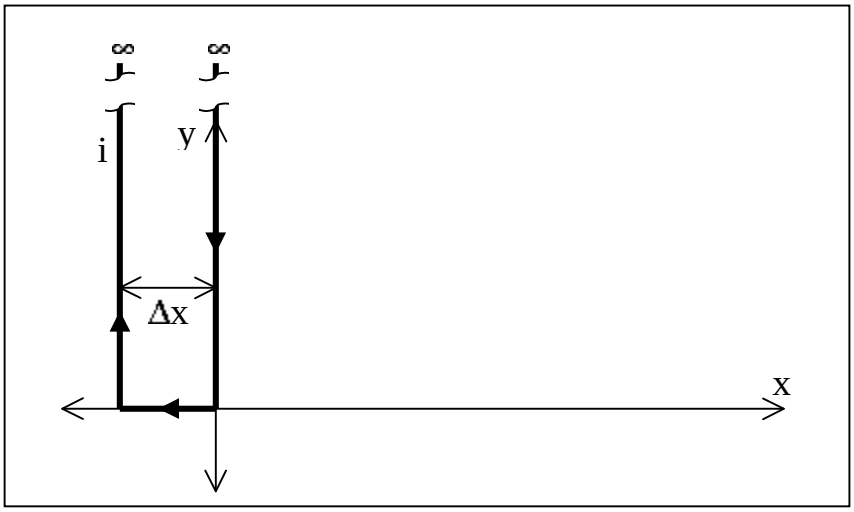
$$\sum N_i = 1$$

$$H_{eff} = H_{app} + H_D$$

$$E = mc^2 \text{ (just in case...)}$$

Problem 1. [35 pts]

(a) Derive the relationship for the magnetic field as a function of distance, x, from the following current loop, along the line y = 0. The origin of x and y is on the lower right corner of the loop, and you need only consider the B fields for points along the positive x-direction. The two vertical legs can be considered infinite in the positive y-direction. Be sure to use a right handed system (out of the paper is positive) [20 pts]



(b) Sketch a *log B* vs. *log x* curve, and indicate the numerical value of the slope of this graph in the limit of very small and very large x. [15 pts]

Problem 2. [20 pts]

Consider a sphere of magnetic material, initially demagnetized, and having the property that its magnetization is linearly proportional to the *total* field experienced by the material. This constant of proportionality is referred to as the magnetic susceptibility, χ, and satisfies the following relationship up to the point of saturation:

$$M = \chi H$$

For the sphere in this problem, χ = 1000, and the material has a saturation magnetization of 1000 kA/m. Suppose that an external field, H_z, were applied in the z-direction.

(a) Derive and expression relating the applied field, H_z to the effective field felt by the material, H_{eff}. [5 pts]
(b) Using the definition of χ, above, and the result from (a), find a relationship between H_z and M_z [5 pts]
(c) Plot M_z as a function of H_z. [5 pts]
(d) Label the point at which saturation occurs and indicate the numerical value of the applied field, H_z, at this point. [5 pts]

Problem 3. [45 pts]

Suppose you are trying to transfer all of the data from a disk drive to a linear tape drive, each of which have the specifications given below.

a) How many parallel channels on the tape system are needed such that the transfer process is limited by the data rate of the disk drive? [15 pts]
b) How long should the tape be to store all of the information from the disk drive, to the nearest meter? [15 pts]
c) How long, to the nearest minute, would it take to dump all of the data to the tape, given that a) is true, and no other latencies are present in the system. [15 pts]

Disk System	
Sides	8
Inner Radius	1 cm
Outer Radius	4 cm
PW50	250 nm
Max density	2 Bits/PW50
Rotation Rate	10000 RPM
Track Width	1 um
Tape System	
Usable Tape Width	8 mm
PW50	500 nm
Max linear density	2 Bits/PW50
Speed	5 m/s
Track Width	20 um

You may assume the following:

i) This is single zone recording, in which all of the tracks are recorded at a single data rate.

ii) The rotation rate of the disk is constant.

iii) The disk has been recorded with the maximum amount of information that it can hold, subject to the constraints described in i) and ii) and the parameters described in the table.

Solution to 1.

Only the wires directed in the y directions contribute, as the segment in the x-direction, has no projection along the x-axis.

The right and left segments contribute half what in infinite wire would contribute, respectively, where positive is out of the page:

$$B(right) = \frac{\mu_0 i}{4\pi x}$$

$$B(left) = -\frac{\mu_0 i}{4\pi (x + \Delta x)}$$

such that the net field is

$$B(net) = \frac{\mu_0 i \Delta x}{4\pi x (x + \Delta x)}$$

This problem can also be handled by direct integration of the Biot Savart law, as was shown in class for the square loop. The limiting cases are shown below:

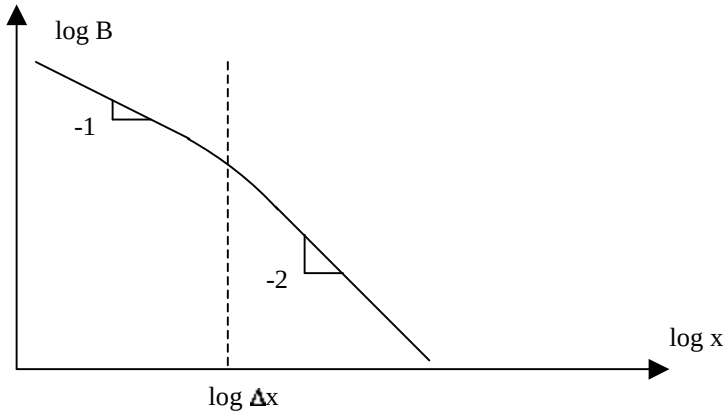
$$\lim_{x \to 0} B(net) = \frac{\mu_0 i}{4\pi x}$$

$$\lim_{x \to 0} \log B(net) = \log \frac{\mu_0 i}{4\pi} - \log x$$

and

$$\lim_{x \to \infty} B(net) = \frac{\mu_0 i \Delta x}{x^2}$$

$$\lim_{x \to \infty} \log B(net) = \log \Delta x \frac{\mu_0 i}{4\pi} - 2 \log x$$



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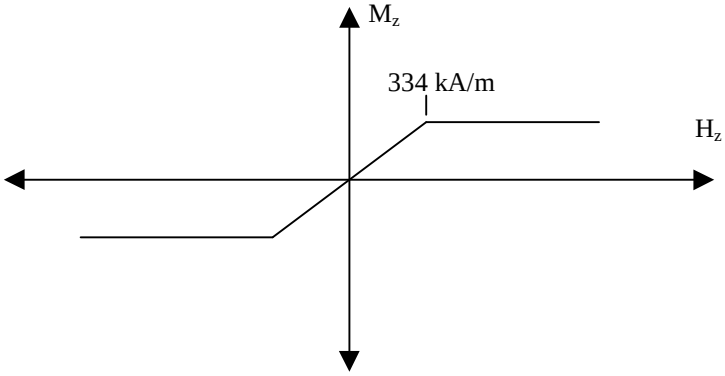
Solution to 2.

a) $M = \chi H_{eff}$

$H_{eff} = H_z + H_D$

$H_D = -N_z \cdot M_z$

$H_z = H_{eff}(1 + N\chi)$



b) $M = H_z \dfrac{\chi}{(1 + N\chi)}$

d) $H_z(sat) = M_s \dfrac{(1 + N\chi)}{\chi} = 334\,\text{kA/m}$

Solution to 3.

Disk System		
Sides	8	
Inner radius	0.01	m
Outer radius	0.04	m
PW50	2.50E-07	m
Max Linear Bit Density	2	bits/PW50
Rotation	10000	RPM
Track Width	1.00E-06	m
Linear Bit Density	8.00E+06	bits/m
Data Rate	8.38E+07	bits/sec
# Tracks	30000	
Total Data	1.21E+11	bits
Dump Time	1440	s
	24	min

Tape System		
Tape Width	0.008	m
PW50	5.00E-07	m
Max Linear Bit Density	2	bits/PW50
Speed	5	m/s
Track Width	2.00E-05	m
Linear Bit Density	4.00E+06	bits/m
Data Rate/Track	2.00E+07	bits/sec
# Tracks	400	
Capacity/Tape Length	1.60E+09	bits/m
Data Rate Ratio	4.19	
Minimum # heads	5	

Minimum Tape Length	75	m