Carnegie Mellon University

Department of Electrical and Computer Engineering

18-200 **Fall 2000**

Mathematical Foundations of Electrical Engineering

Problem Set 5

Issued: Tuesday, October 17
Due: Thursday, October 26

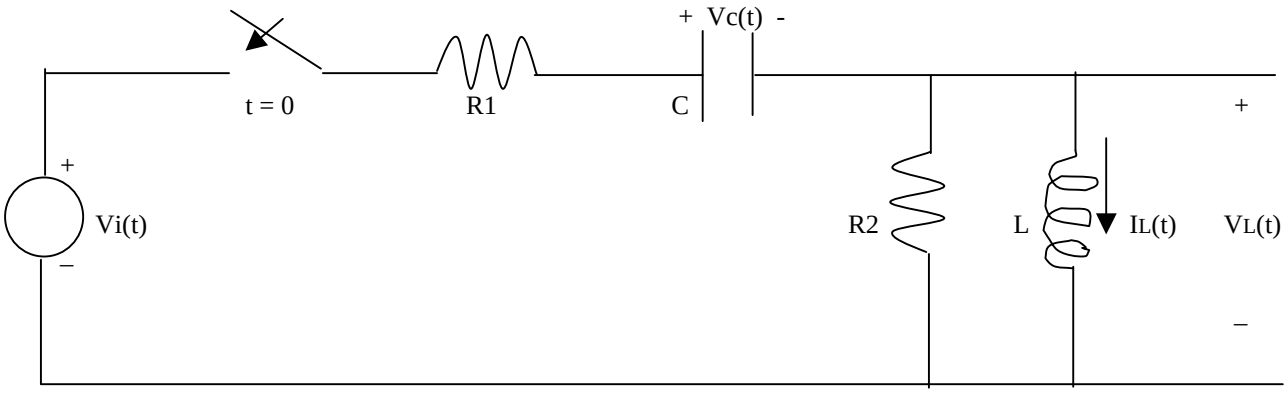
Note: Exam II will be held during lecture time on October 31, 2000.

1. For this problem, (sinusoidal driving function), please evaluate the *particular* solution using:
   a) the phasor method that we've discussed in class and,
   b) the method of undetermined coefficients (assume a solution of the form Kcos(3t) + Msin(3t)).
   And show that you get the same result doing this using both of these methods.

   c) Also solve for the homogeneous solution and find the total solution for y(t).

$y'' + 4y' + 4y = -5 \sin(3t) + 12 \cos(3t); \quad y(0) = 1; y'(0) = 3$

2. This next problem is an L-R-C circuit with a sinusoidal driving function (voltage source) attached.



For this problem, we are going to solve for the voltage in the capacitor, Vc(t), and current in the inductor, $I_L(t)$. The switch is closed at time t=0, and the initial conditions on Vc(t) and $I_L(t)$ are: $Vc(0) = 0v$; $I_L(0) = 0A$.

a) Write two second-order non-homogeneous ordinary differential equations (ODE) for the circuit above, one for Vc(t) and one for $I_L(t)$. Your answers should be in terms of R1, R2, L, C, Vi(t) and Vi'(t) for both equations, and either Vc(t) (and its derivatives), or $I_L(t)$ (and its derivatives) depending on the quantity for which you are writing the ODE.

To help you do this, you'll need to do the following things:

i) Use Ohm's law to write an equation for the current through R1 (also the current through C) in terms of R1 and voltages Vi(t), Vc(t), and $V_L(t)$.
ii) Write an equation for the current through the capacitor (C) in terms of C and Vc(t)
iii) Write an equation for the voltage across the inductor (L) in terms of L and the current through it.
iv) Write a KCL equation at the node where C, L, and R2 come together.
v) Use Ohm's law to write an equation for the current through R2 in terms of $V_L(t)$ and R2.

Do some equation combining to get a second-order equation with terms of Vi(t) and Vi'(t) (the derivative of Vi(t)) as the right-hand side. (Remember, the characteristic equation must be the same for both the Vc(t) and I_L(t) ODE's.)

For Vc(t) I got:

L [1 + R1/R2] *Vc"(t) + [R1 + L/(R2*C)] *Vc'(t) + [1/C] *Vc(t) = [L/(C*R2)] *Vi'(t) + [1/C]*Vi(t)

And for I_L(t), I got:

L [1 + R1/R2] * I_L"(t) + [R1 + L/(R2*C)] * I_L'(t) + [1/C] * I_L(t) = Vi'(t)

b) Solve the characteristic equation for the above ODE and express your λ's in terms of the circuit components: L, C, R1, and R2.

c) As in lecture, let L = 100 microhenrys and C = 100 picofarads. If R2 = infinity (open circuit, or removing R2 from the circuit), what range(s) of R1 make the system "over damped" and "under damped"? What is the value of R1 you need to make the system "critically damped"?

Note: If you remove R2 from this circuit, you just have the series R-L-C circuit like we solved in class, with R=R1. You should note that the two ODE's that we derived in part a) reduce to the case we did in class if you let R2 go to infinity.

d) If the values of L and C are as in part c) and R1 = 0 ohms (short circuit), what range(s) of R2 make the system "over damped" and "under damped"? What is the value of R2 you need to make the system "critically damped"?

e) Let the values of L and C be as in part c), let R1 = 0 ohms, and let R2 = 2000 ohms. Let Vi(t) = 10v. Solve the non-homogeneous ODE's with these values and the initial conditions given in the problem statement to find final function's for the voltage Vc(t) and current I_L(t). (You may use MATLAB or your calculator to help you with the math, but set up the equations so we can see what you've done.) Use MATLAB to plot the voltage Vc(t) and current I_L(t) in the range of t = 0 to 10 **micro**-seconds. (use a fairly small step size so you can see what is going on, like 0.05 microseconds or less).