HW SET #5 (DUE BEFORE CLASS ON FEB 24, WED)

Problem 1 (10 points) Consider an alphabet with two symbols $\{A, B\}$, where $P(A) = x$ and $P(B) = 1 - x$. Plot the entropy as a function of x.

Problem 2 (15 points) Consider an alphabet with three symbols $\{A, B, C\}$, where $P(A) = x$, $P(B) = y$ and $P(C) = 1 - x - y$. Plot the entropy as a function of x and y.

Problem 3 (10 points) From the results of Problem 1 and Problem 2, we can expect that the entropy of a source reaches its maximum when all symbols are equally probable. Please prove this formally for a alphabet with *N* symbols.

Problem 4 (15 points) Based on the RGB-to-YUV conversion in Lecture Notes on H.261, derive the YUV-to-RGB conversion in the matrix/vector form.

Problem 5 (20 points) This is an exercise of arithmetic coding. Given the following probabilities of symbols *A*, *B*, *C*, and *D* at different time instants:

	$t = 1$	$t = 2$	$t = 3$	$t = 4$	$t = 5$
A	0.5	0.4	0.5	0.4	0.3
B	0.3	0.2	0.2	0.4	0.3
C	0.1	0.2	0.2	0.1	0.2
D	0.1	0.2	0.1	0.1	0.2

we want to encode the sequence *"BCAAD"* using Implementation #1 as described in class. Consider *"D"* as the EOF symbol. You don't need to write a program to do this. Simply compute the interval `[low,high)` after each symbol is processed, and summarize the result in a table. Note that all `cum_freq[i]` are time-varying. At the end of the five symbols, pick a `value` in the range `[low,high)` and send it to the decoder. Then, based on `value`, the decoder can recover the symbols. Compute the interval `[low,high)` after each symbol is processed and summarize the result in a table. Verify that the interval `[low,high)` at the encoder varies in synchronization with the interval `[low,high)` at the decoder.

Problem 6 (30 points) Repeat Problem 5 with Implementation #2. Use the following numbers:

```
C = 8
cum_freq[0] = 10
```

Again, at the encoder, compute the interval `[low,high]` and the bits to output, after each symbol is processed. At the decoder, compute `value`, `cum`, and `[low,high]` after each symbol is decoded.