
 HW SET #3 (DUE BEFORE CLASS ON FEB 10, WED)

Problem 1 (20 points) Consider the 2D vector quantization. Suppose there are three code vectors in a codebook: (0,0), (1,1), and (0,2). Draw the optimal cells when L_2 is used. What if L_1 is used? How about L_∞ ? The L_1 norm of two points (x_1, y_1) and (x_2, y_2) is defined as $|x_1 - x_2| + |y_1 - y_2|$, and the L_∞ norm is $\max(|x_1 - x_2|, |y_1 - y_2|)$. (Hint: There may areas that belong to multiple cells.)

Problem 2 (20 points) We showed in class that pyramid coding increases the number of samples. If the number of stages (filtering/downsampling) is infinity, the number of samples that need to be coded and transmitted is 4/3 of the number of original samples for the 2D case. What is this ratio for the 1D case? How about the 3D case? Please derive a formula for the ratio for the n -dimensional case.

Problem 3 (20 points) Suppose we apply subband coding to an image by applying the lowpass-highpass decomposition recursively to the LL subband (This is also referred to as the *wavelet* decomposition). If we apply a two-level decomposition, i.e., splitting the LL band of the first level further into LL, LH, HL, and HH, there are altogether seven subbands. Now, let us try to extend subband coding to n -dimensional signals. Suppose we apply an L -level decomposition to an n -dimensional signal, how many subbands do we get?

Problem 4 (40 points) We mentioned in class that a Huffman code is uniquely decodable because of its prefix property, i.e., no codeword is a prefix of other codewords. However, a Huffman code is generally not *reversible*, i.e., a bitstream can not be uniquely decoded backward. For example, the code

- A 0
- B 10
- C 110
- D 111

is a prefix code so it is uniquely decodeable. If we decode the following bitstream from the beginning

1111101110...

it can be uniquely decoded into DCDA... However, if we try to decode

...1111101110

from the last bit, we may get either ...(11)DADA or ...(1)DBDA. Can you think of a systematic way to modify a Huffman code to construct a reversible code (in addition to being prefix)? In other words, can you modify the codewords of a Huffman code so that no codeword is a *postfix* of other codewords? Do not worry about the symbol probabilities. You are given a Huffman table to begin with, not a sequence of probabilities. (Hint: There is no unique solution to this, so use your imagination.)