

## (Lec05) BDDs Applied: Finite State Machine Verific

### ▼ What you know

- ▶ Representations: Cube lists, BDDs
- ▶ Manipulations: URP attacks on cubes, implementation for BDDs
- ▶ Useful computations
  - ▷ Are these 2 blocks of logic doing the same thing for all inputs?
  - ▷ Build a BDD for each and see if they are identical

### ▼ What you don't know

- ▶ Cool ways people apply BDDs out in real world
- ▶ Example: Verifying logic with any kind of **time-varying** behavior
- ▶ Important application: Finite State Machines
  - ▷ I give you 2 different FSM implementations
  - ▷ You tell me: are they *behaviorally equivalent*...
  - ▷ ...ie, for same input stream, will they make identical outputs?
- ▶ This is a whole new problem...

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## Where Are We?

▼ Something *really* new, made possible by BDDs: *Formal Verif.*

	M	T	W	Th	F	
Aug	27	28	29	30	31	1
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Introduction  
Advanced Boolean algebra  
JAVA Review  
**Formal verification**  
2-Level logic synthesis  
Multi-level logic synthesis  
Technology mapping  
Placement  
Routing  
Static timing analysis  
Electrical timing analysis  
Geometric data structs & apps

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## Handouts

### ▼ Physical

- ▶ This lecture -- Lec05 Formal Verification

### ▼ Electronic

- ▶ Project I (will be shortly...) on the web site. In project I, we give you the skeleton of a BDD package in JAVA, and you get to complete it, and then try to apply it to a portfolio of common gate-level logic test/verification problems.
- ▶ HW2 is also (still out) on the web site. HW2 covers lectures 3, 4 (BDD basics and internals) but not lecture 5 (FSM verification).
  - ▷ Some HW2 bug fixes will also appear shortly

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## Readings

### ▼ De Micheli

- ▶ Doesn't really do much on formal verification

### ▼ Bryant's Symbolic Analysis Paper from Comp Surveys

- ▶ Does a lot of this material...
- ▶ ...but it's very dense. Go thru the lecture, work thru all the algebra, go back and look at Bryant's paper.

## Some Terminology

### ▼ Verification

- ▶ You give me a logic implementation of some design, and some specification you guarantee is correct
- ▶ I figure out -- somehow -- whether the implementation is correct

### ▼ Strategy: Simulation

- ▶ Validate that the logic implementation works for all the inputs that you simulate
- ▶ Problem: it might NOT work for an input you DON'T simulate!

### ▼ Strategy: Formal Verification

- ▶ Prove that there DOES NOT exist an input (or a sequence of inputs over time) that causes the logic to make a wrong answer
- ▶ Or, FIND an input that causes it to make a wrong answer

# Formal Verification: Who Cares...?

## Computer Stocks Tumble Over Chip Flaw

By Bloomberg Business Week  
 Shares of computer companies tumbled yesterday because of concerns about a flaw in the Intel Corporation's top-of-the-line Pentium chip. Although the flaw, which was disclosed last week, affects only complex mathematical calculations and

Intel is faulted for poor P.R. in its Pentium problem.

satisfaction," said John Latta, a computer analyst at Paine Webber. In a report, Intel said it would not begin shipping new chips without the flaw for several weeks. "This is a public relations disaster, and it puts Intel in an embarrassing situation," said Fries

## Flurry of Lawsuits Filed Against Intel Over Pentium Flaw

By RICHARD B. SCHMITT  
 Staff Reporter of THE WALL STREET JOURNAL  
 Lawsuits against Intel Corp. over its flawed Pentium chip are rapidly multiplying, even as debate continues over how much harm the device's inability to do certain complex math calculations will cause.

The legal assault, including at least 10 suits in three states as of late yesterday, accuses the Santa Clara, Calif., chip maker

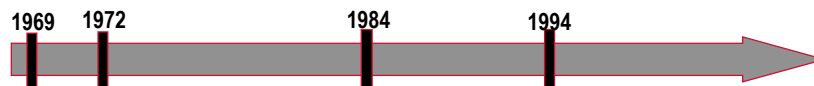
### ▼ Intel, for one...

- ▶ They simulated the divider a whole lot
- ▶ They still missed some inputs that made errors
- ▶ Result, the Pentium FDIV bug, lots of bad press, lots and lots of money lost
- ▶ Formal verification techniques are now capable of finding errors like these in complex designs

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# RAR's Amazing (and Coincidental) Link to This..



**Dan Atkins:**  
 Grad student at U Illinois publishes paper in IEEE Trans. Computers on how to do this division alg.

In 1972 becomes Prof at U Michigan, and later Dean of Engineering.  
 His last Phd student: **Rob Rutenbar**



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**Intel FDIV Bug:**  
 Intel, in a mistaken attempt to save Si area on a Pentium, over-simplifies a crucial piece of logic in Atkins' *High-Radix Quotient-digit* algorithm.  
 Result: "the Pentium bug"

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## Formal Verification: This Lecture

### ▼ Pick *one* significant problem: FSM verification

- ▶ Review finite state machines (from basic digital designs)
- ▶ Show why this is a hard problem
- ▶ Show a clever attack on the verification problem that exercises a lot of what you know about BDDs and their capabilities

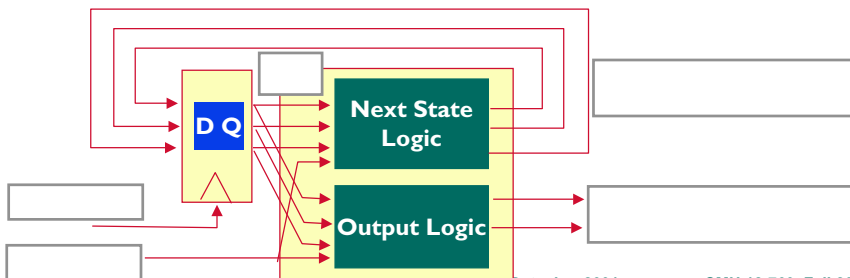
### ▼ New stuff here

- ▶ Dealing with temporal behavior: we want to know the FSM works for all patterns of inputs, over all future clock ticks
- ▶ Representing this temporal behavior using Boolean functions
- ▶ Reasoning about this behavior
- ▶ Turning the whole shebang into a sequence of symbolic computations you could do with C code + a good BDD package

## FSM Verification: Reminders

### ▼ What's in an FSM?

- ▶ States -- unique bit pattern represents each state; FFs store state
- ▶ External inputs: inputs from the outside world
- ▶ Next state logic: from current state and external inputs, this makes the inputs to the FFs that determine the next state at the next clock tick
- ▶ Output logic: from states (and maybe the inputs) makes combinational outputs for the FSM
- ▶ Clock: synchronizes everything; only states changes on clock tick



## FSM Verification: What's the Problem?

### ▼ Several scenarios

- ▶ You have a “trusted” implementation of the machine that you know is correct (but maybe not optimal as hardware) and a “real” implementation as gates. Are they the same?
- ▶ You have an “old” implementation in one technology, and a “new” implementation in another technology (eg, a different tech library). Is old == new?

### ▼ Why is this hard?

- ▶ Because of what “same” means here
- ▶ Has to do with behavior over time -- this **temporal dependence** is new for us...
- ▶ We need to be more precise

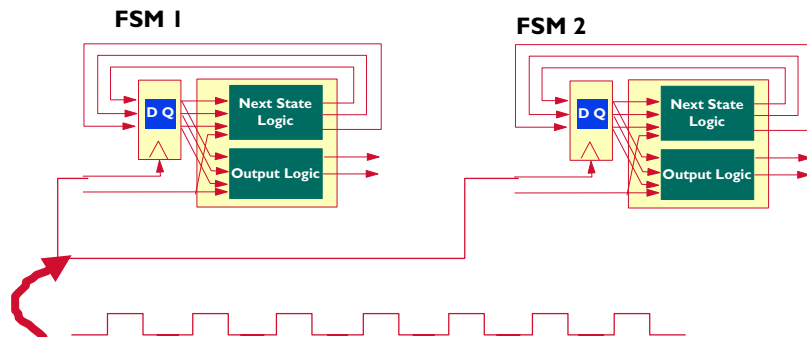
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## Equivalent FSMs

### ▼ Means this:

- ▶ Start them in some known “equivalent” states; clock starts running
- ▶ For **every** possible combination of inputs over future clock ticks, two machines will have *identical* outputs
- ▶ (Doesn't say anything about logic delays or low level stuff like that; just think about ideal clock ticks here...)



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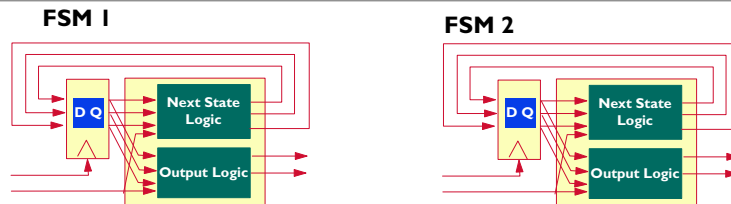
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## FSM Equivalence: Easy Case

### ▼ Sometimes this isn't too hard: special case

- ▶ Suppose 2 FSMs have identical state encodings: same #bits, same unique bit pattern for each state, same kinds of FFs

### ▼ Then this reduces to combinational equiv. checking



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## Combinational Equivalence Checking

### ▼ Reminder

- ▶ This is the easiest “formal verification” sort of problem
- ▶ You have 2 different implementations of the same function
- ▶ They have identical input and output variables
- ▶ Your task: determine if they give identical outputs over all possible inputs, or find a counterexample where the outputs differ

### ▼ Why is this easy?

- ▶ Build a BDD for each function. If they are identical, you get the identical same BDD pointer for each one.
- ▶ If not identical, build BDD for  $(\text{function } F) \oplus (\text{function } G)$  and find satisfying inputs for this new function. These inputs make  $F \neq G$ !
- ▶ Easy since it uses all the standard BDD stuff.
- ▶ No notion of time in here, no sequences of inputs over clock ticks

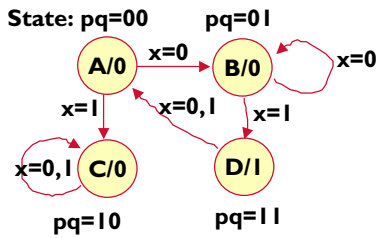
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# FSM Equivalence: Hard Case

▼ FSMs not always so easy to check. More general case:

► 4 states. 2 state variables p,q. 1 input x. 1 output z. D FFs.



Input x	PS	NS
	pq	p+q+
0	00	
	01	
	10	
	11	
1	00	
	01	
	10	
	11	

x \ pq	00	01	11	10
0				
1				

p+ (= D input on p FF)

x \ pq	00	01	11	10
0				
1				

q+ (= D input on q FF)

p \ q	0	1
0		
1		

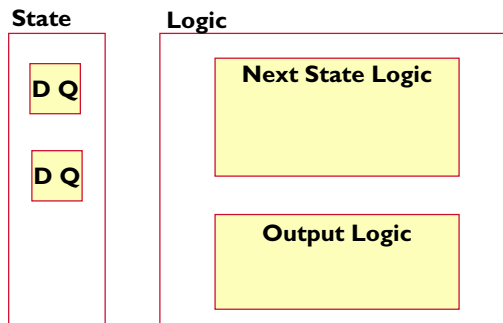
z

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# FSM1 Implementation

▼ It looks like this...



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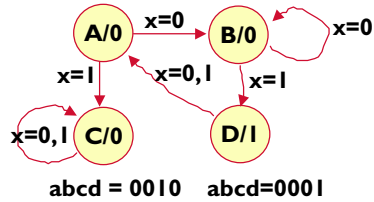


## FSM2: Same Behavior, Different Implementation

▼ Let's implement it as a 1-hot machine

► Now, 4 state variables: abcd; again D FFs, but now 4 of them

State: abcd=1000    abcd=0100



Can just read off the NS logic

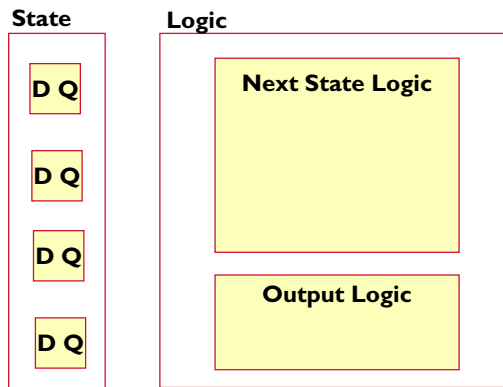
1 hot example transition...

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## FSM 2 Implementation

▼ It looks like this



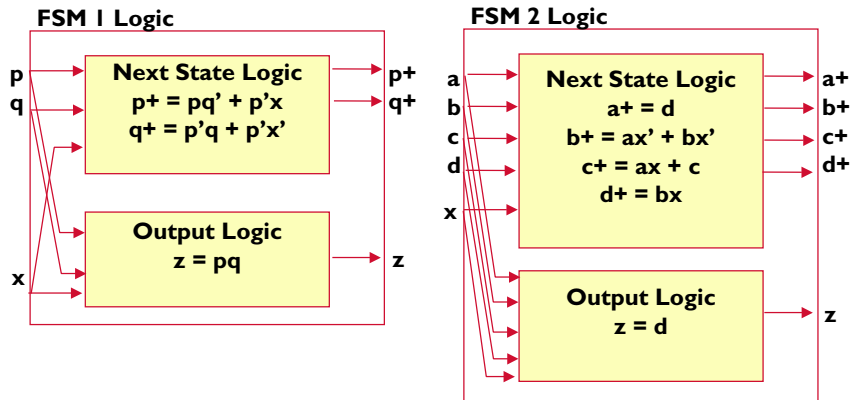
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## FSM Equivalence Checking

### ▼ This is why it's hard

- ▶ Can't just look at the logic now and say "yeah, they're the same"
- ▶ Need a whole new, systematic method that deals with temporal aspect of things here, and the possible differences in encodings



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## FSM Formal Verification Strategy

### ▼ 4 Big Ideas

#### ▼ Sets as Boolean functions

- ▶ Use BDDs to represent sets of things

#### ▼ Symbolic representation of FSMs

- ▶ Represent them as sets of allowable transitions

#### ▼ Reachability analysis

- ▶ Represents the sets of FSM states you can get to from the start state on 0 clock ticks, 1 clock tick, 2 ticks, etc.

#### ▼ Cross-product FSMs

- ▶ Take 2 FSMs you want to compare for equivalence, and make 1 single new special machine, on which reachability analysis == verification

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# 1. Sets as Boolean Functions

▼ We already saw this idea on an early HW

- ▶ Suppose your objects in your sets are: a b c d e f
- ▶ Assume these are now Boolean vars; var = 1 means “an object in set”
- ▶ Represent set as function; value = 1 for patterns that == **one** obj in set

let  $S = \{a, b, f\}$

a b c d e f S

0 0 0 0 0 0 0

0 0 0 0 0 1 1

0 0 0 0 1 1 0

0 0 0 1 0 0 0

.

0 1 0 0 0 0 1

.

1 0 0 0 0 0 1

1 1 0 0 0 0 0

.

.

.



Since it's a Boolean function,  
we can also represent this as  
a BDD, even for very large sets

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# Sets as Boolean Functions

▼ BDD representation lets you do some neat stuff

- ▶ Suppose  $A(a,b,c,d,e,f)$ ,  $B(a,b,c,d,e,f)$  are sets represented as BDDs

What is  $A \cup B$ ?

What is  $A \cap B$ ?

Is  $A \subset B$ ?

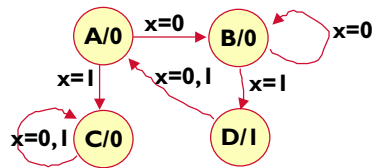
Is A the empty set?

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## 2. Symbolic Representation of FSMs

▼ Idea is to represent the set of all legal transitions



Legal transitions

FROM	ON Input	TO

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## Symbolic FSM Representation

▼ How do we do this

- ▶ First, what **NOT** to do: can't do it by enumerating all transitions and "adding logic" to represent each one
- ▶ A machine with 100 FFs + 10 inputs has approx.  $1000 \cdot 2^{100}$  transitions!

▼ What do we want?

- ▶ A new Boolean function, called  $\delta()$ , the "transition relation"

$$\delta(\text{state vars for current state, input vars, state vars for next state}) = \begin{cases} 0 & \text{if transition not legal} \\ 1 & \text{if transition is legal} \end{cases}$$

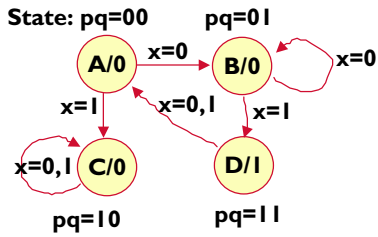
- ▶ Let's look at some examples

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## Transition Relation Examples

▼ Look at FSM 1: it will be  $\delta(p, q, x, p+, q+)$



$\delta(p=0, q=0, x=0, p+=0, q+=1) =$   
 $\delta(p=0, q=0, x=1, p+=1, q+=0) =$   
 $\delta(p=1, q=1, x=1, p+=1, q+=1) =$   
 $\delta(p=0, q=1, x=1, p+=0, q+=1) =$

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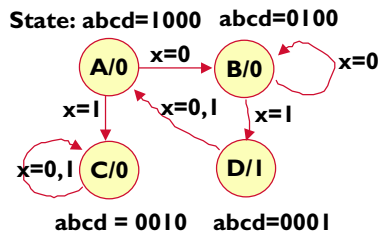
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## Transition Relation Examples

▼ OK, now FSM 2: it will be:  $\delta(a,b,c,d,x,a+,b+,c+,d+)$

$\delta(a=0, b=0, c=0, d=1, x=0, a+=0, b+=0, c+=0, d+=1) =$

$\delta(a=1, b=0, c=0, d=0, x=0, a+=0, b+=1, c+=0, d+=0) =$



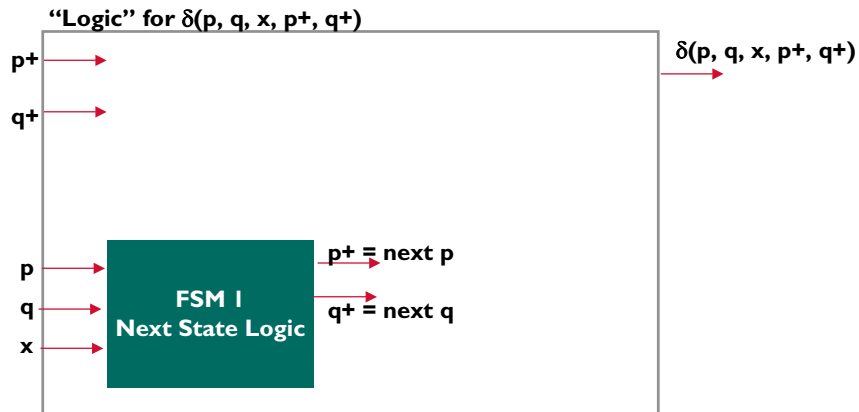
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## Constructing the Transition Relation

### What do we want? a BDD for $\delta()$

- ▶ There is a great, simple trick here
- ▶ The next state logic is already most of the logic you need



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## About the Transition Relation

### Bryant reports that, as a BDD, it can be *very big*

- ▶ Much bigger than the next state logic itself, which is inside of it
- ▶ Various tricks to get around having to build the whole thing

### For us though, we'll just press on

- ▶ Knowing how to do this in the most straightforward, “frontal assault” is fine for now.

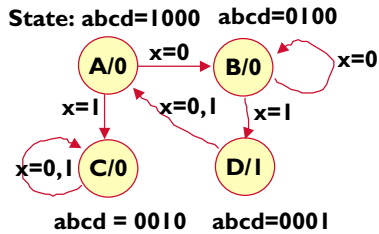
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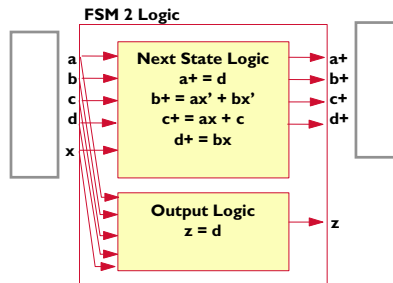
# About the Transition Relation

## One subtlety: May be some “bogus” stuff in $\delta$

- ▶ What happens if not all bit patterns are legal states?
- ▶ Will find also all the  $\delta$ ( illegal old state, input, illegal new state) too
- ▶  $\delta$  tells you *everything* about FSM. Ignores legal start state assumptions.



Legal = 1000, 0100, 0010, 0001  
All others illegal



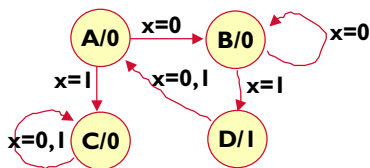
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# 3. Reachability Analysis

## What's the idea?

- ▶ Build a Boolean function that represents all the FSM states you can reach in up to  $K$  clock ticks, starting from a known initial state at clock tick 0
- ▶ Sets called “reachability” sets:  $R_0, R_1, R_2, \dots, R_K$
- ▶ Example of what we expect to happen.



$R_0 =$  fixed initial state = { A }

$R_1 =$  states you can reach on 0 or 1 tick = { }

$R_2 =$  states you can reach on 0, 1, or 2 ticks = { }

$R_3 =$  states you can reach on 0, 1, 2 or 3 ticks = { }

$R_4 =$  0, 1, 2, 3 or 4 ticks = { } =

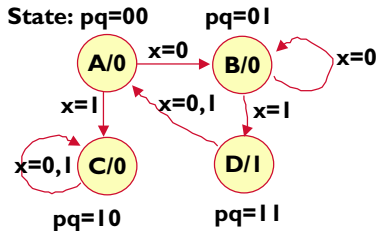
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# Reachability Analysis

## Observations

- ▶  $R_k$  is a function only of the state variables, since all you want to know is if the state bit pattern you input can be reached from the start state in not more than  $k$  clock ticks



Reachability functions

- ▶  $R_0$  is easy to compute, since there is only 1 state in it, the assumed initial (reset) state for the FSM.

Assume A is initial state; then  $R_0$  for this FSM =

# Reachability Analysis

## The hard part: going from $R_k$ to $R_{k+1}$

### What do we know to start? Ex: FSM 1

- ▶ Transition relation:  $\delta(p, q, x, p+, q+)$  (we have a BDD for this)
- ▶  $R_0 = R_0(p+, q+) = (p+)' \cdot (q+)'$  (we have a BDD for this too)

### What do we want to do?

- ▶ Compute  $R_1(p+, q+)$  using  $R_0$  and  $\delta(p, q, x, p+, q+)$
- ▶ Then, compute  $R_2(p+, q+)$  using  $R_1(p+, q+)$  and  $\delta(p, q, x, p+, q+)$
- ▶ Then  $R_3(p+, q+)$  from  $R_2(p+, q+)$  and  $\delta(p, q, x, p+, q+)$  ...
- ▶ ...keep going until  $R_{k+1} = R_k$  for some  $k$ , and we are done
- ▶ Note: easy to tell if  $R_k = R_{k+1}$  given BDDs!

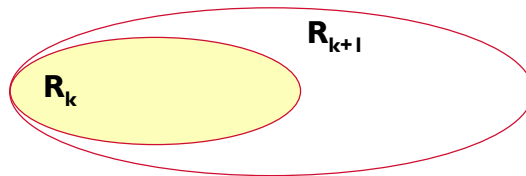
### Called *iterated reachability analysis*



## Iterated Reachability Analysis

### ▼ Mechanically, how?

- ▶ Need to look close at the structure of the R sets
- ▶ Think about sets like Venn diagrams: what's inside what?
- ▶ If you know  $R_k$ , what else is there to get to  $R_{k+1}$ ...

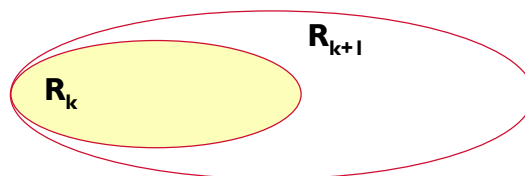


Observation #1

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## Iterated Reachability Analysis

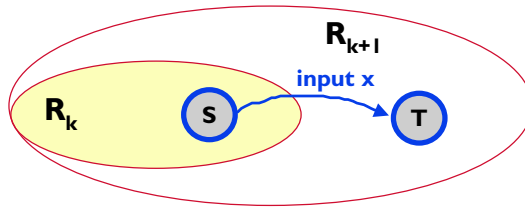


Observation #2

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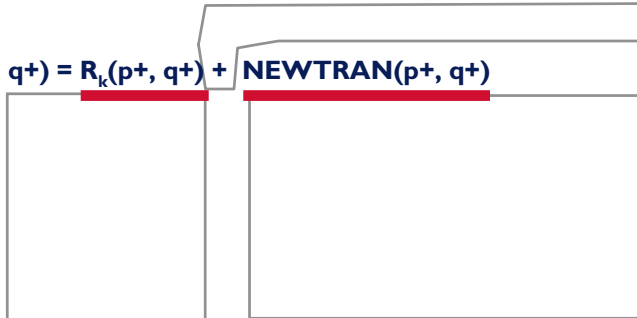
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# Iterated Reachability Analysis

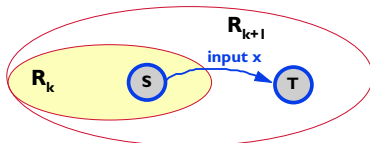


▼ So, for FSM1 example we can write  $R_k$  like this:

$$R_{k+1}(p+, q+) = R_k(p+, q+) + \text{NEWTRAN}(p+, q+)$$



# Iterated Reachability Analysis



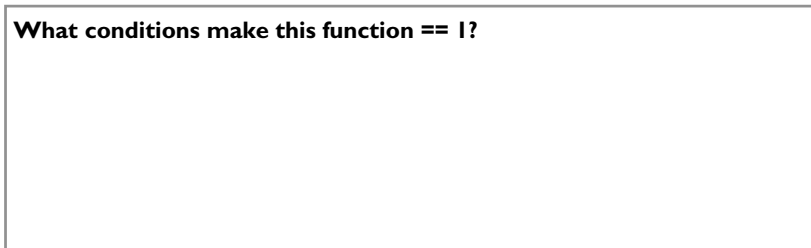
$$R_{k+1}(p+, q+) = R_k(p+, q+) + \text{NEWTRAN}(p+, q+)$$

▼ Focus on the hard part here: what is “NEWTRAN” here

► Consider this function, made of pieces we know:

$$R_k(p, q) \cdot \delta(p, q, x, p+, q+)$$

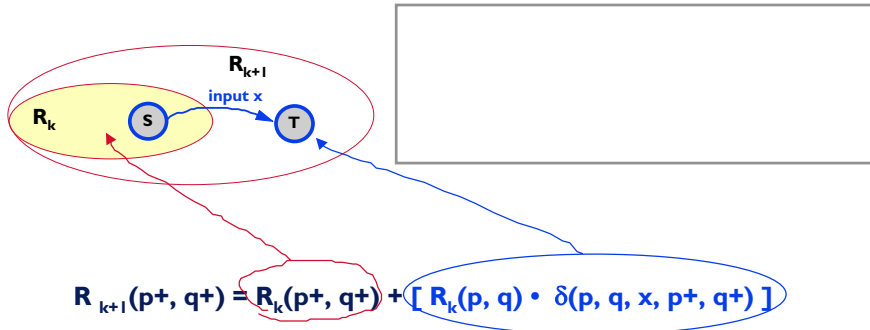
What conditions make this function == 1?



## Iterated Reachability Analysis

▼ OK, this guess is close, but...

- ▶ Why can't  $[ R_k(p, q) \cdot \delta(p, q, x, p^+, q^+) ]$  be the piece we want?
- ▶ It has to depend only on the variables  $p^+, q^+$
- ▶ In other words, we want a function that  $\equiv I$  just for the new state patterns that are reachable in 1 transition from  $R_k$ ...
- ▶ ... we don't care from which exact state, or with which input



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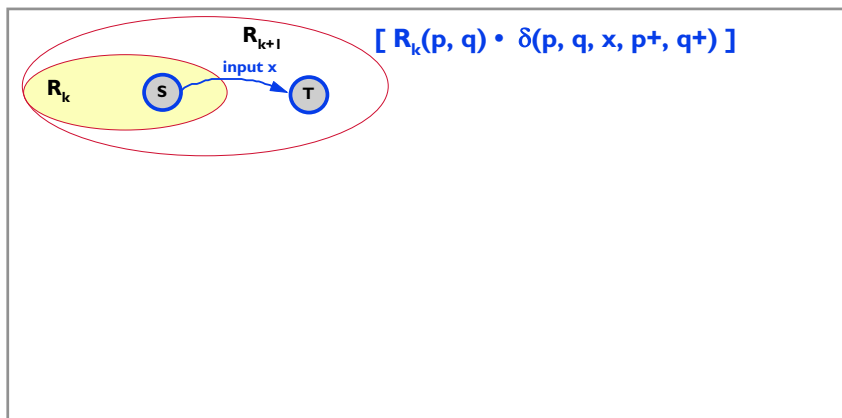
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## Iterated Reachability Analysis

▼ Quantification to the rescue!

- ▶ (Another reason we keep hammering this elusive yet powerful idea...)

▼ Say in English what we want:

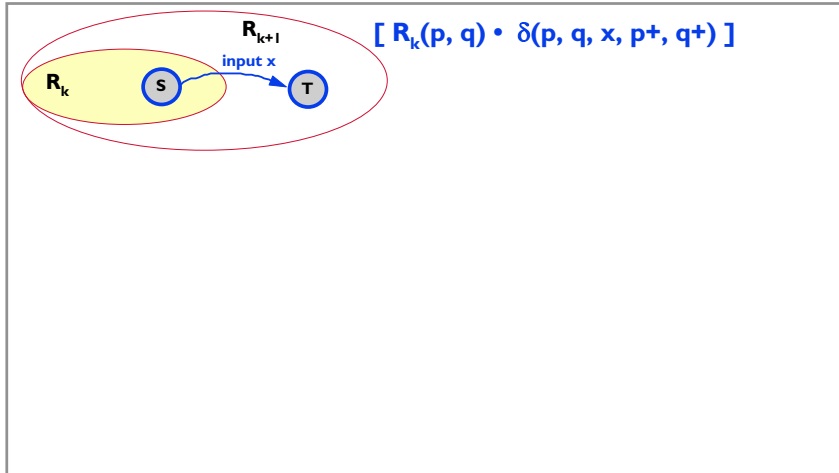


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## Iterated Reachability Analysis

▼ OK, now say it precisely, *mathematically*



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## Iterated Reachability Analysis

▼ General solution for FSM 1

$$R_{k+1}(p+, q+) = R_k(p+, q+) + \{ (\exists p, q, x) [ R_k(p, q) \cdot \delta(p, q, x, p+, q+) ] \} (p+, q+)$$

▼ In general...

$R_{k+1}$ (next state vars) =

$R_k$ (next state vars)

+  $\{ (\exists \text{ current state, inputs}) [ R_k(\text{current state vars}) \cdot$

$\delta(\text{current state, inputs, next state}) ] \}$

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## Iterated Reachability Analysis: BDD Subtlety

▼ Look close at  $R_k$  functions

$$R_{k+1}(p+, q+) = R_k(p+, q+) + \{(\exists p, q, x) [R_k(p, q) \cdot \delta(p, q, x, p+, q+)]\}(p+, q+)$$

This is the same function  $R_k()$ , but with 2 *different* sets of input variables

When you make the BDDs, you get 2 separate BDDs here, one representing  $R_k(p+, q+)$ , and the other representing  $R_k(p, q)$

You can do this kind of thing with the “composition” op from the homework

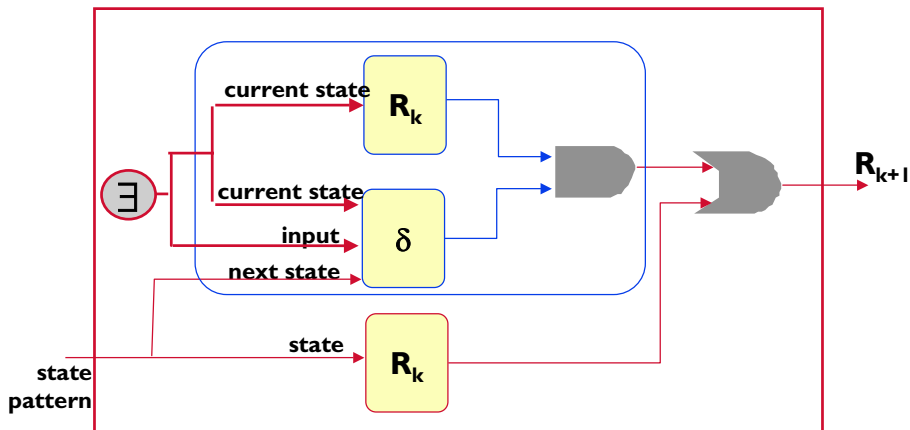
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## Iterated Reachability Analysis

▼ Randy Bryant suggests we draw it like this...

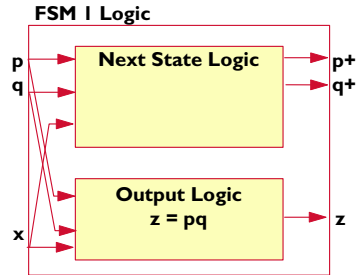
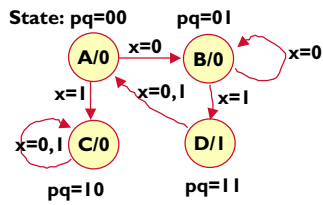
► Actually think about it again like a big piece of hardware



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# Iterated Reachability: Example FSM1



▼ What's  $\delta(p,q,x,p+,q+)$ ?

▼ What's  $R_0$ ? Assume start state is A

# Example: FSM1

▼ Do the messy quantification part of the eqn

$$R_1(p+, q+) = R_0(p+, q+) + \{ (\exists p, q, x) [ R_0(p, q) \cdot \delta(p, q, x, p+, q+) ] \}$$

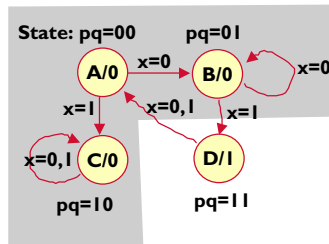
	p q x	$[p' q'] \cdot [p+ \oplus (pq' + p'x)] \cdot [q+ \oplus (p'q + p'x) ]$	==
cofactors	0 0 0		
	0 0 1		
	0 1 0		
	0 1 1		
	1 0 0		
	1 0 1		
	1 1 0		
	1 1 1		

$$(\exists p, q, x) [ R_0(p, q) \cdot \delta(p, q, x, p+, q+) ] = \text{OR all 8 of these}$$

## Example: FSM1

### ▼ So, it works!

- ▶ Result is  $R_1(p+, q+) = (p+ \cdot q+) + (p+ \cdot q+) + (p+ \cdot q+)$



- ▶ If you do it again, and compute  $R_2$ , will find == 1 (do it!). Why?
- ▶ Also find all subsequent R's ==  $R_2$ . Why?

## 4. Cross Product Machine

### ▼ Where are we?

- ▶ We can compute a Boolean function (represented as a BDD) that tells us all states in the FSM reachable in at most K ticks of the clock
- ▶ Can do this mechanically using BDDs, and the next state logic

### ▼ Where do we want to be?

- ▶ Given 2 different FSMs, can we tell if they are *equivalent*

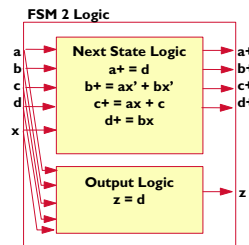
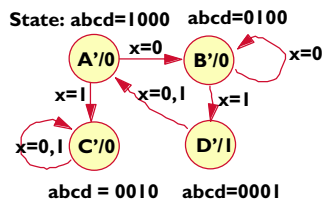
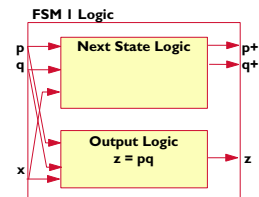
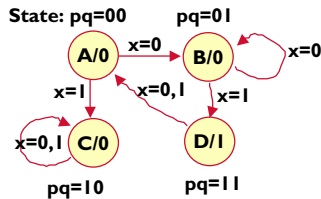
### ▼ Now what?

- ▶ Make a new FSM that combines the 2 FSMs we which to check for equivalence....
- ▶ ...and we do the construction so that reachability analysis gives us the verification that we want.
- ▶ Construction == **cross-product machine**

# Cross Product Machine

▼ Given 2 FSMs to check...

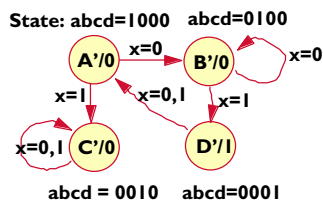
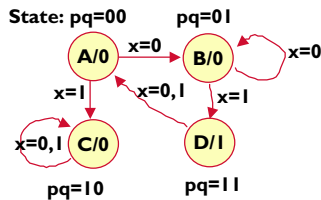
► Just FSM1, FSM2, with FSM2's states given new names...



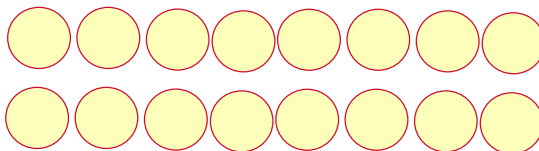
# Cross Product Machine: FSM1 × FSM2

▼ What are the states in FSM1 × FSM2? (× == “cross”)

► All pairs of possible states (FSM1 state, FSM2 state)



FSM1 × FSM2 has 4\*4=16 states

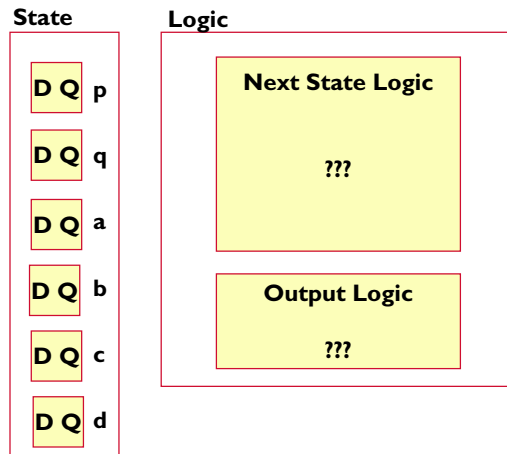




## Cross Product Machine: $FSM1 \times FSM2$

### What are the state variables for $FSM1 \times FSM2$

- Easy, just "all" the state vars from  $FSM1$  and from  $FSM2$



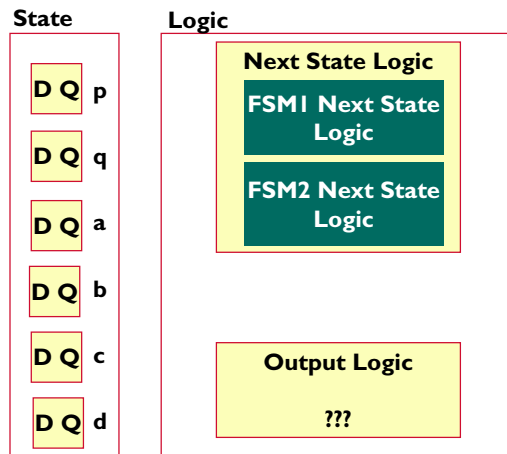
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## Cross Product Machine: $FSM1 \times FSM2$

### So, what then is the next-state logic for $FSM1 \times FSM2$ ?

- Easy again: just the 2 separate next-state blocks from  $FSM1$ ,  $FSM2$



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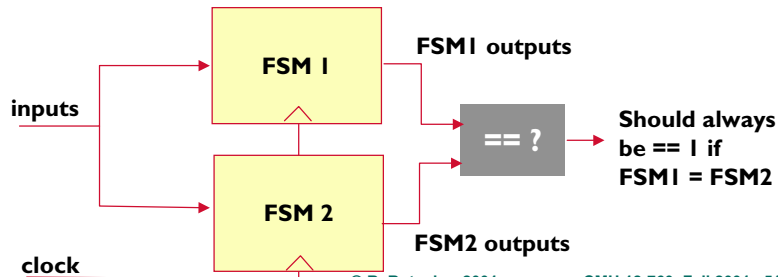
## Cross Product Machine: $FSM1 \times FSM2$

▼ So, what is the output logic for  $FSM1 \times FSM2$

► *Now, it's different.*

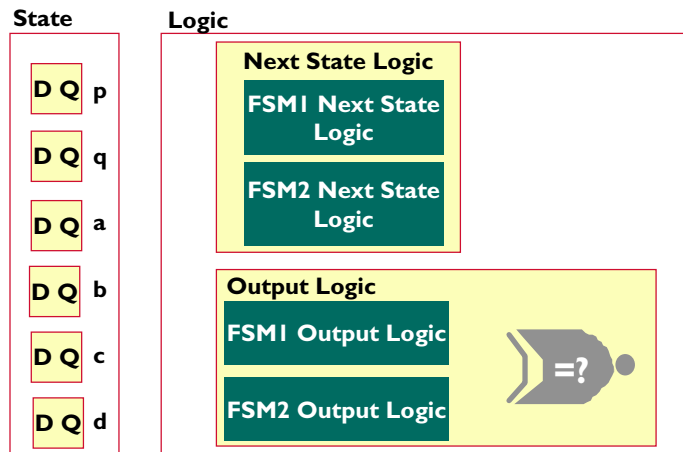
► The cross-product machine is really (up to now) just the 2 separate machines  $FSM1$  and  $FSM2$  running side-by-side

► What we really want to know is, if we start them both in the *same* "equivalent" state, will we ever reach a "combined" state ( $FSM1$  state vars,  $FSM2$  state vars) where the outputs of the 2 machines are actually *different*



## Cross Product Machine: $FSM1 \times FSM2$

▼ So, what is the output logic for  $FSM1 \times FSM2$ ?



## Cross Product Machine Reachability Analysis

### ▼ Where are we?

- ▶ We can build, mechanically, the cross product machine

### ▼ What good is this?

- ▶ FSM1 and FSM2 are equivalent if, when started in the same initial “equivalent” states, it is **NEVER POSSIBLE** to reach a state where the output of the cross product machine == 0
- ▶ Put another way: output of cross product machine should always be ==1, for any combined state (FSM1 state, FSM2 state) we can reach from (FSM1 start, FSM2 start)

### ▼ How do we do this?

- ▶ We already know how...

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## Cross Product Reachability Analysis

### ▼ Build the cross product machine

- ▶ Means build the next state logic, and the output logic for it

### ▼ Do reachability analysis on [ FSM1 × FSM2 ]

- ▶ In our little example, start with A=00, A' = 1000 states:

$$R_0(p+,q+,a+,b+,c+,d+) =$$

- ▶ Build  $R_1, R_2, \dots, R_K$  until it stops changing
- ▶ At this point, you know pairs of states it is possible to reach from (A, A') start state...

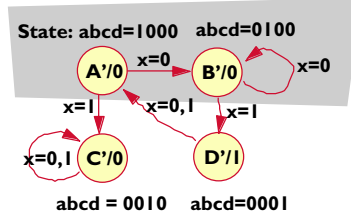
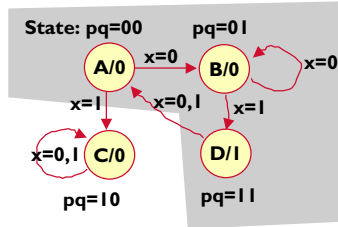
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# Cross Product Reachability Analysis

## What does this tell you?

- Pairs of states you can get to from (A, A')
- Example



Example: 2 ticks, A->B->D

Example: 2 ticks, A'->B'->B'

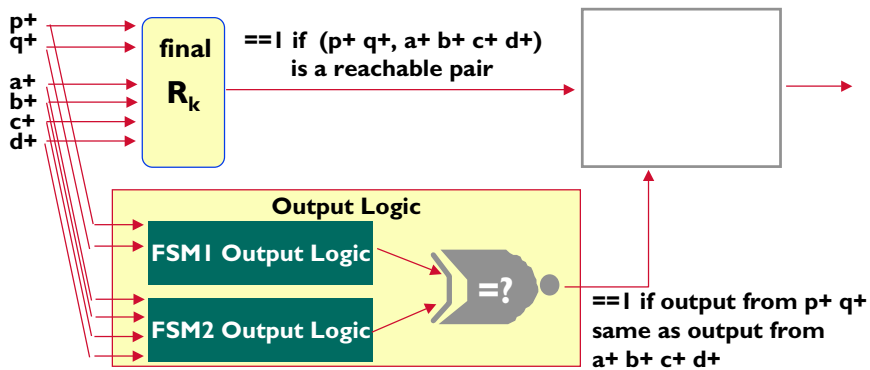
So, in  $R_0$  we have (A, A'), and in  $R_2$  we will have...

# Verification via Reachability

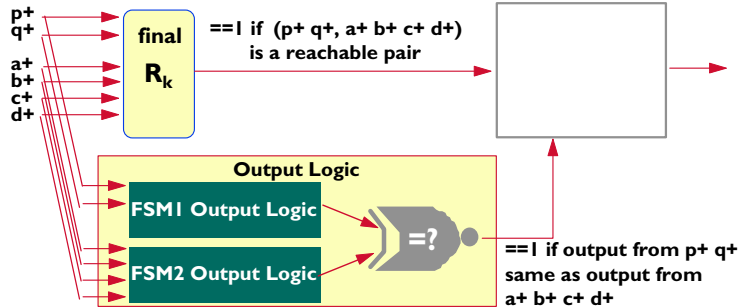
## OK, we can build $R_k$ for this $FSM1 \times FSM2$

## Now what?

- Want to know if we can reach a combined state where cross-product output ==0; consider this logic...



## Verification via Reachability



### ▼ Look what happens

- ▶ Build a BDD for this ckt; it has 6 inputs  $p+ q+ a+ b+ c+ d+$ , has 1 output
- ▶ If the 2 FSMs are equivalent, then this output  $==$  constant 0 BDD
- ▶ If these 2 FSMs NOT equivalent, this is some BDD with a 1 node in it
- ▶ All we have to do is see if this BDD  $!=$  constant 0 BDD, and we are done!

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## Verification via Reachability

### ▼ What did we do here?

- ▶ We turned the complex temporal problem of verification of equivalence for all inputs over all subsequent clock ticks...
- ▶ ...into a **series** of BDD exercises, ending in a satisfiability check on a single (big, nasty) BDD; if any pattern of inputs makes this BDD  $== 1$ , then machines not equivalent

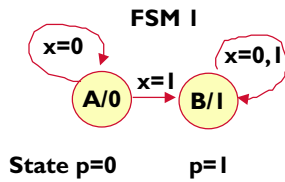
### ▼ Amazingly cool result

- ▶ Doable mechanically with a good BDD package
- ▶ Definitely NOT something you want to try to do by hand.
- ▶ On the HW, you get to do it for a similar pair of machines using the KBDD package to do the nasty Boolean manipulations

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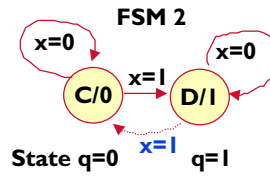
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## Simpler Example



Next state:  
 $p^+ = p+x$

Output  
 $z = p$



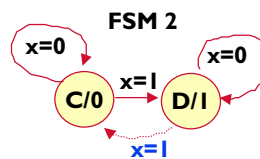
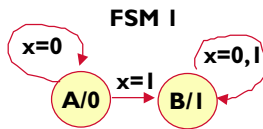
Next state:  
 $q^+ = q \oplus x$

Output  
 $z = q$

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## Simple Example: Cross Product Reachability



▼ What do we *expect* for reachability on FSM1 X FSM2?

$R_0 = \{ (A,C) \}$

$R_1 =$  reachable on 0 or 1 clock tick =

$R_2 =$  reachable on 0, 1, 2 clock ticks =

$R_3 =$  reachable on 0, 1, 2, 3 clock ticks =

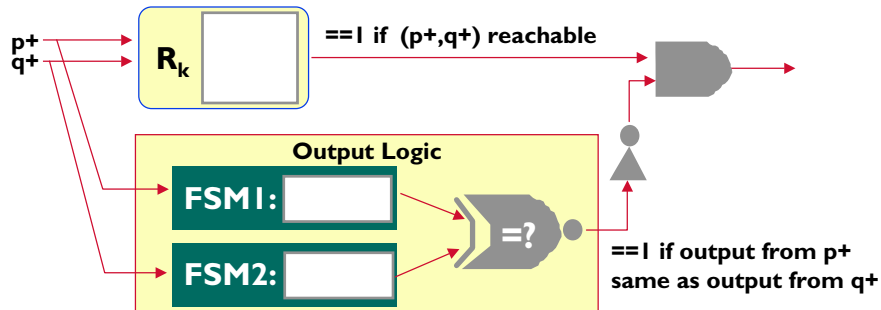
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## Cross Product Reachability Analysis

▼ So, we get  $R_k = \{ AC, BD, BC \}$  ... what is Boolean func?

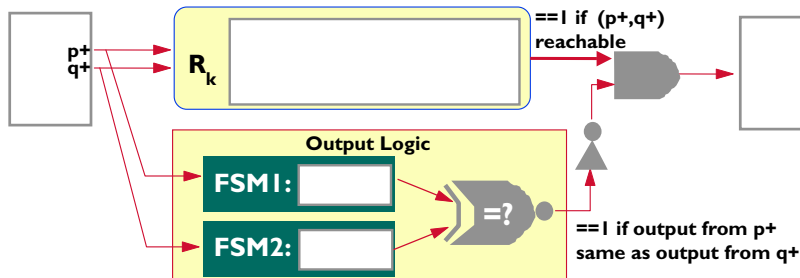
▼ OK, let's make the cross product satisfiability logic



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## Cross Product Reachability: Satisfiable?



▼ Well, can you get a 1 out of this...?

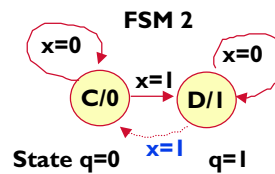
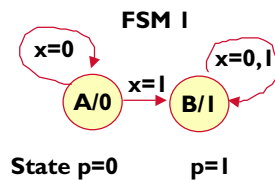
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## Cross Product Reachability: Satisfiable?

### ▼ Aha!

- ▶ Pattern  $p=1$   $q=0$  satisfies this BDD, makes it  $\neq 1$
- ▶ Since you can get a 1 here, FSMs NOT equivalent
- ▶ What does  $p=1$   $q=0$  signify here?



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## Summary

### ▼ FSM verification is important, but different

- ▶ Have to worry about **time**, about equivalent outputs over all possible **future** inputs
- ▶ Not just simple block-to-block combinational equivalence

### ▼ Big ideas

- ▶ Symbolic representation of FSMs -- transition relation
- ▶ Reachability analysis
- ▶ Cross product machine with “special” satisfiability output
- ▶ Transforms the temporal problem into another series of do-able BDD exercises

### ▼ Hugely important application of BDD analysis out in the real world these days...

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