Carnegie Mellon University

Department of Electrical and Computer Engineering

18-200 **Fall 2000**

Mathematical Foundations of Electrical Engineering

Problem Set 6 – Solution

Section 6.1 p. 309-310 You are encouraged to check your work with MATLAB

$$A = \begin{bmatrix} 2 & 1 \\ 1 & 7 \end{bmatrix} \qquad B = \begin{bmatrix} -2 & 5 \\ 0 & 8 \end{bmatrix}$$

1. Problem 2

$$4A - 8B = 4\begin{bmatrix} 2 & 1 \\ 1 & 7 \end{bmatrix} - 8\begin{bmatrix} -2 & 5 \\ 0 & 8 \end{bmatrix} = \begin{bmatrix} 8 & 4 \\ 4 & 28 \end{bmatrix} - \begin{bmatrix} -16 & 40 \\ 0 & 64 \end{bmatrix} = \begin{bmatrix} 8+16 & 4-40 \\ 4-0 & 28-64 \end{bmatrix} = \begin{bmatrix} 24 & -36 \\ 4 & -36 \end{bmatrix}$$

$$4(2B - A) = 8B - 4A = -(4A - 8B) = -\begin{bmatrix} 24 & -36 \\ 4 & -36 \end{bmatrix} = \begin{bmatrix} -24 & 36 \\ -4 & 36 \end{bmatrix}$$

$$8B - 4A = -(4A - 8B) = \begin{bmatrix} -24 & 36 \\ -4 & 36 \end{bmatrix}$$

2. Problem 7

$$(B^T - A^T)^T = (B^T)^T - (A^T)^T = B - A = \begin{bmatrix} -2 & 5 \\ 0 & 8 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} -4 & 4 \\ -1 & 1 \end{bmatrix}$$

$$A - A^T = \begin{bmatrix} 2 & 1 \\ 1 & 7 \end{bmatrix} - \begin{bmatrix} 2 & 1 \\ 1 & 7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

Section 6.2 p. 319-320 You are encouraged to check your work with MATLAB

$$a = \begin{bmatrix} 1 \\ 4 \\ 3 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix}, \quad d = \begin{bmatrix} 4 & 3 & 0 \end{bmatrix}$$

1. Problem 1

Since,

Number of rows of **a** (3) ≠ Number of columns of **B** (2)

the product **Ba** is not defined.

$$a^T B = [1 \quad 4 \quad 3]\begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} = [1\cdot 2 + 4\cdot 0 + 3\cdot 0 \quad 1\cdot(-3) + 4\cdot 2 + 3\cdot 1] = [2 \quad 8]$$

Since,

Number of rows of **B** (3) ≠ Number of columns of **a** (1)

the product **aB** is not defined.

2. Problem 8

$$a^T BB^T d^T = (a^T B)B^T d^T = [2 \quad 8]B^T d^T = [2 \quad 8]\begin{bmatrix} 2 & 0 & 0 \\ -3 & 2 & 1 \end{bmatrix}\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$

$$= [2\cdot 2 - 8\cdot 3 \quad 2\cdot 0 + 8\cdot 2 \quad 2\cdot 0 + 8\cdot 1]\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix} = [-20 \quad 16 \quad 8]\begin{bmatrix} 4 \\ 3 \\ 0 \end{bmatrix}$$

$$= [-20\cdot 4 + 16\cdot 3 + 8\cdot 0] = -32$$

$$dBB^T a = \left[(dBB^T a)^T\right]^T = \left[a^T BB^T d^T\right]^T = [-32]^T = -32$$

$$CB = \begin{bmatrix} 4 & 6 & 2 \\ 6 & 0 & 3 \\ 2 & 3 & -1 \end{bmatrix}\begin{bmatrix} 2 & -3 \\ 0 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4\cdot 2 + 6\cdot 0 + 2\cdot 0 & -4\cdot 3 + 6\cdot 2 + 2\cdot 1 \\ 6\cdot 2 + 0\cdot 0 + 3\cdot 0 & -6\cdot 3 + 0\cdot 2 + 3\cdot 1 \\ 2\cdot 2 + 3\cdot 0 - 1\cdot 0 & -2\cdot 3 + 3\cdot 2 - 1\cdot 1 \end{bmatrix} = \begin{bmatrix} 8 & 2 \\ 12 & -15 \\ 4 & -1 \end{bmatrix}$$

$$B^T C^T = (CB)^T = \begin{bmatrix} 8 & 2 \\ 12 & -15 \\ 4 & -1 \end{bmatrix}^T = \begin{bmatrix} 8 & 12 & 4 \\ 2 & -15 & -1 \end{bmatrix}$$

Section 6.3 p. 329-330

The Gauss elimination problem will use the following notations:

(1) R_i <–> R_j means "interchange i^{th} and j^{th} rows".

(2) R_i –> aR_i + bR_j means "replace i^{th} row by a times i^{th} row plus b times j^{th} row". The plus could well be replaced by a minus.

1. Problem 4

$$\begin{bmatrix} 0 & 7 & 3 \\ 2 & 8 & 1 \\ -5 & 2 & -9 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -12 \\ 0 \\ 26 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 0 & 7 & 3 & -12 \\ 2 & 8 & 1 & 0 \\ -5 & 2 & -9 & 26 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 2 & 8 & 1 & 0 \\ 0 & 7 & 3 & -12 \\ -5 & 2 & -9 & 26 \end{array}\right] \qquad R_1 <-> R_2$$

$$\left[\begin{array}{ccc|c} 2 & 8 & 1 & 0 \\ 0 & 7 & 3 & -12 \\ 0 & 44 & -13 & 52 \end{array}\right] \qquad R_3 -> 5R_1 + 2R_3$$

$$\left[\begin{array}{ccc|c} 2 & 8 & 1 & 0 \\ 0 & 7 & 3 & -12 \\ 0 & 0 & -223 & 892 \end{array}\right] \qquad R_3 -> 7R_3 - 44R_2$$

$$\left[\begin{array}{ccc|c} 2 & 8 & 1 & 0 \\ 0 & 7 & 3 & -12 \\ 0 & 0 & 1 & -4 \end{array}\right] \qquad R_3 -> \frac{R_3}{-223}$$

$$\left[\begin{array}{ccc|c} 2 & 8 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -4 \end{array}\right] \qquad R_2 -> \frac{R_2 - 3R_3}{7}$$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 0 \\ -4 \end{bmatrix}$$

$\therefore \quad x = 2, \quad y = 0, \quad z = -4$

2. Problem 5

$$\begin{bmatrix} 1 & 1 & -1 \\ 0 & 8 & 6 \\ -2 & 4 & -6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ -6 \\ 40 \end{bmatrix}$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ -2 & 4 & -6 & 40 \end{array}\right]$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 6 & -8 & 58 \end{array}\right] \qquad R_3 -> R_3 + 2R_1$$

$$\left[\begin{array}{ccc|c} 1 & 1 & -1 & 9 \\ 0 & 8 & 6 & -6 \\ 0 & 0 & 1 & -5 \end{array}\right] \qquad R_3 -> \frac{8R_3 - 6R_2}{-100}$$

$$\left[\begin{array}{ccc:c} 1 & 1 & -1 & 9 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right] \qquad R_2 -> \frac{R_2 - 6R_3}{8}$$

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -5 \end{array}\right] \qquad R_1 -> R_1 - R_2 + R_3$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ -5 \end{bmatrix}$$

$\therefore \quad x = 1, \quad y = 3, \quad z = -5$

3. Problem 9

$$\begin{bmatrix} 0 & 4 & 3 \\ 2 & 0 & -1 \\ 3 & 2 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 8 \\ 2 \\ 5 \end{bmatrix}$$

$$\left[\begin{array}{ccc:c} 0 & 4 & 3 & 8 \\ 2 & 0 & -1 & 2 \\ 3 & 2 & 0 & 5 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 2 & 0 & -1 & 2 \\ 0 & 4 & 3 & 8 \\ 3 & 2 & 0 & 5 \end{array}\right] \qquad R_1 <-> R_2$$

$$\left[\begin{array}{ccc:c} 2 & 0 & -1 & 2 \\ 0 & 4 & 3 & 8 \\ 0 & 4 & 3 & 4 \end{array}\right] \qquad R_3 -> 2R_3 - 3R_1$$

$$\left[\begin{array}{ccc:c} 2 & 0 & -1 & 2 \\ 0 & 4 & 3 & 8 \\ 0 & 0 & 0 & -4 \end{array}\right] \qquad R_3 -> R_3 - R_2$$

A contradiction is produced here. This shows that this system has no solution.

4. Problem 13

$$\begin{bmatrix} 0 & 5 & 5 & -10 \\ 2 & -3 & -3 & 6 \\ 4 & 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 4 \end{bmatrix}$$

$$\left[\begin{array}{cccc:c} 0 & 5 & 5 & -10 & 0 \\ 2 & -3 & -3 & 6 & 2 \\ 4 & 1 & 1 & -2 & 4 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 2 & -3 & -3 & 6 & 2 \\ 0 & 5 & 5 & -10 & 0 \\ 4 & 1 & 1 & -2 & 4 \end{array}\right] \qquad R_1 <-> R_2$$

$$\left[\begin{array}{cccc:c} 2 & -3 & -3 & 6 & 2 \\ 0 & 1 & 1 & -2 & 0 \\ 4 & 1 & 1 & -2 & 4 \end{array}\right] \qquad R_2 -> \frac{R_2}{5}$$

$$\left[\begin{array}{cccc:c} 2 & -3 & -3 & 6 & 2 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 1 & 1 & -2 & 0 \end{array}\right] \qquad R_3 -> \frac{R_3 - 2R_1}{7}$$

$$\left[\begin{array}{cccc:c} 2 & -3 & -3 & 6 & 2 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad R_3 -> R_3 - R_2$$

$$\left[\begin{array}{cccc:c} 1 & 0 & 0 & 0 & 1 \\ 0 & 1 & 1 & -2 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \qquad R_1 -> \frac{R_1 + 3R_2}{2}$$

Thus,

$$w = 1$$
$$x + y - 2z = 0$$

Therefore, we can determine the value of w and find an equation for x, y, and z. We can arbitrarily set values for two of x, y, and z and find the value of the third one. So there are infinite solutions for this system.

5. Problem 14

$$\begin{bmatrix} 2 & -2 & 4 & 0 \\ -3 & 3 & -6 & 5 \\ 1 & -1 & 2 & 0 \end{bmatrix} \begin{bmatrix} w \\ x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 15 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{cccc:c} 2 & -2 & 4 & 0 & 0 \\ -3 & 3 & -6 & 5 & 15 \\ 1 & -1 & 2 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cccc:c} 0 & 0 & 0 & 0 & 0 \\ -3 & 3 & -6 & 5 & 15 \\ 1 & -1 & 2 & 0 & 0 \end{array}\right] \qquad R_1 -> R_1 - 2R_3$$

$$\left[\begin{array}{cccc:c} 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 1 & -1 & 2 & 0 & 0 \end{array}\right] \qquad R_2 -> \frac{R_2 + 3R_3}{5}$$

Thus,

$$z = 3$$
$$w - x + 2y = 0$$

Therefore, we can determine the value of z and find an equation for w, x, and y. We can arbitrarily set values for two of w, x, and y and find the value of the third one. So there are infinite solutions for this system.

6. Problem 17

KVL for up loop: $4i_1 + i_3 = 16$

KVL for bottom loop: $4i_2 + i_3 = 32$

KCL for the middle point of the left side: $i_1 + i_2 - i_3 = 0$

Thus we have

$$\begin{bmatrix} 4 & 0 & 1 \\ 0 & 4 & 1 \\ 1 & 1 & -1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 16 \\ 32 \\ 0 \end{bmatrix}$$

$$\left[\begin{array}{ccc:c} 4 & 0 & 1 & 16 \\ 0 & 4 & 1 & 32 \\ 1 & 1 & -1 & 0 \end{array}\right]$$

$$\left[\begin{array}{ccc:c} 4 & 0 & 1 & 16 \\ 0 & 4 & 1 & 32 \\ 0 & 4 & -5 & -16 \end{array}\right] \qquad R_3 -> 4R_3 - R_1$$

$$\left[\begin{array}{ccc:c} 4 & 0 & 1 & 16 \\ 0 & 4 & 1 & 32 \\ 0 & 0 & 1 & 8 \end{array}\right] \qquad R_3 -> \frac{R_3 - R_2}{-6}$$

$$\left[\begin{array}{ccc:c} 4 & 0 & 1 & 16 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 8 \end{array}\right] \qquad R_2 -> \frac{R_2 - R_3}{4}$$

$$\left[\begin{array}{ccc:c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 6 \\ 0 & 0 & 1 & 8 \end{array}\right] \qquad R_1 -> \frac{R_1 - R_3}{4}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} i_1 \\ i_2 \\ i_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \\ 8 \end{bmatrix}$$

$$\therefore \quad i_1 = 2\,(A), \quad i_2 = 6\,(A), \quad i_3 = 8\,(A)$$