Differentially Private Data Analysis of Social Networks via Restricted Sensitivity

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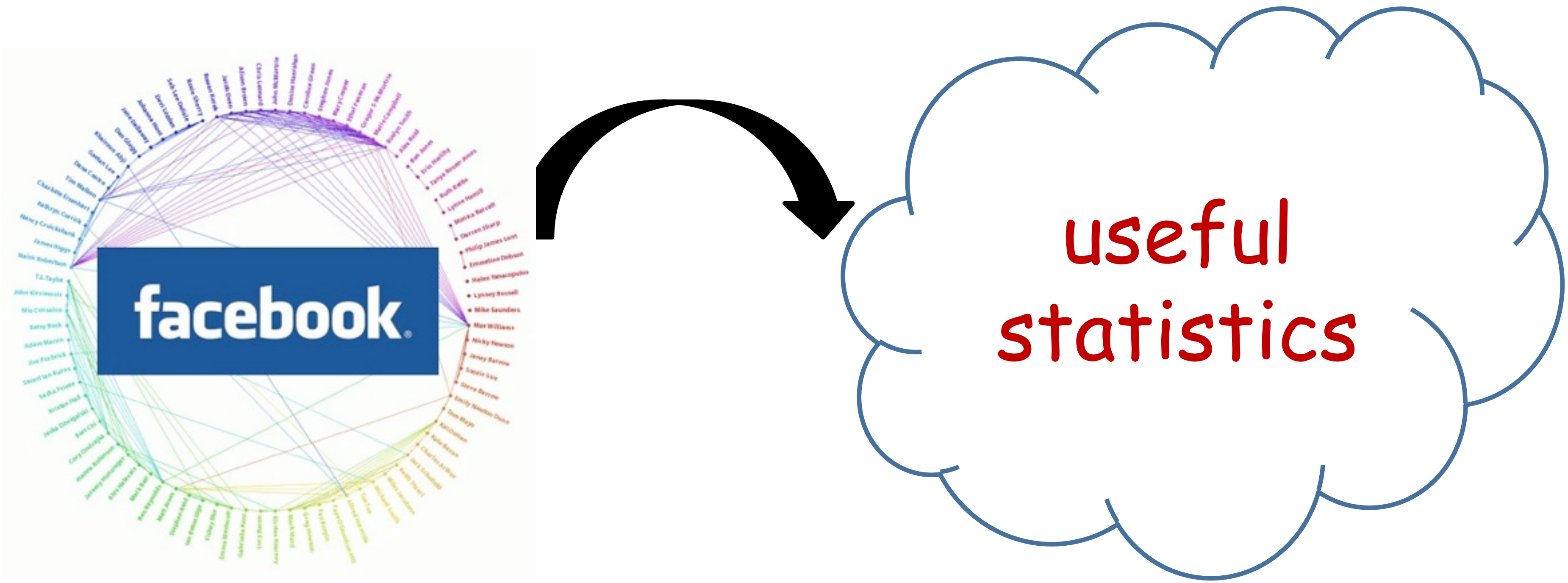
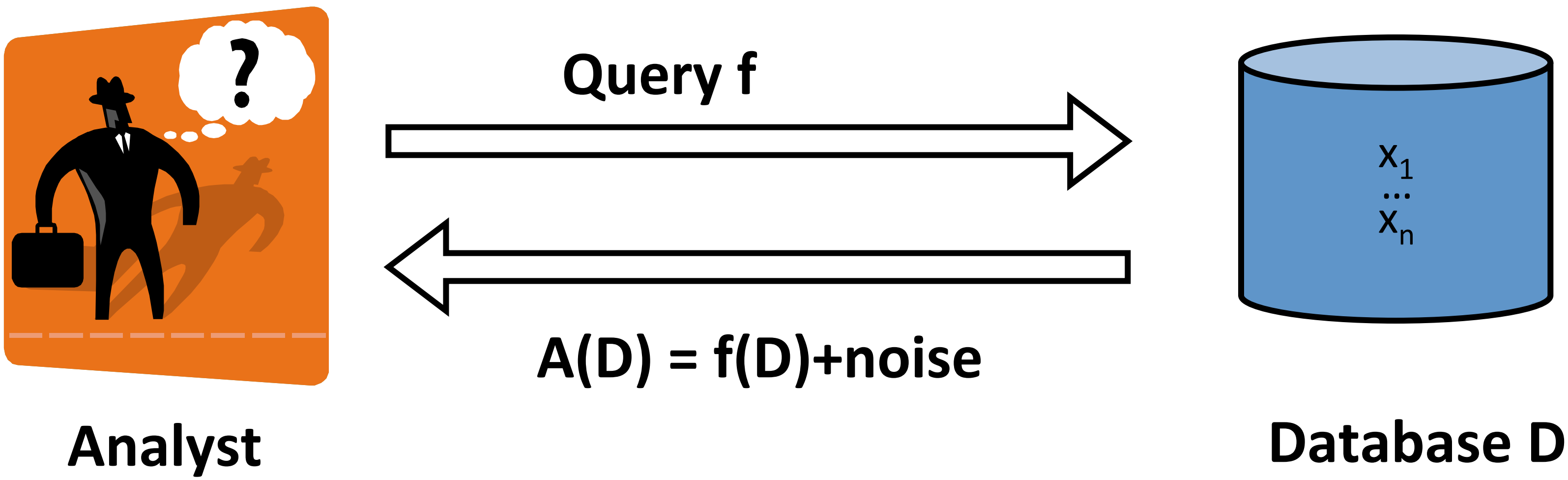
Foundations of Privacy, Fall 2014

Goal



Preserve Privacy and Release Useful Statistics

Usual Differential Privacy Setting



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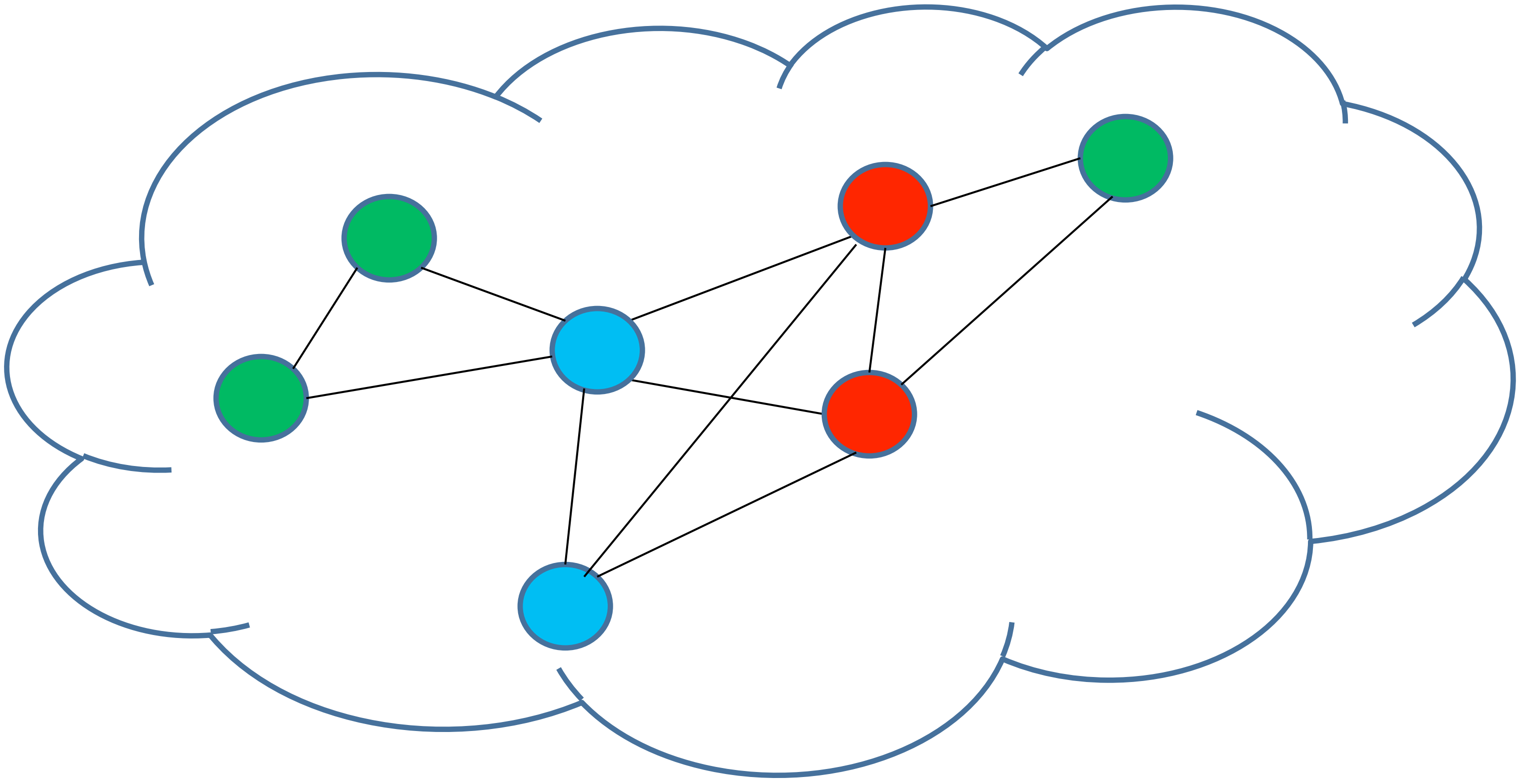
Outline

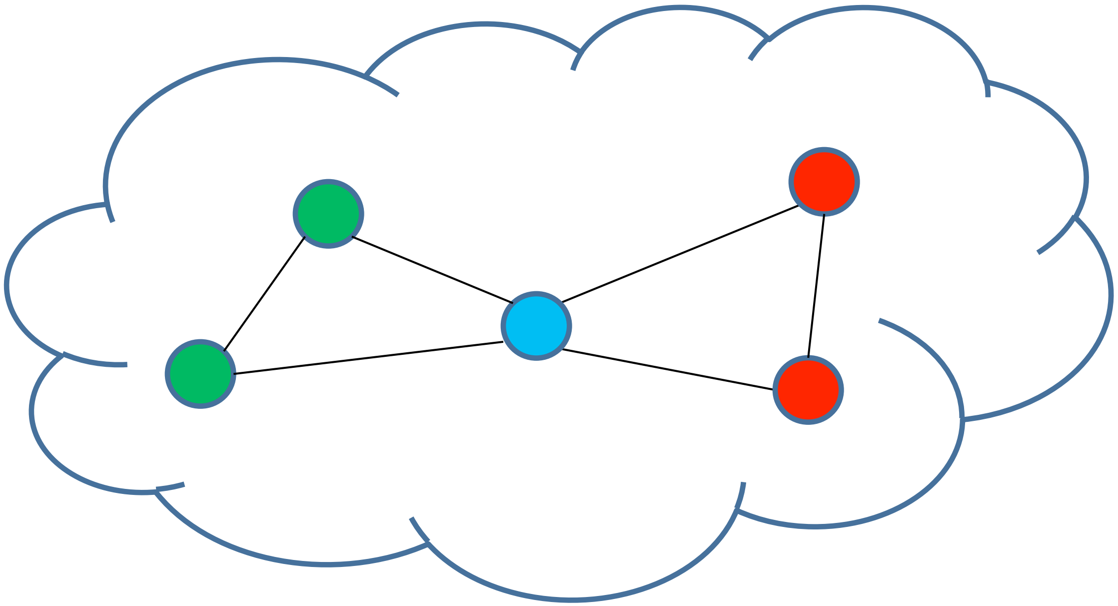
- **Background**
  - Social Networks
  - Differential Privacy
- The Problem
- Restricted Sensitivity
- Algorithms

Social Network



Vertices in a social network G [illegible] G are labeled (e.g., doctor, lawyer, professor).

Local Profile Query



How many people know 2 lawyers who know each other and 2 doctors who know each other, but the lawyers aren't friends with the doctors?

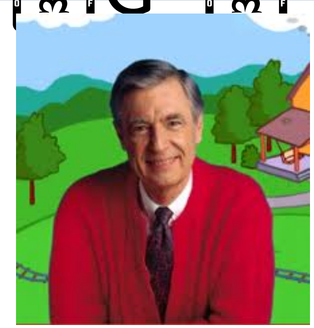
Differential Privacy (Dwork et al)

An algorithm A satisfies (□,□)-differential privacy for social networks if for any S □ Range(A)

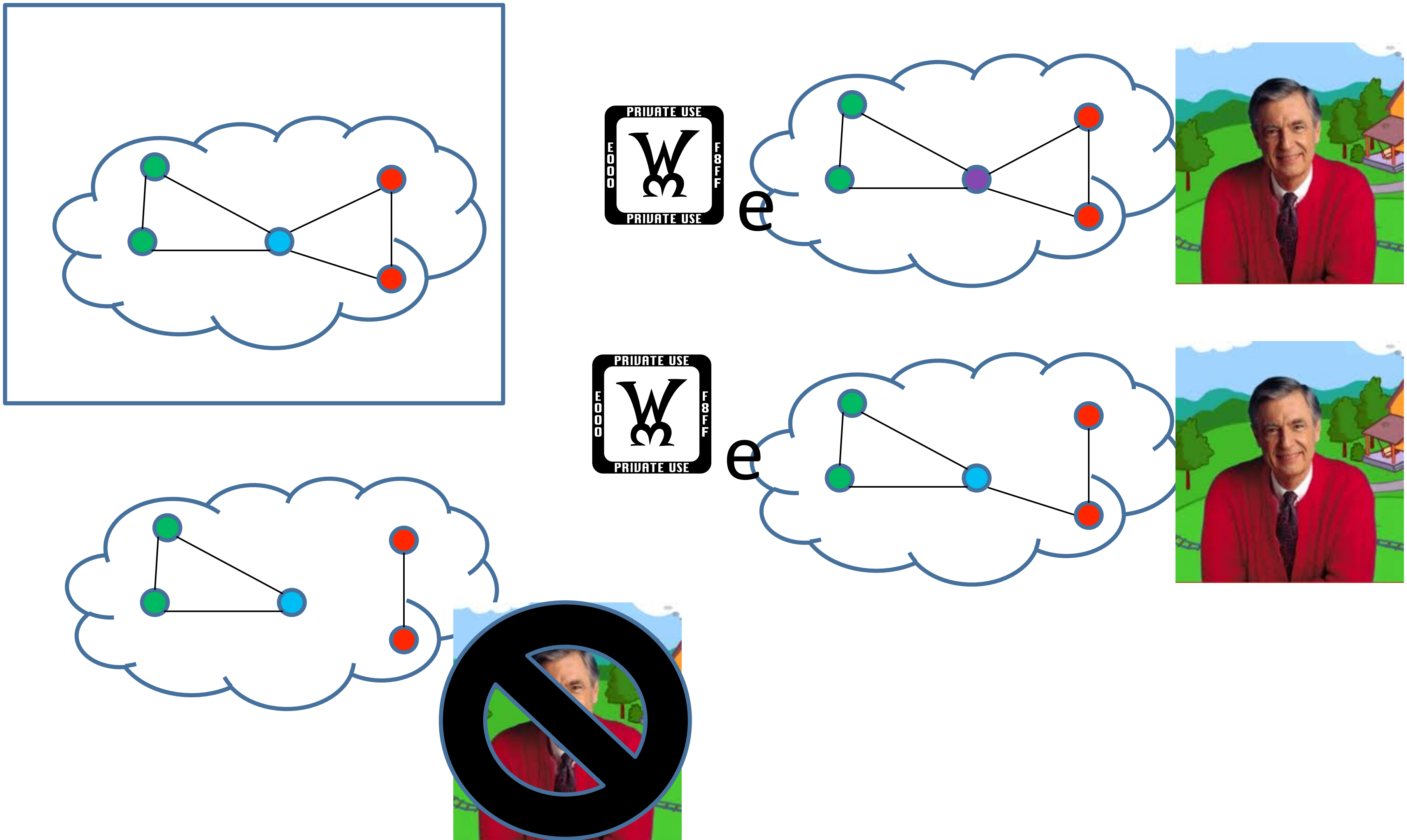
Pr[A(G)□S] ≤ e^□ Pr[A(G')□S] + □

for every *neighboring* social networks G□G' □ G.

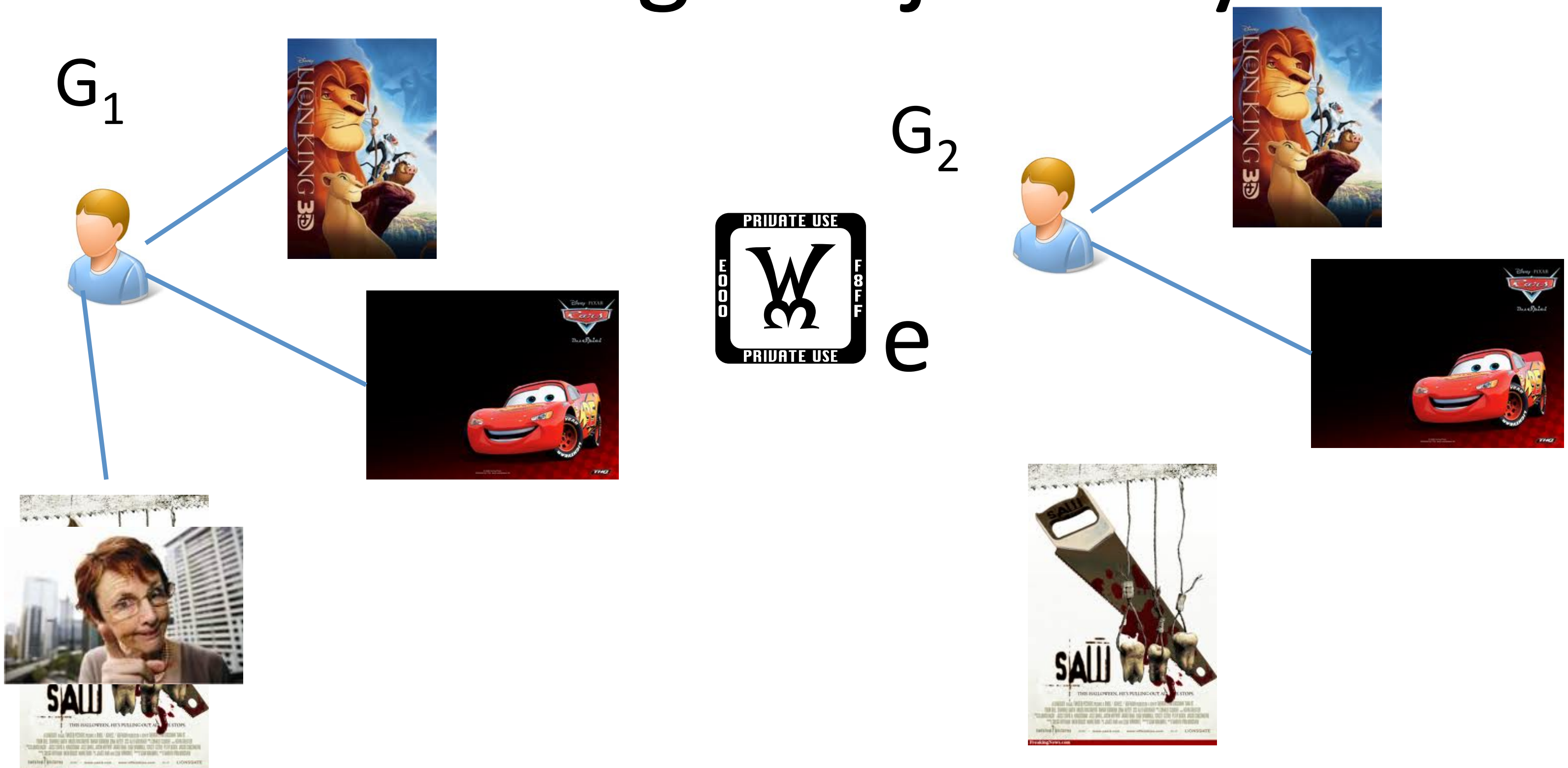
Important Question: When are G and G' are neighbors?



Edge Adjacency

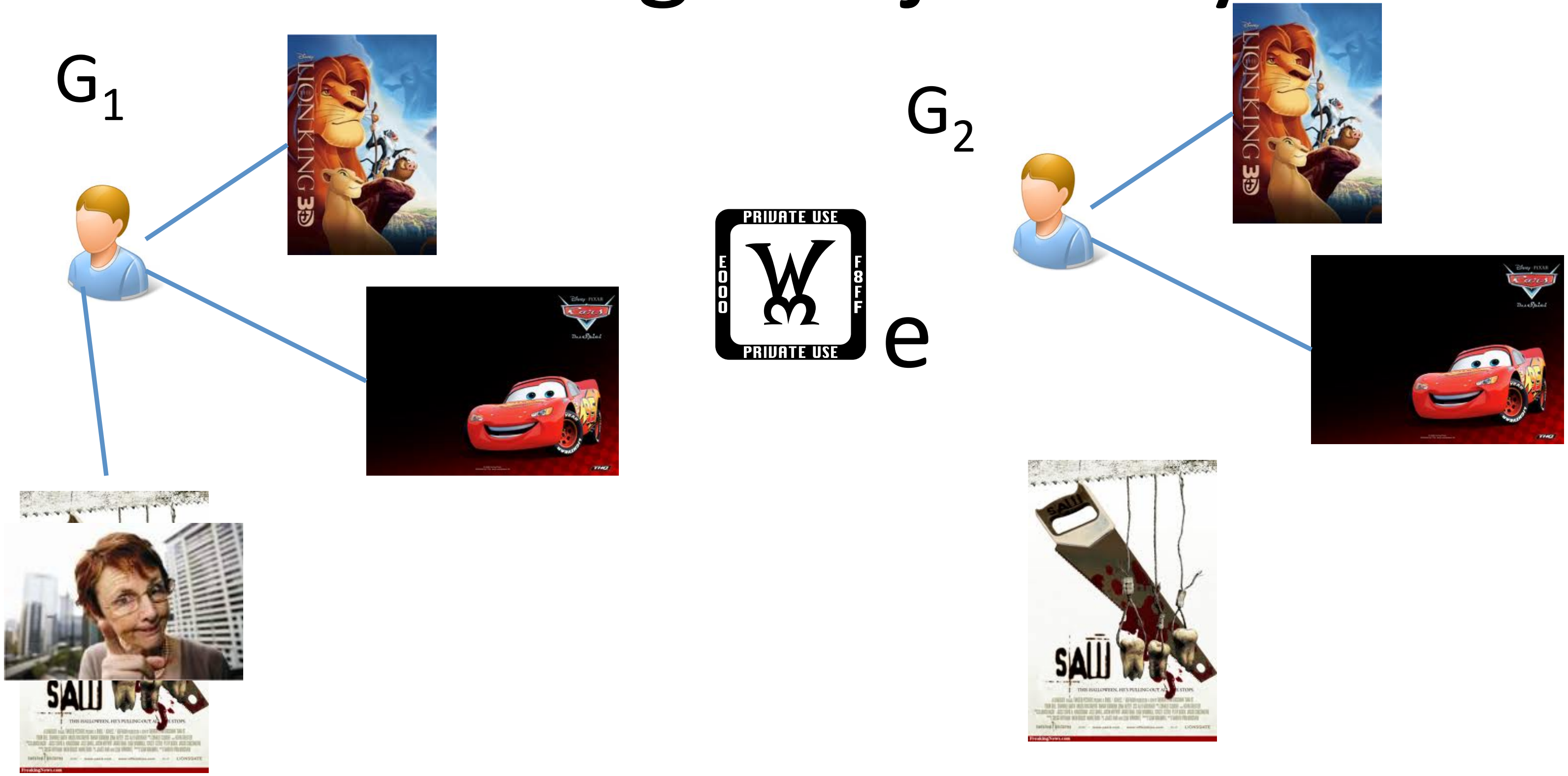


Edge Adjacency



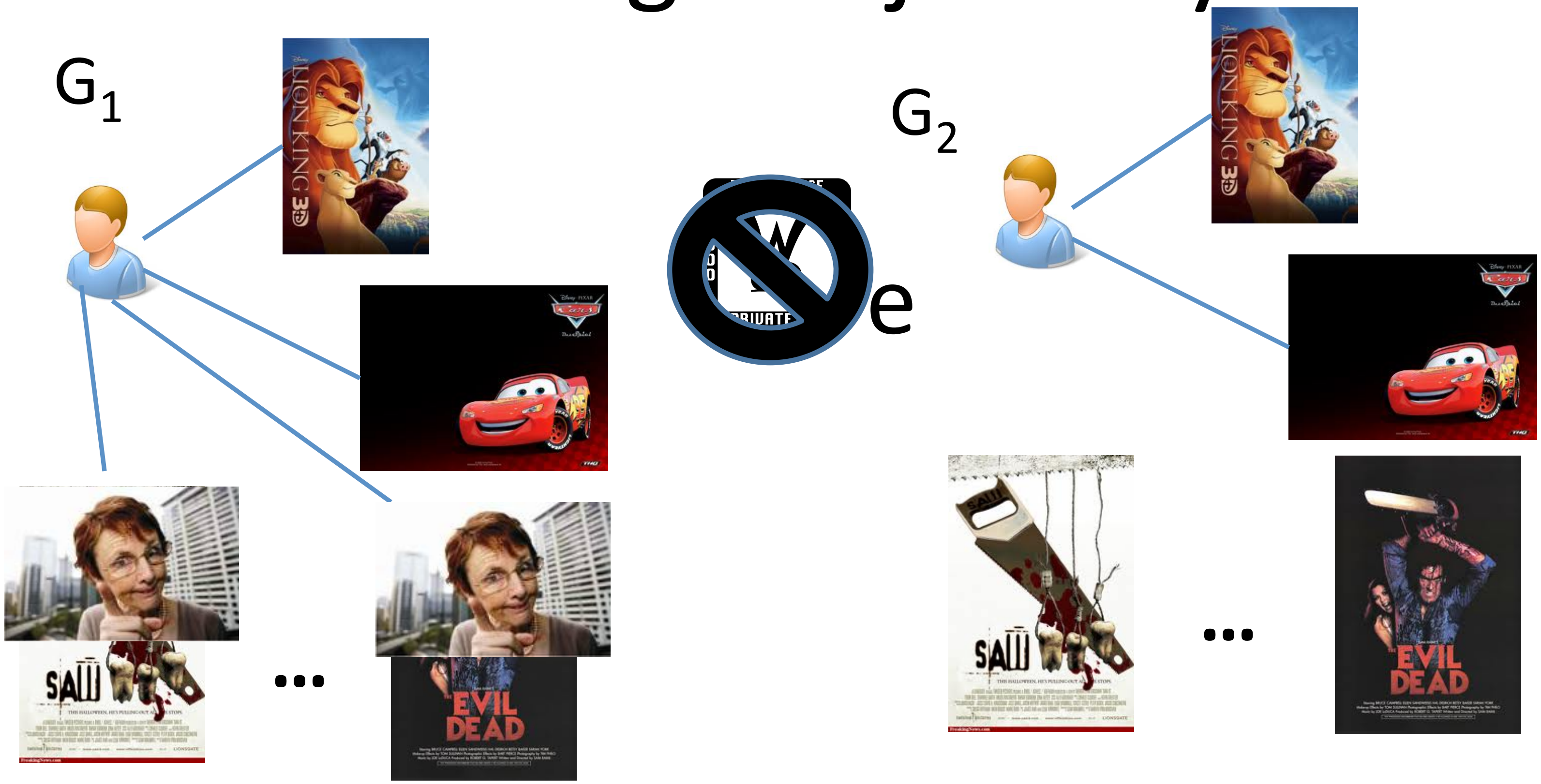
Pr[A(G1)= ] ≤e[PRIVATE USE]Pr[A(G2)= ] + [PRIVATE USE]

Edge Adjacency



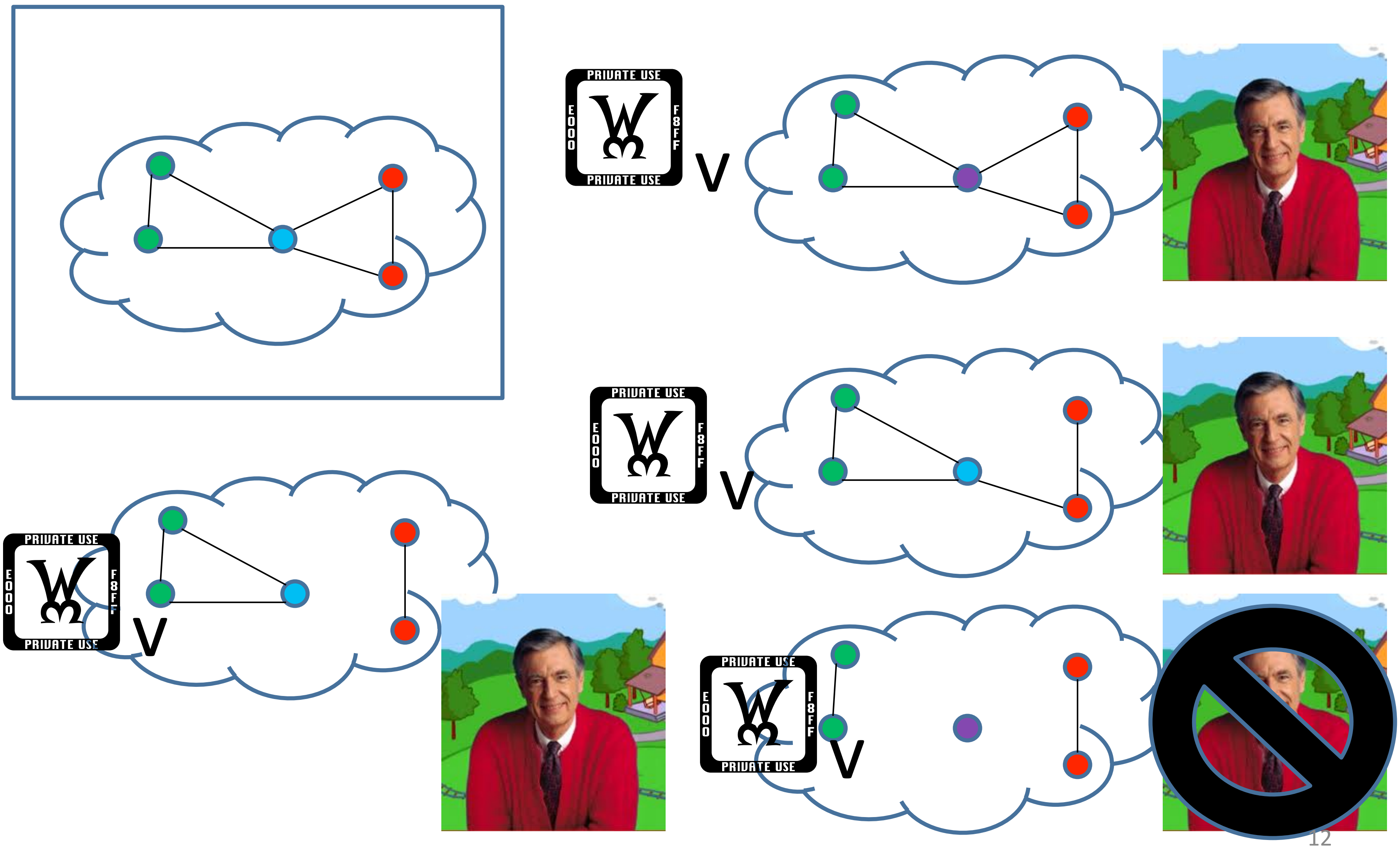
Johnny’s mom cannot tell if he watched Saw.

Edge Adjacency

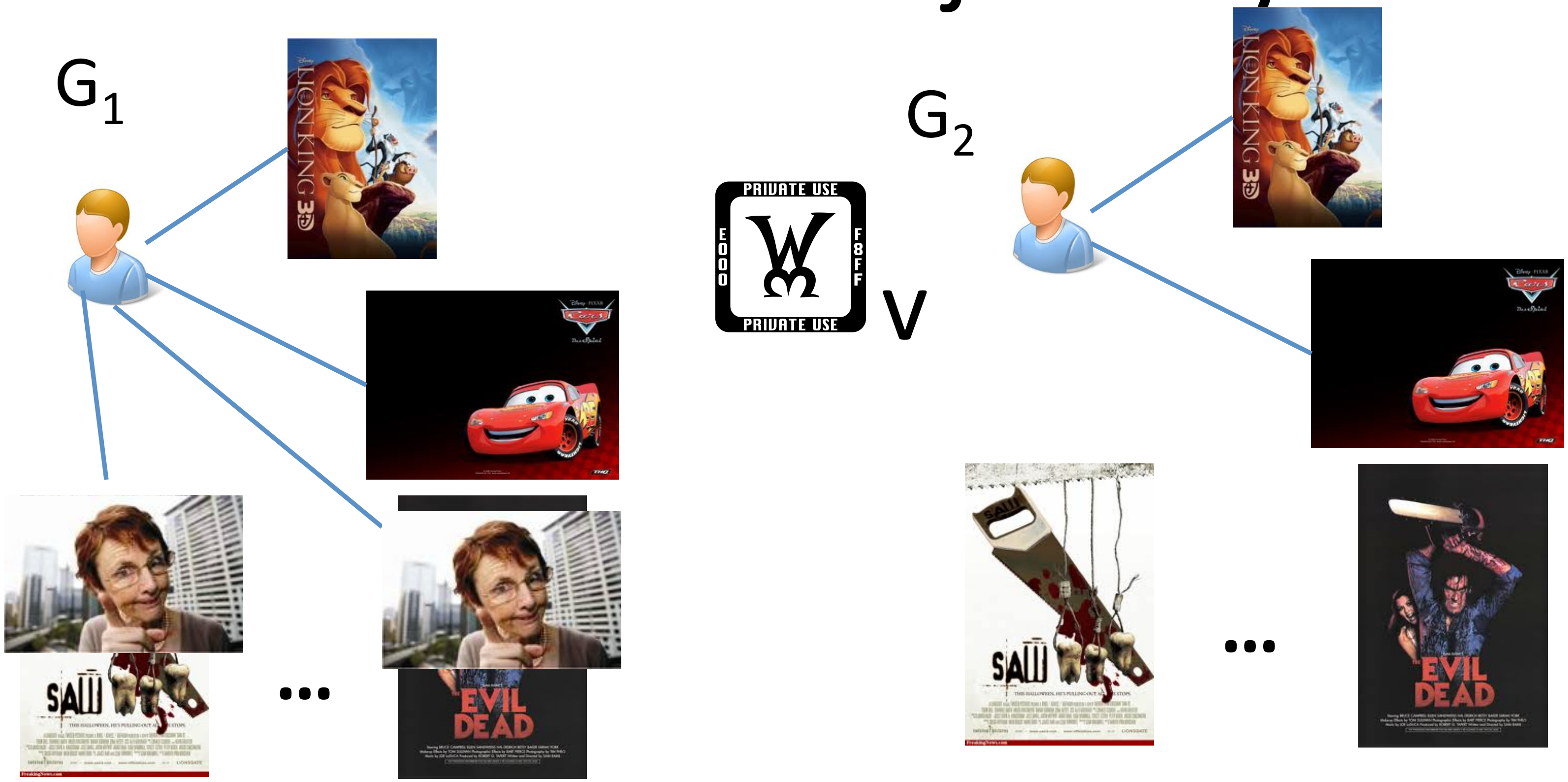


Johnny's mom may be able tell if he watches R-rated movies.

Vertex Adjacency

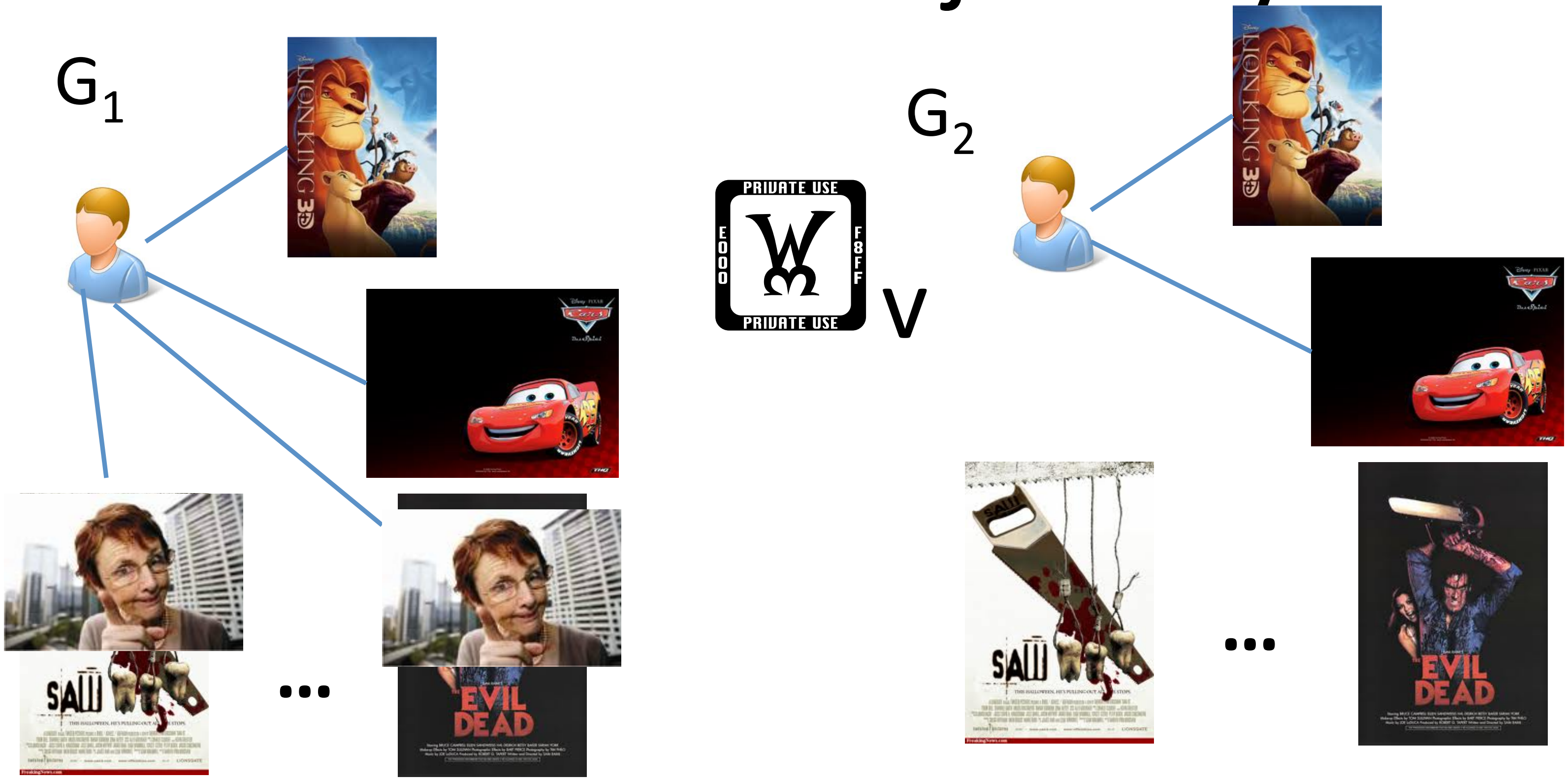


Vertex Adjacency



$\Pr[A(G_1)=$  $] \leq e^{[illegible]} \Pr[A(G_2)=$  $] + [illegible]$

Vertex Adjacency



Johnny’s mom cannot tell if he regularly watches R-rated movies.

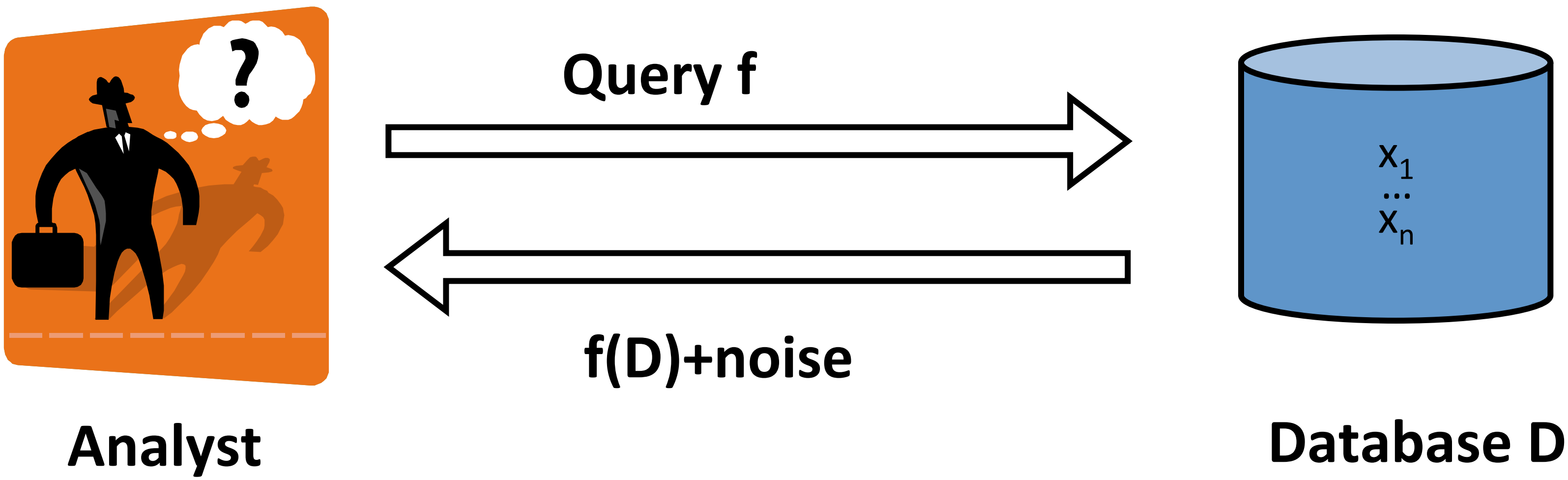
Differential Privacy (Dwork et al)

An algorithm A satisfies (ϵ, δ)-differential privacy for social networks if for any S $\subseteq$ Range(A)

$$\Pr[A(G) \in S] \leq e^{\epsilon} \Pr[A(G') \in S] + \delta$$

for every pair of *vertex adjacent* social networks G and G’

Usual Differential Privacy Setting



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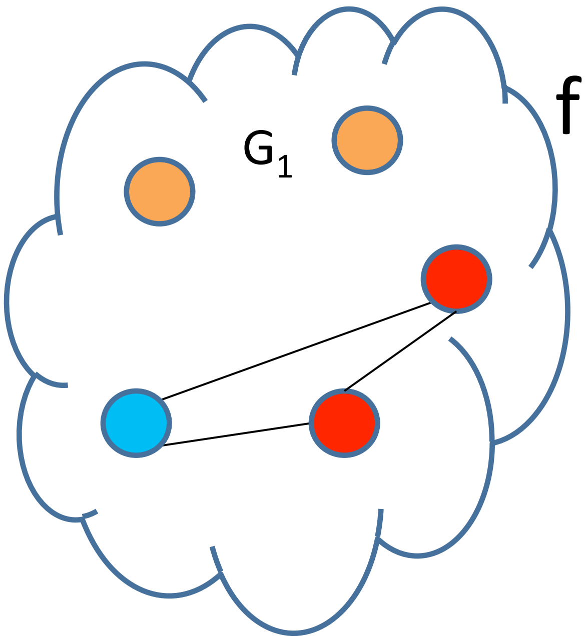
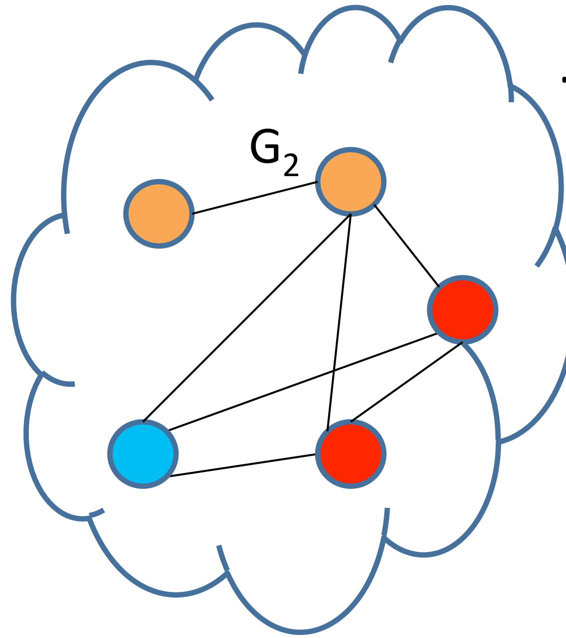
Challenge: High Sensitivity

f(G) = “how many people in G know a pianist?”



$f(G_1)=0$

v



$f(G_2)=n$

Previous Work

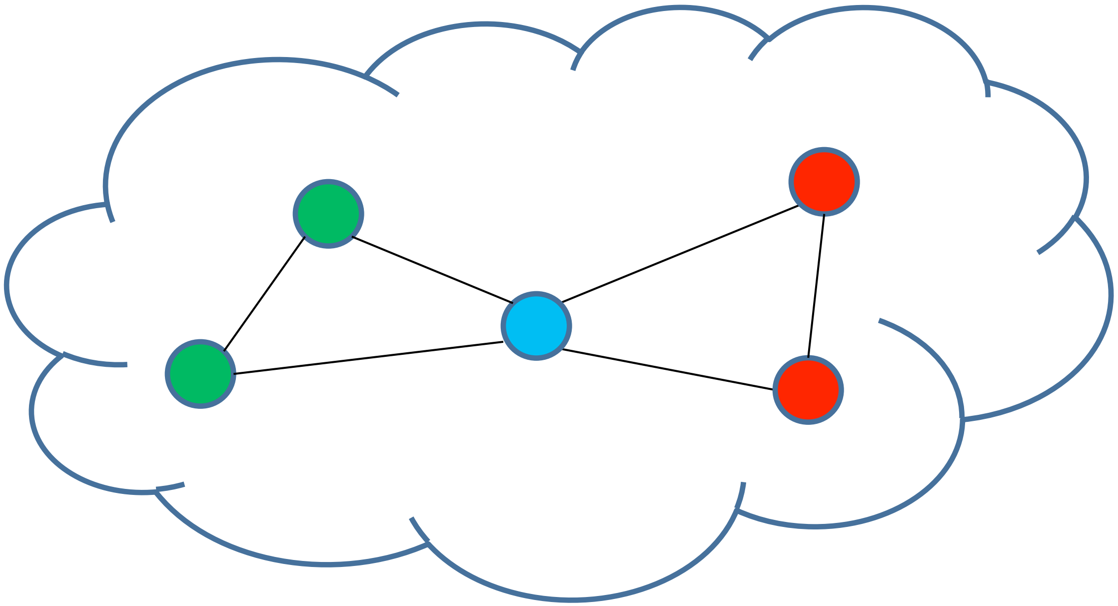
- Edge Adjacency Model
  - Degree Distribution [HLM'09]
  - Subgraph Counting [KRSY'11]
  - Cut Queries [GRU'12, BBDS'12]

- This Work: Vertex Adjacency
  - Node-level differential privacy [KNRS'12]
    (Concurrent independent results)

Outline

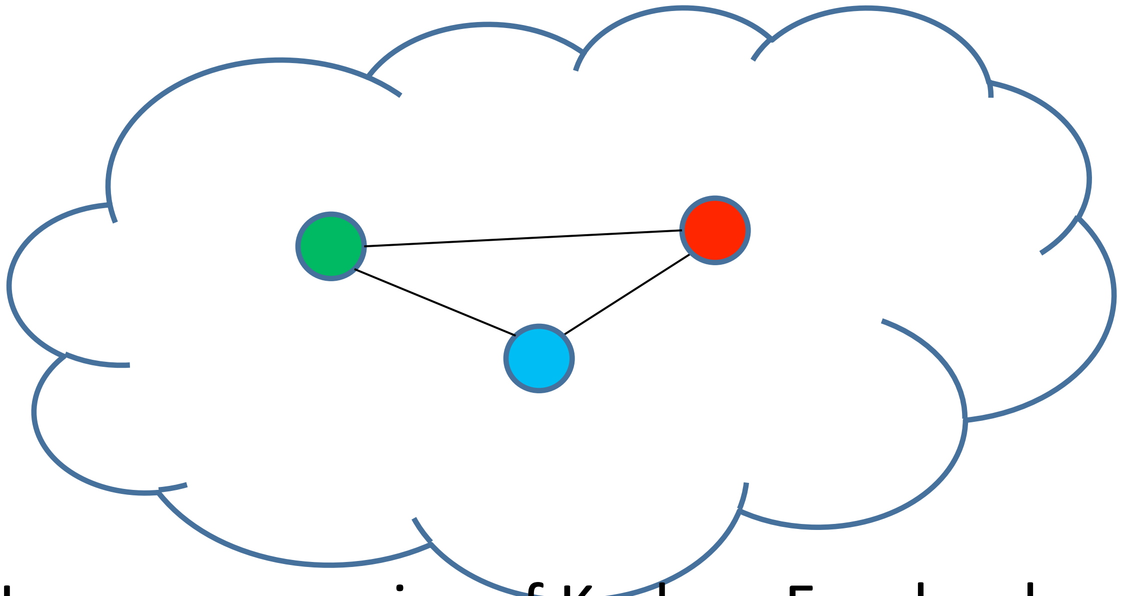
- Background
- **The Problem**
  - Interesting Queries
  - Sensitivity of a Query
  - Interesting Queries have High Sensitivity
- Restricted Sensitivity
- Algorithms

Local Profile Query



How many people know 2 lawyers who know each other and 2 doctors who know each other, but the lawyers aren't friends with the doctors?

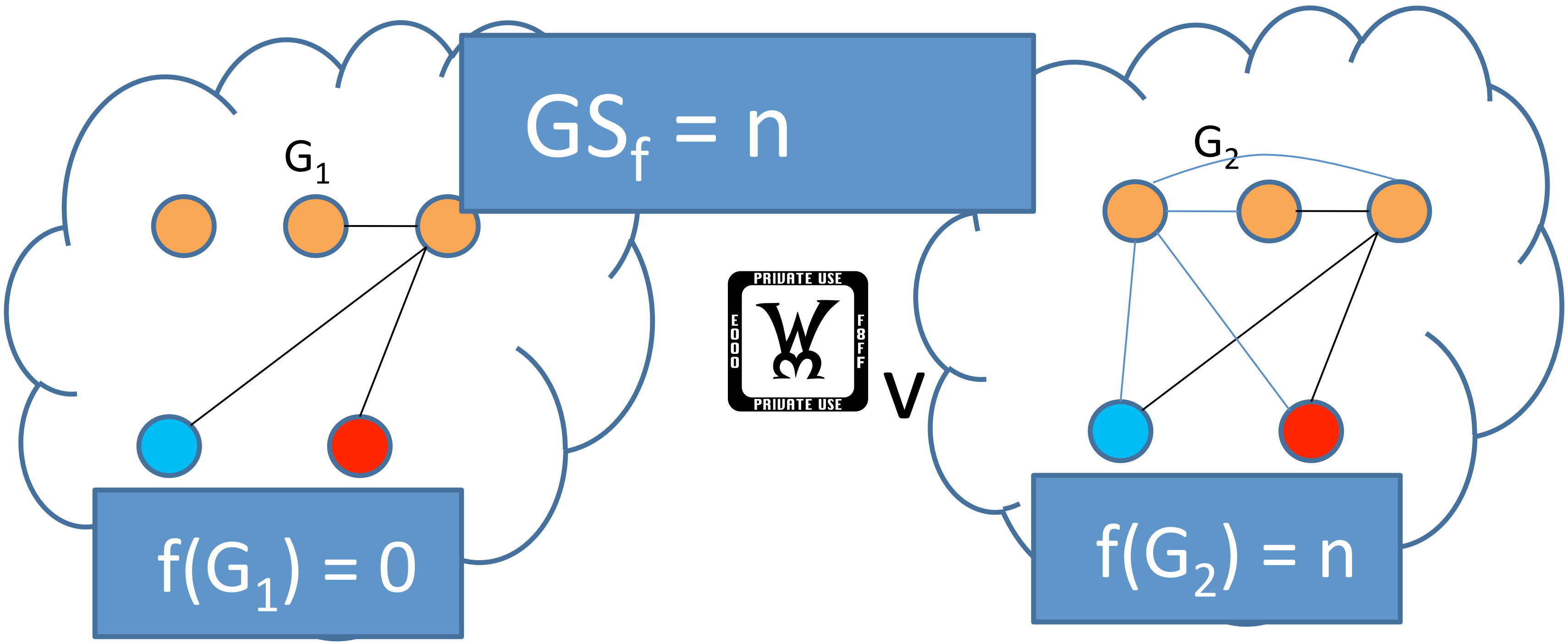
Subgraph Counting Queries



How many copies of K_3 does Facebook contain where one node is a doctor, one node is a professor and one node is a lawyer?

Global Sensitivity

f(G) = "how many people in G know two pianists?"



Global Sensitivity

Global Sensitivity of f:

$$GS_f = \max_{G_1, G_2} \left(\frac{\left| f(G_1) - f(G_2) \right|}{d(G_1, G_2)} \right)$$

Local Profile Queries (f):

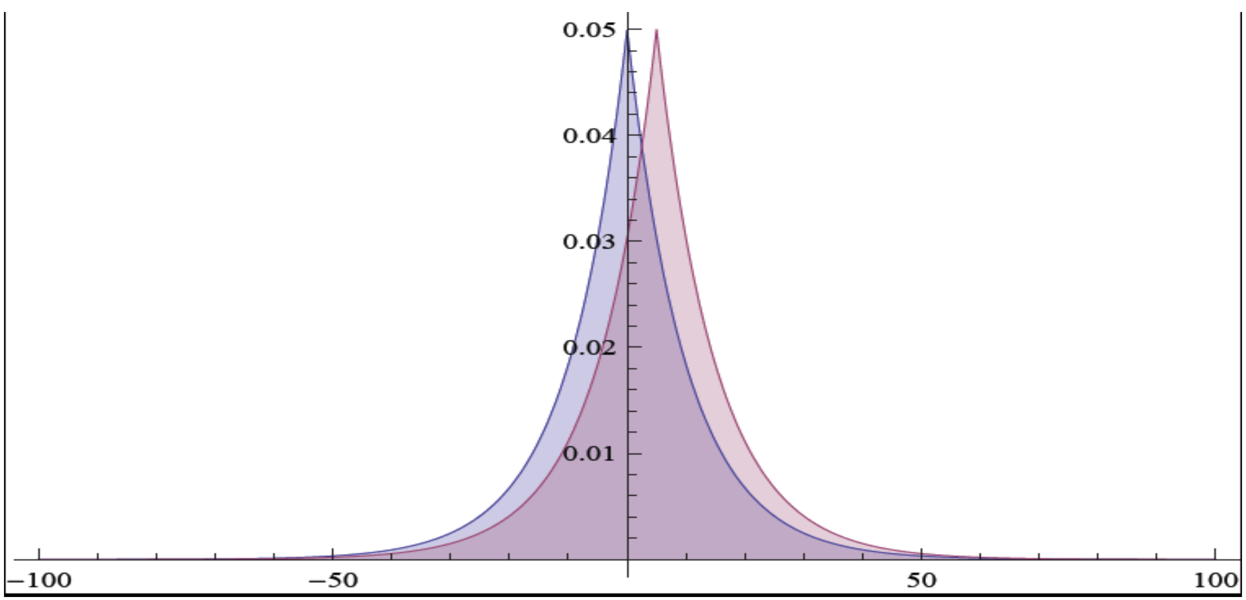
$$GS_f = n$$

Laplacian Mechanism

The mechanism

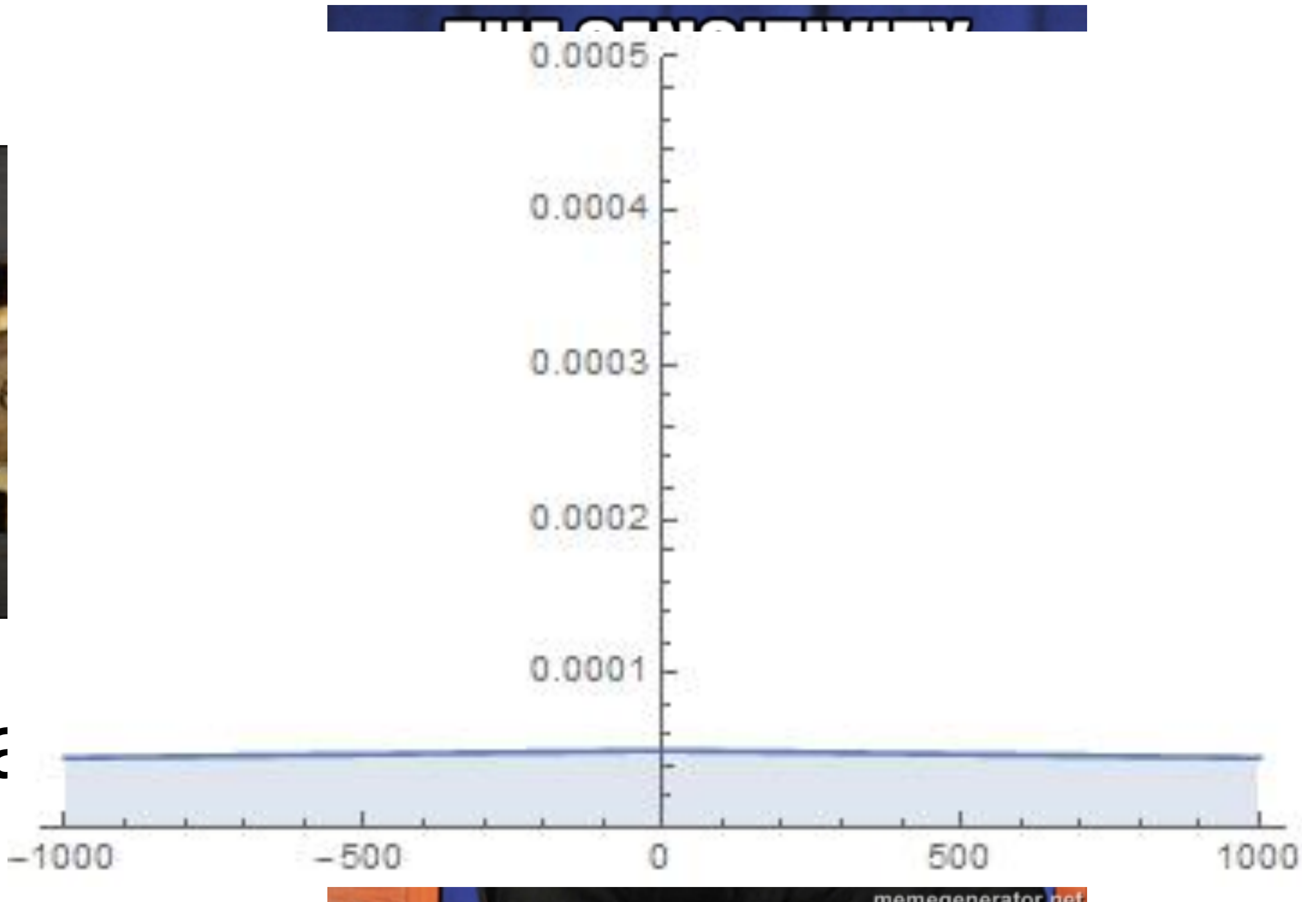
$$A(G) = f(G) + Lap(GS_f/\epsilon),$$

satisfies (ϵ,0)-differential privacy.



Privacy vs. Accuracy

$$A(G) = f(G) + \mathrm{Lap}(n/\epsilon)$$

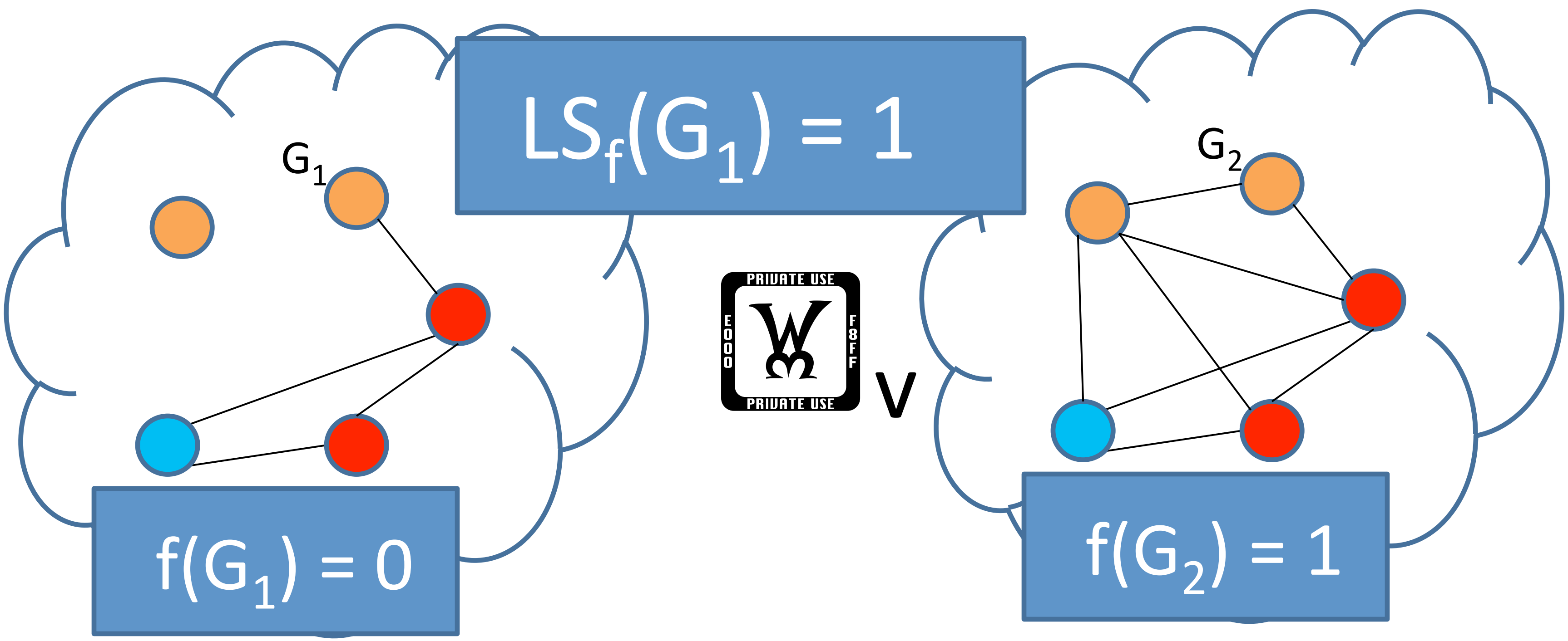


Priva

rate!

Local Sensitivity

f(G) = “how many people in G know two pianists?”



Local Sensitivity

Local Sensitivity of f at G:

$$LS_f(G) = \max_{G\sim G'} |f(G)-f(G')|$$

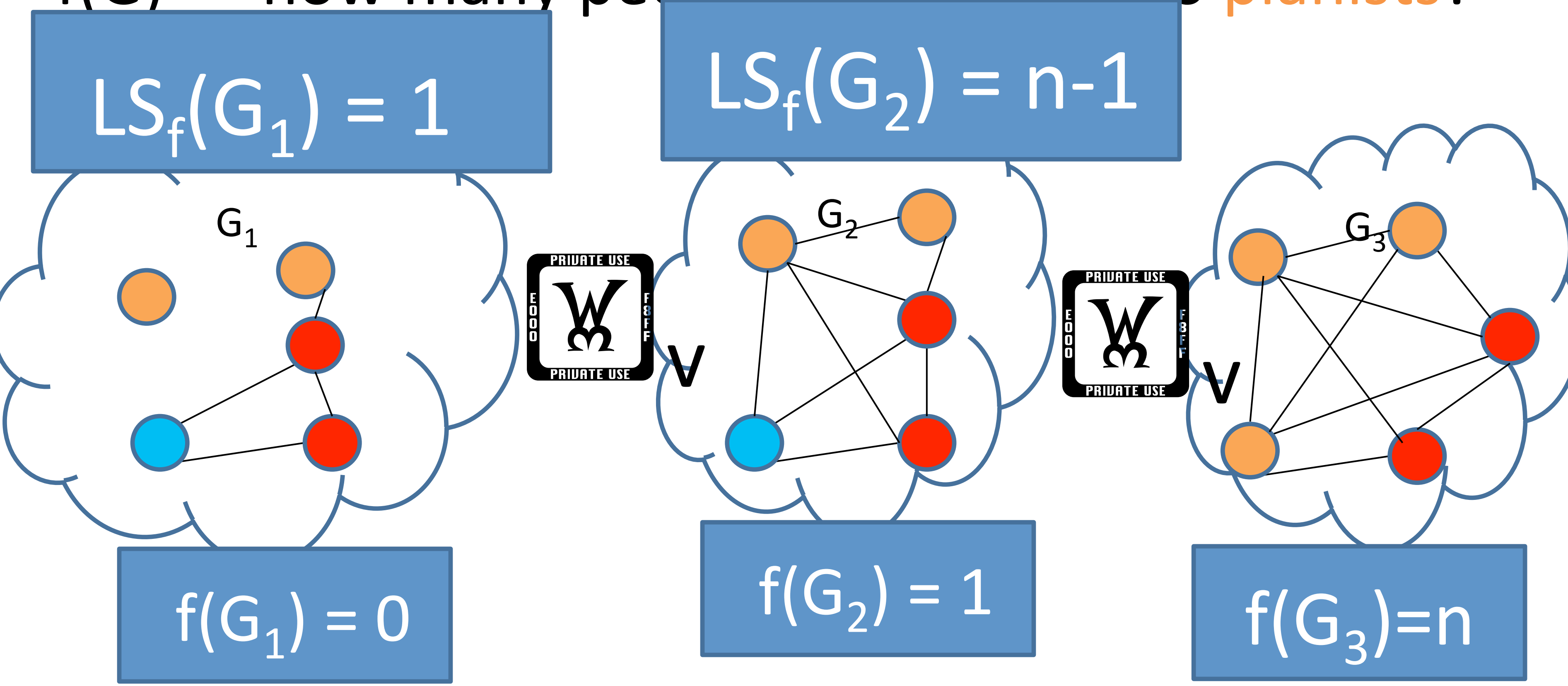
The mechanism

$$A(G) = f(G) + Lap(LS_f(G)/\epsilon)$$

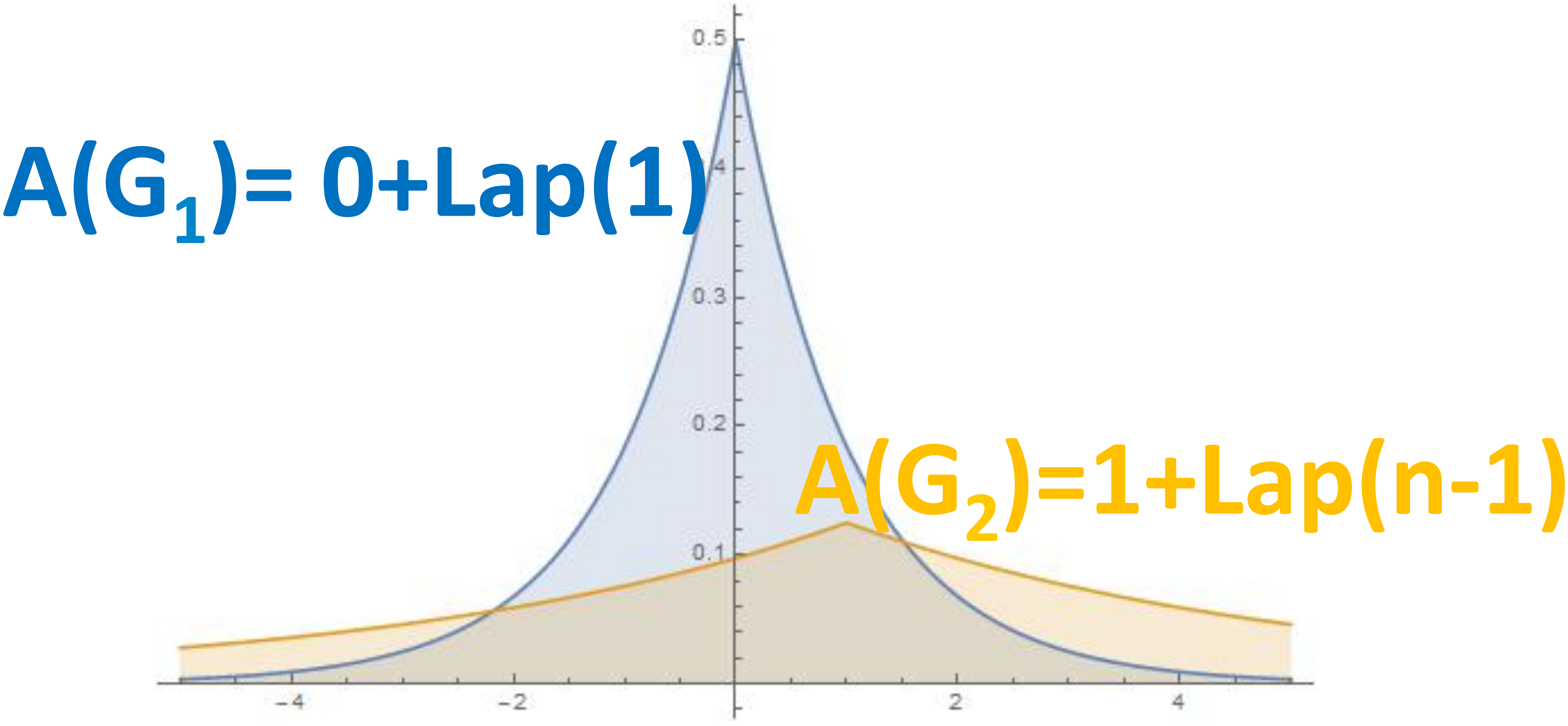
does not satisfy differential privacy.

The Problem

f(G) = “how many people in G know two pianists?”

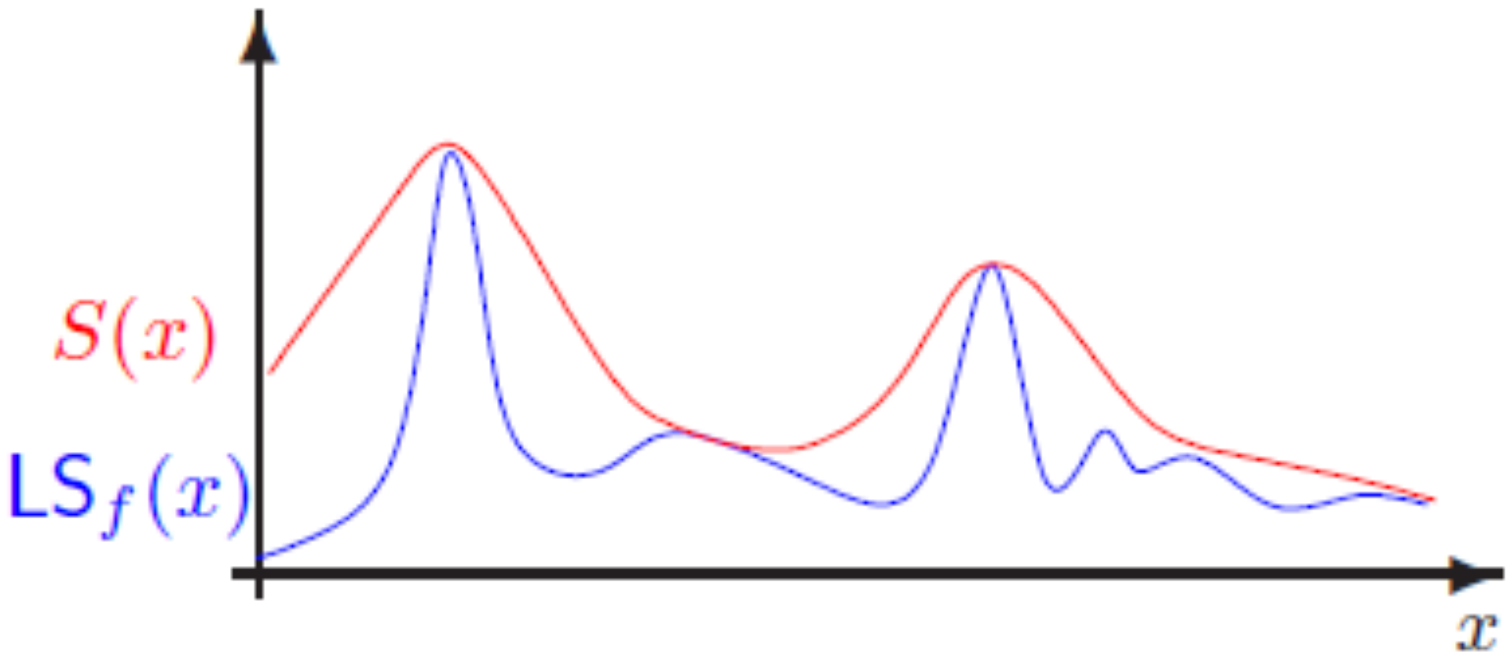


PDFs with $\varepsilon = 1$, n=5



Smooth Sensitivity (Nissim et al)

Problem: $LS_f(G)$ itself could be highly sensitive!



Smooth Sensitivity (Nissim et al)

A □-smooth upper bound on the local sensitivity $S_{f,□}$ satisfies

1. $S_{f,□}(G)$ □ $LS_f(G)$ for all G □ G.
2. $S_{f,□}(G) \leq e^{□} S_{f,□}(G')$ for all G'~G.

Theorem: The mechanism $A(G) = f(G) + Lap(2\, S_{f,□}(G) / □)$
satisfies (□, □)-differential privacy with □=-□/2 ln □.

Smooth Sensitivity

A β-smooth upper bound on the local sensitivity $S_{f,\beta}$ satisfies

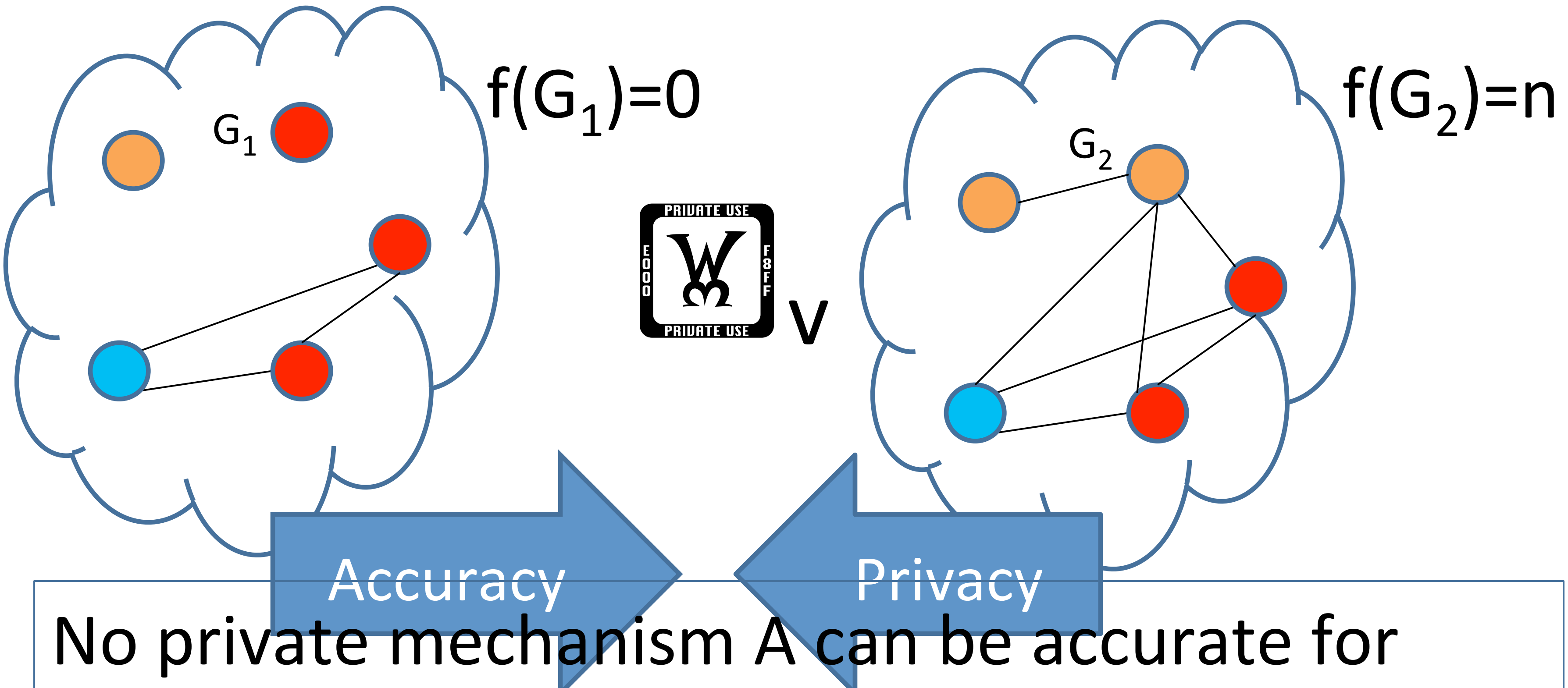
1. $S_{f,\beta}(G) \geq LS_f(G)$ for all $G \in \mathcal{G}$.
2. $S_{f,\beta}(G) \leq e^{\beta} S_{f,\beta}(G')$ for all $G' \sim G$.

When is $S_{f,\beta}(G)$ small?

1. $LS_f(G)$ must be small.
2. For any nearby graph G’, $LS_f(G')$ must also be small.

Even the Smooth Sensitivity is High!

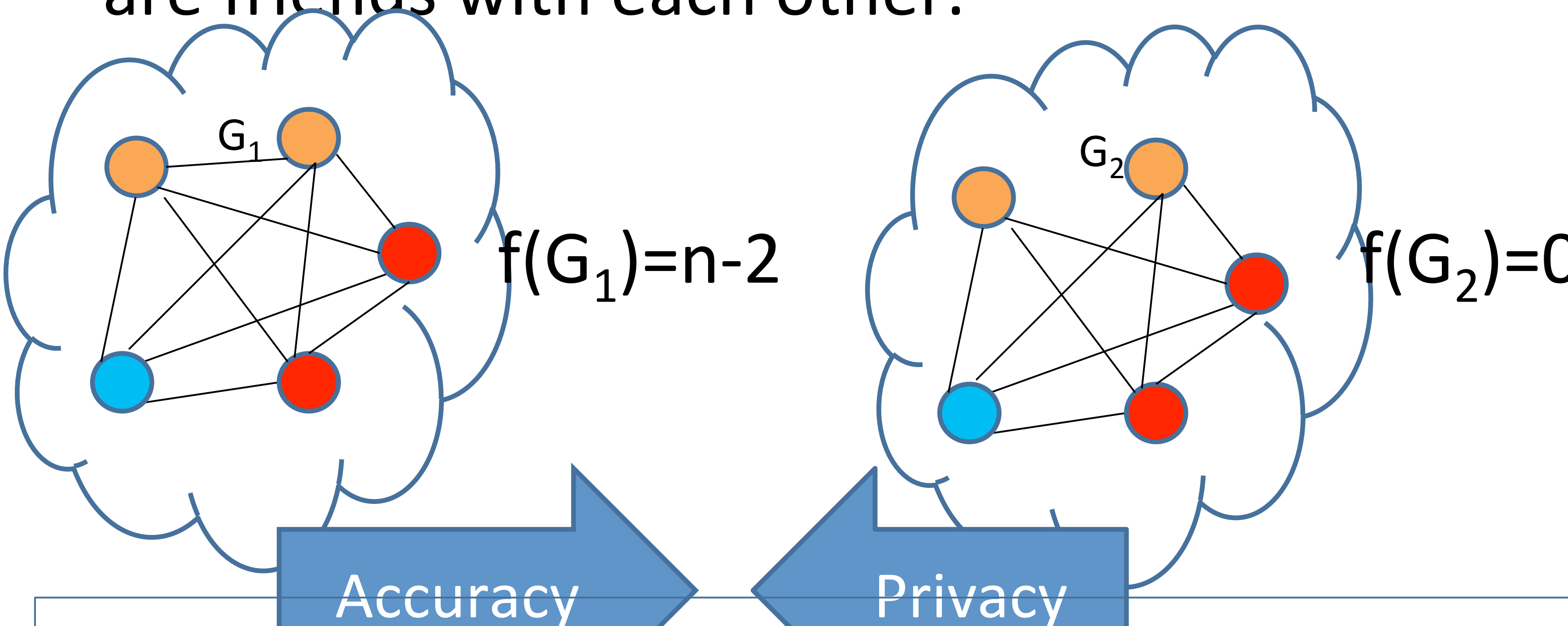
f(G) = "how many people in G know a pianist?"



No private mechanism A can be accurate for both G_1 and G_2!

Hopeless?

f(G) = "how many people know two pianists who are friends with each other."

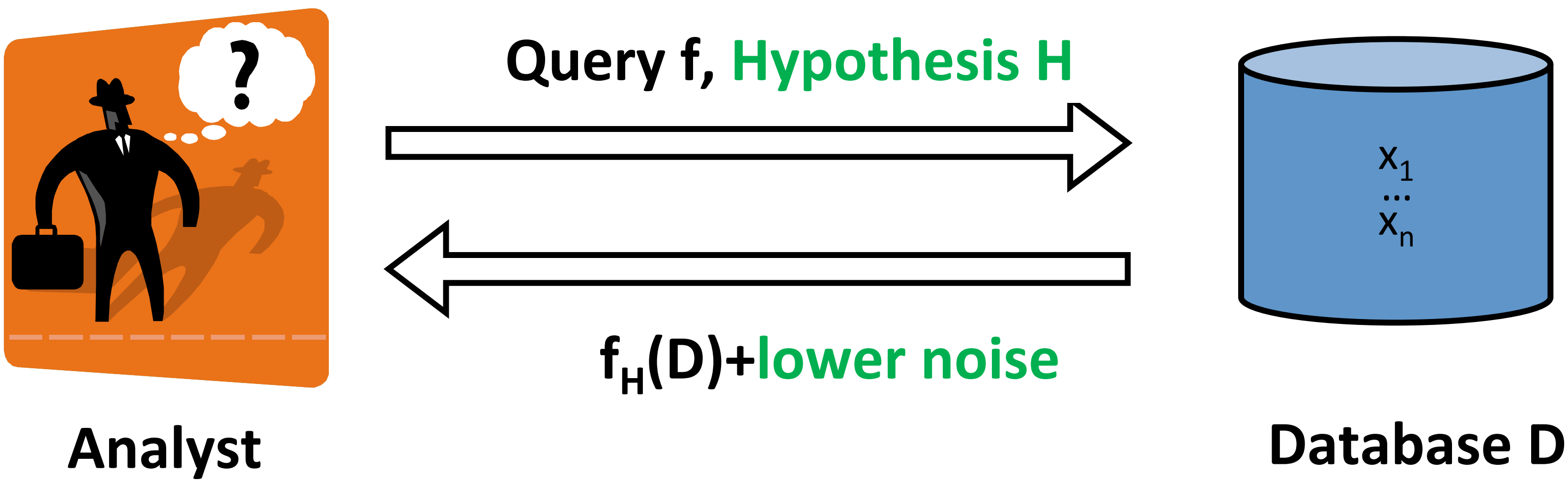


No private mechanism A can be accurate for both G_1 and G_2!

Outline

- Background
- The Problem
- **Restricted Sensitivity**
  - Lower Sensitivity for Interesting Queries
  - Challenges in Designing Mechanisms
  - Relaxed Accuracy Goal
- Algorithms

Our Setting

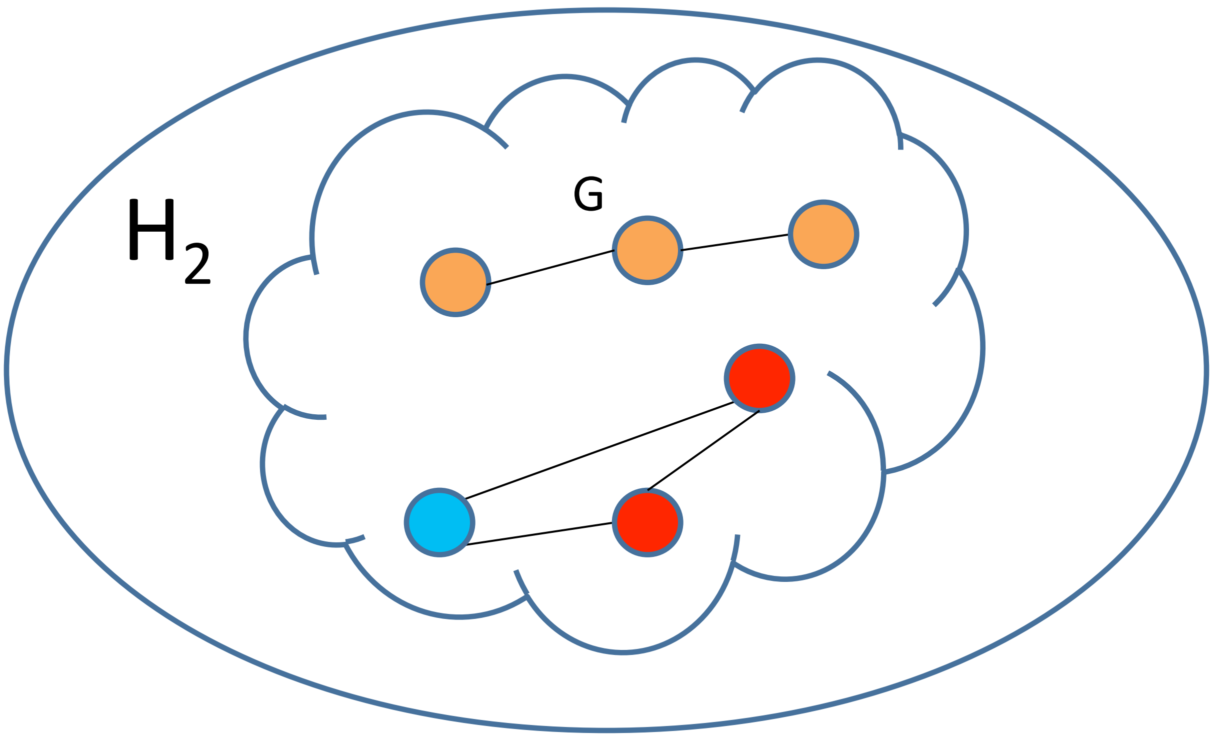


- Accurate for D in H
- Differential Privacy

Bounded Degree Hypothesis

Bounded Degree Hypothesis:

$H_k = \{ G \mid \max_{v \in V(G)} \deg(v) \leq k \}$

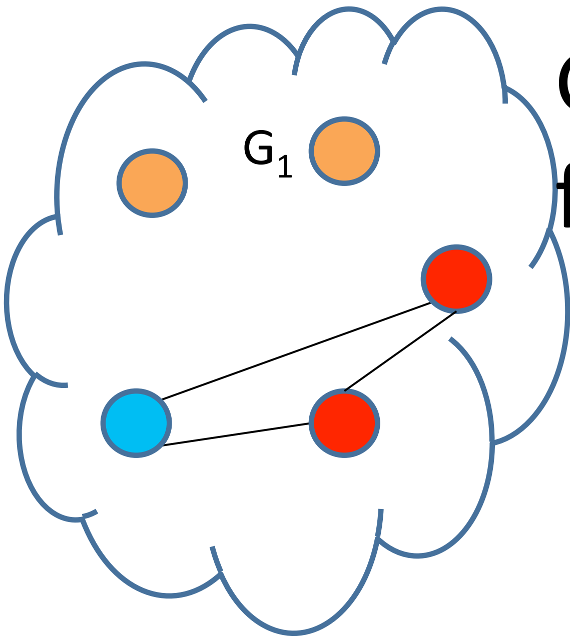


Typical:

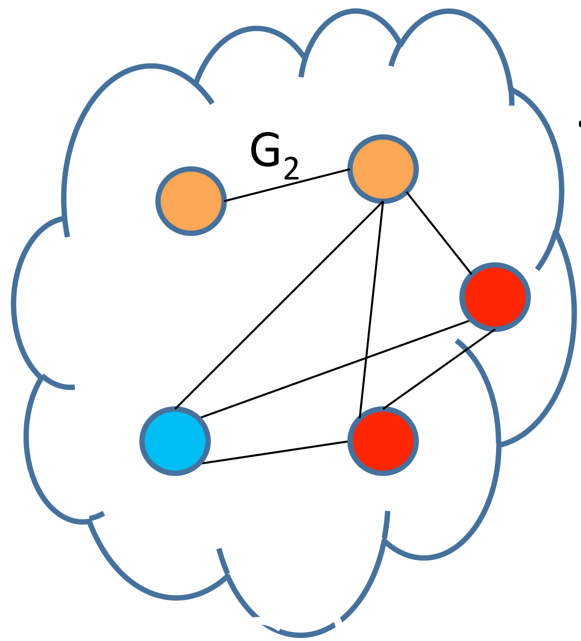
$k << n$

Challenge Revisited

f(G) = "how many people in G know a pianist?"



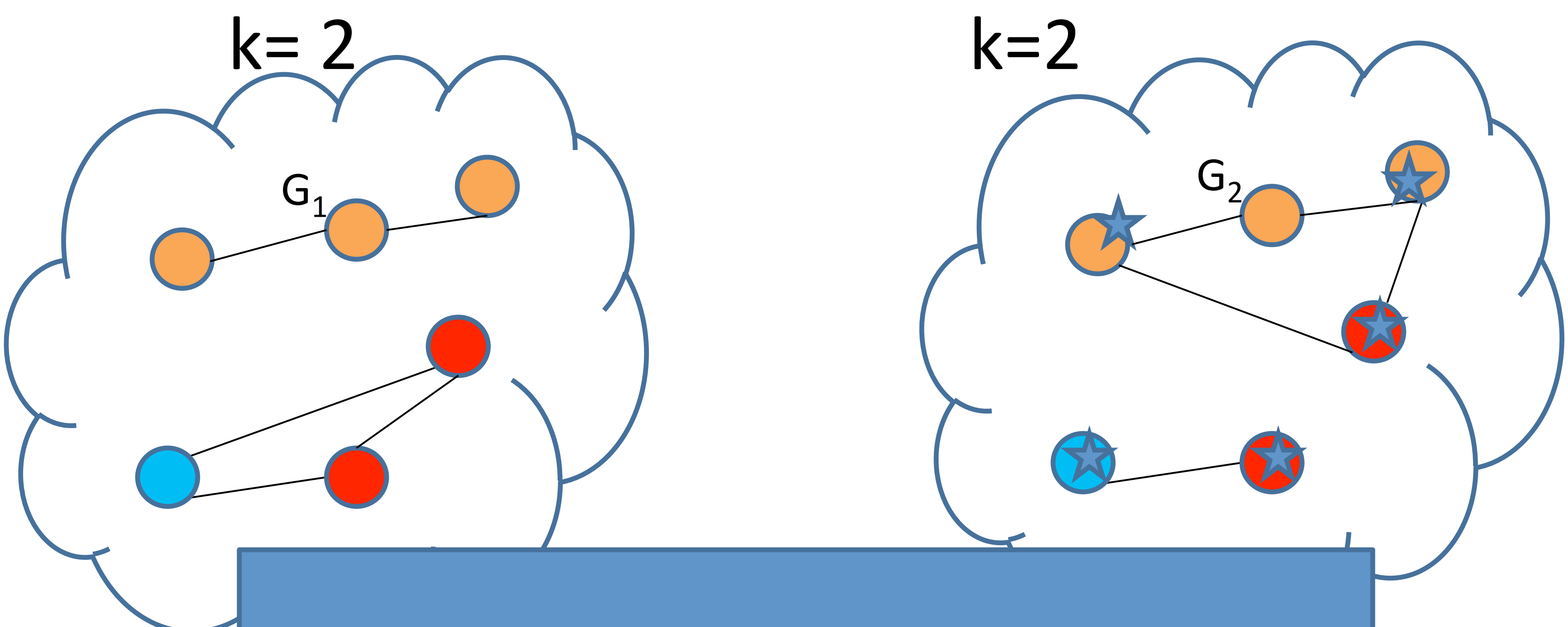
G_1 □ H_k.
$f(G_1) = 0$



G_2 □ H_k.
$f(G_2)=n$

Restricted Sensitivity $RS_f(H_k)$

Fact: For local profile queries f, $RS_f(H_k) \leq 2k+1$



$|f(G_1) - f(G_2)| \leq 5 = RS_f(H_2)$

Restricted Sensitivity

Hypothesis: H [illegible] G

$$GS_f = \max_{G_1, G_2} \frac{\left| f(G_1) - f(G_2) \right|}{d(G_1, G_2)}$$

$$RS_f(H) = \max_{G_1, G_2 \in H} \frac{\left| f(G_1) - f(G_2) \right|}{d(G_1, G_2)}$$

Sensitivity over H_k

	Local Profile Query		Subgraph Counting Query (P)	
Adjacency	**Smooth**	**Restricted**	**Smooth**	**Restricted**
Edge	k+1	k+1	$O(\|P\| k^{\|P\|-1})$	$O(\|P\| k^{\|P\|-1})$
Vertex	n	2k+1	$O(n^{\|P\|-1})$	$O(\|P\| k^{\|P\|-1})$

Two "Strawman"-Algorithms

A(G) = f(G)+	$Lap(GS_f/$[illegible]$)$	$Lap(RS_f(H_k)/$[illegible]$)$
Accurate on H_k?	NO	YES
Private?	YES	NO

Restricted Sensitivity

For $k \ll n$ the mechanism

$$A(G) = f(G) + Lap(RS_f(H_k)/\epsilon)$$

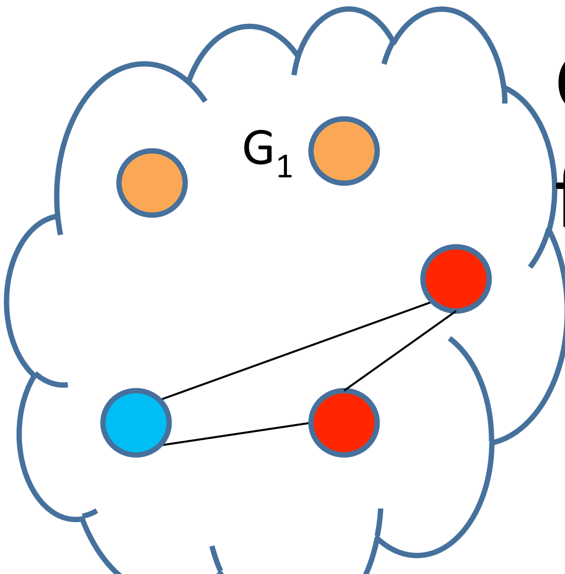
is accurate!

The mechanism

$$A(G) = f(G) + Lap(RS_f(H)/\epsilon)$$

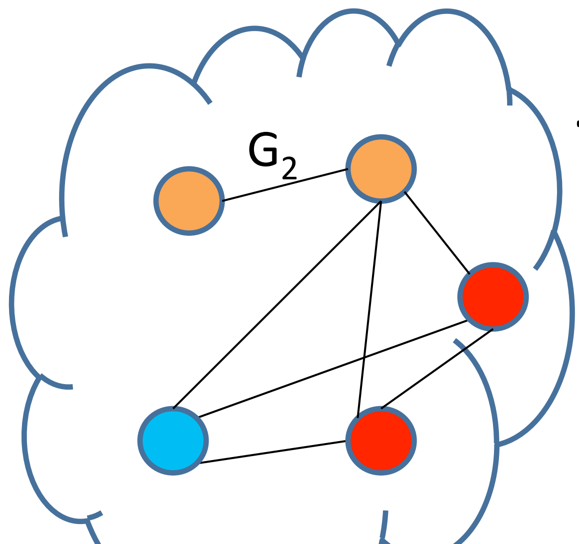
does not satisfy differential privacy for all G.

f(G) = "how many people in G know a pianist?"



G_1 [illegible] H_k.

$f(G_1) = 0$

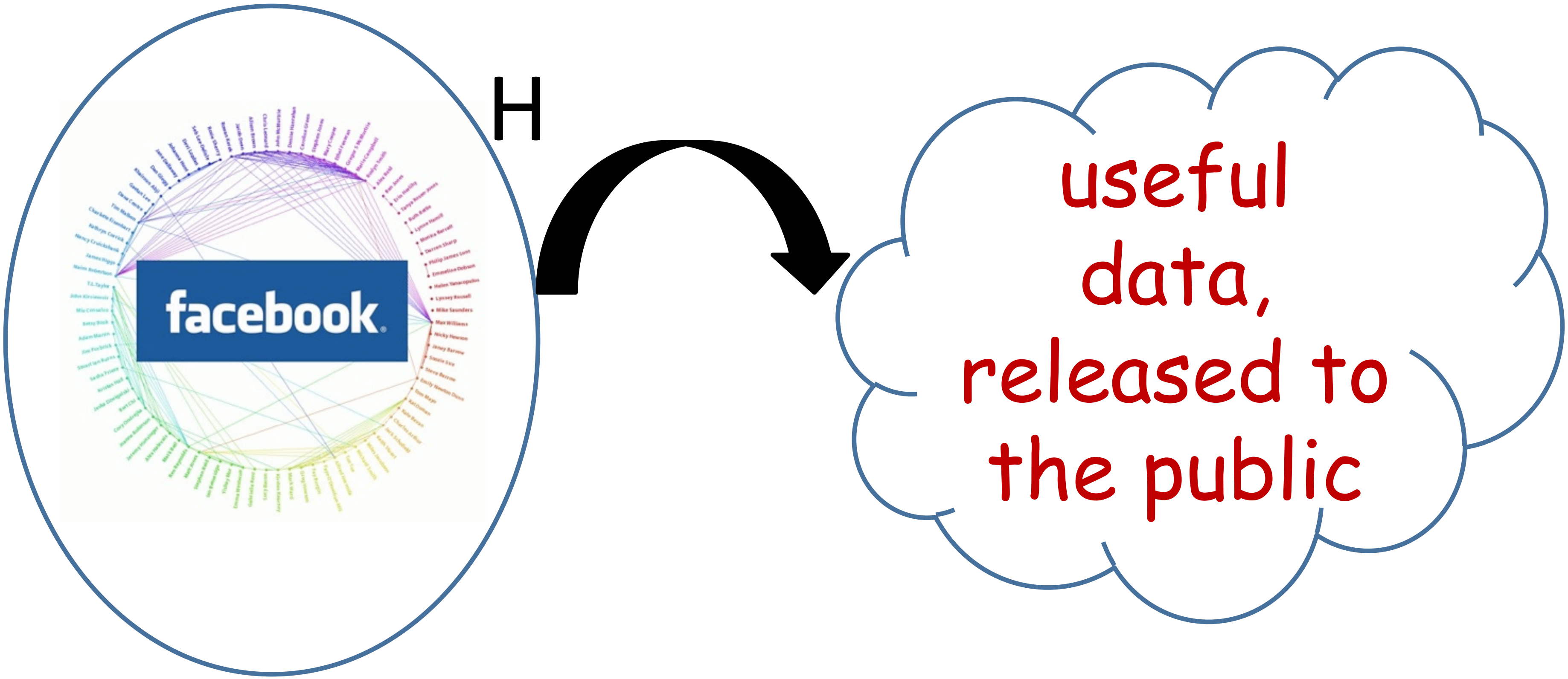


G_2 [illegible] H_k.

$f(G_2)=n$

Pr[A(G_1) [illegible] 0] ≤ e[illegible] Pr[A(G_2) [illegible] 0] +

Privacy for All, Accuracy for Some



1. Privacy for all G
2. Accurate statistics for G [illegible] H

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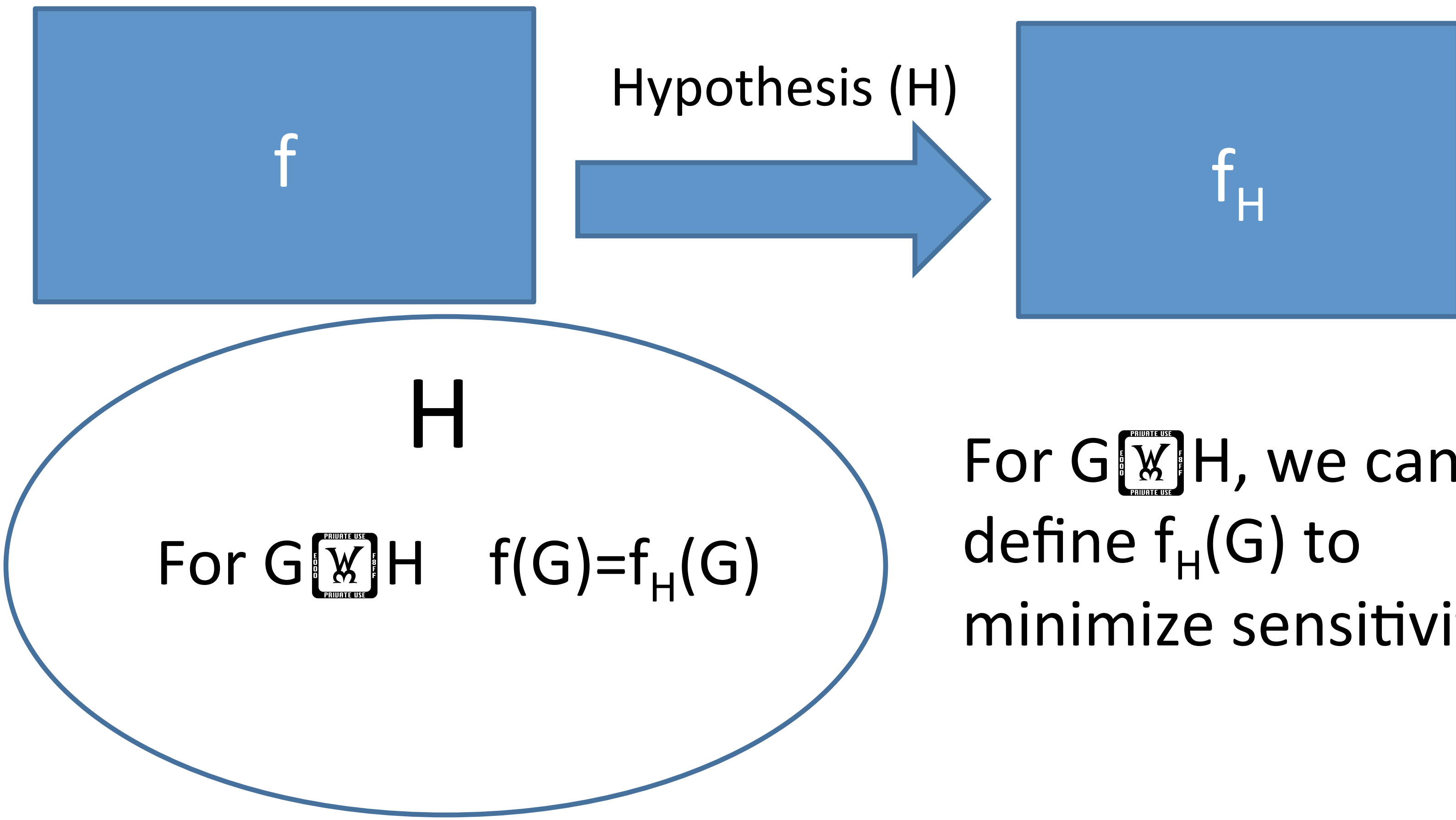
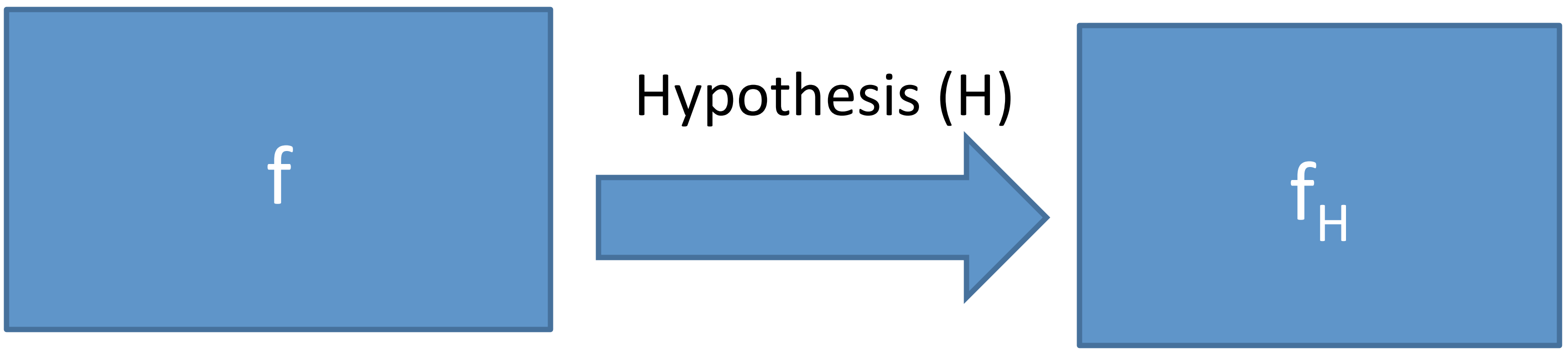
- **Algorithms**
  - **General Template**
  - Possibility: A General Inefficient Algorithm
  - Efficient Algorithms for H_k via Projections

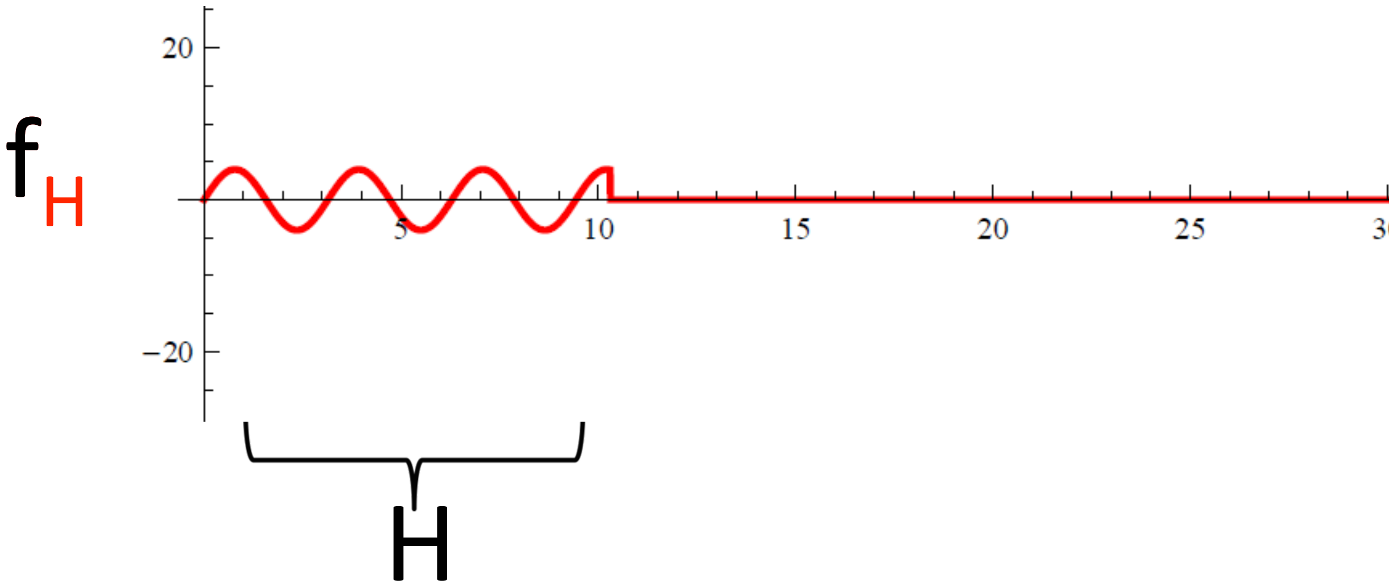
Accuracy for Some



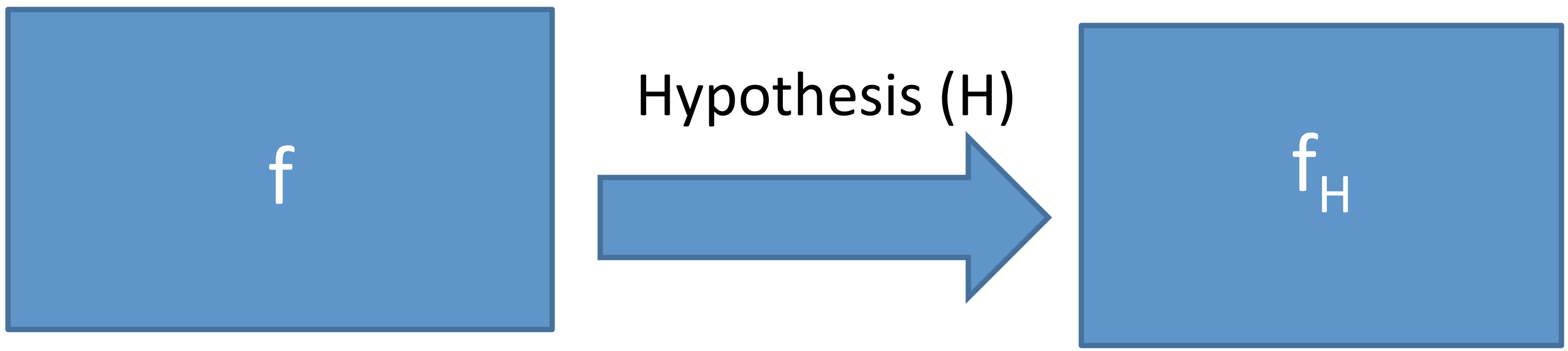
For $G \in H$, we can define $f_H(G)$ to minimize sensitivity.

Accuracy for Some





Privacy For All



Answer f_H in a differentially private manner.

Ideally, we would like to compute f_H efficiently.

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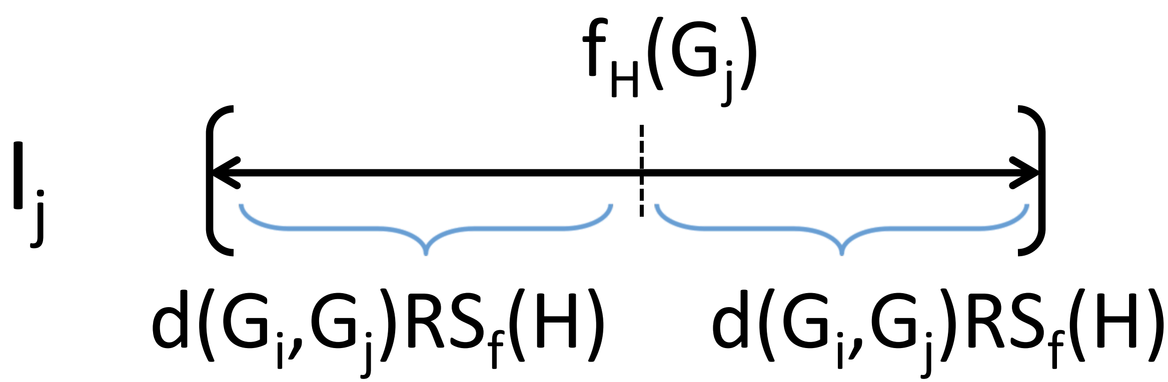
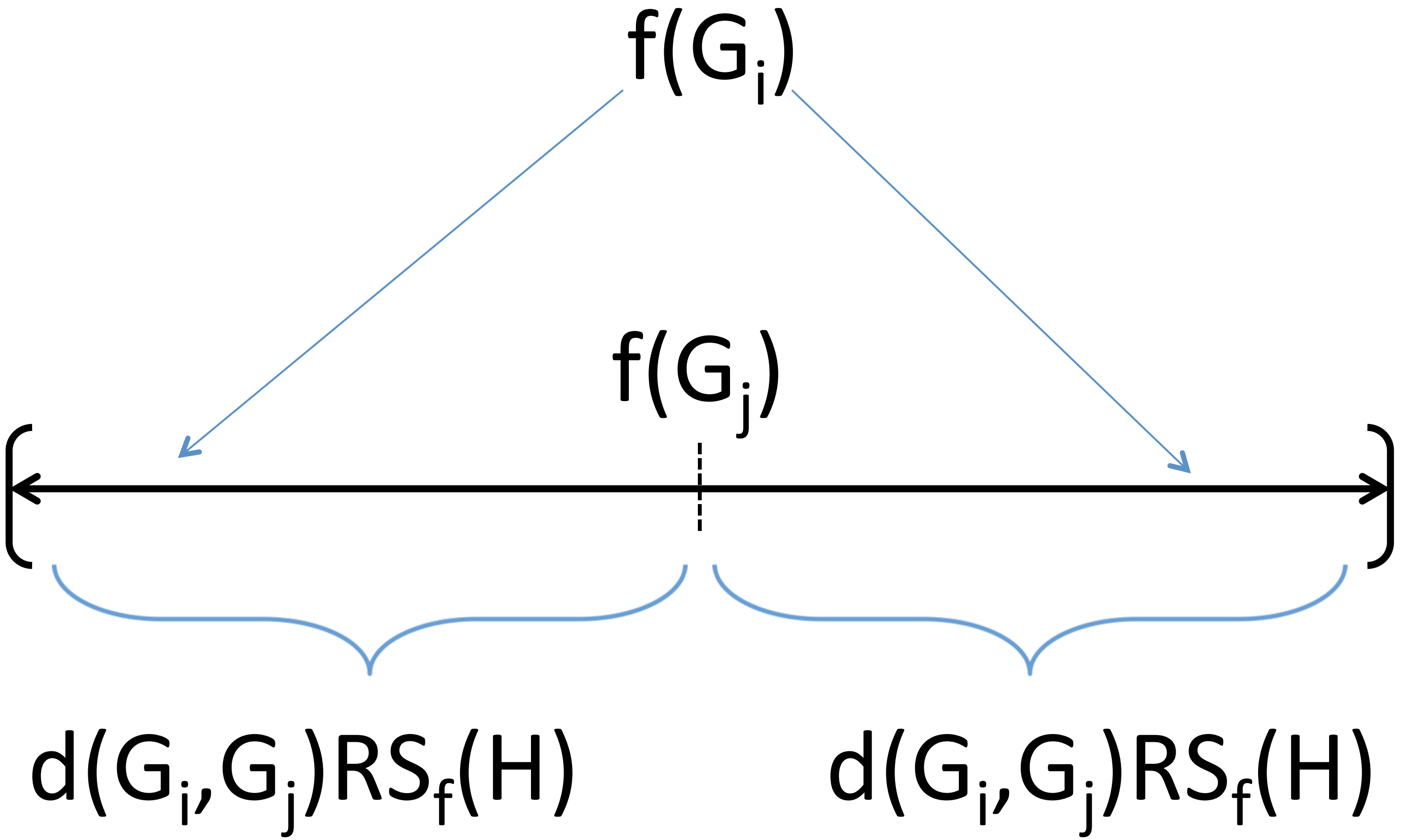
General Construction of f_H

Consider a canonical ordering $G_1, G_2, \ldots$ over all social networks($\mathcal{G}$).

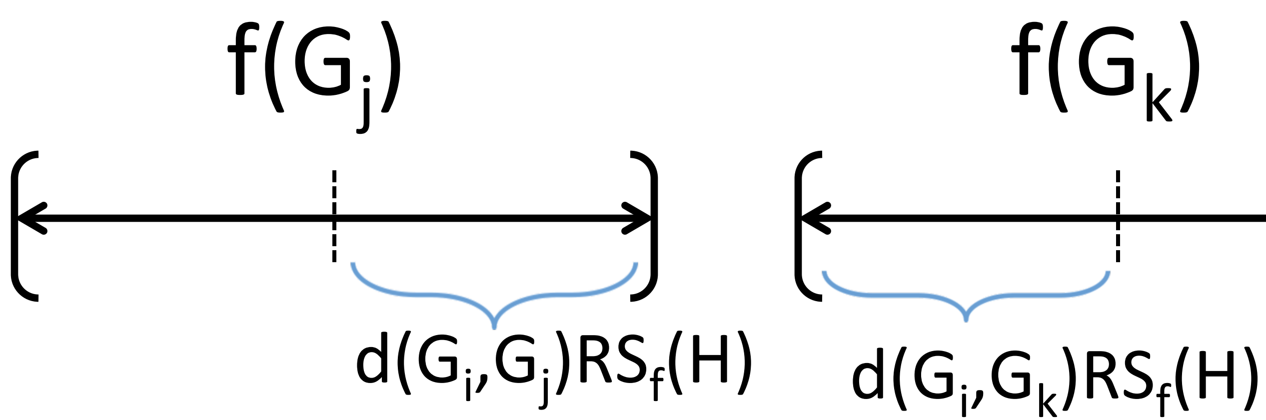
- For $G_i \in H$ set $f_H(G_i) = f(G_i)$
- Find $f_H(G_i) \in \bigcap_{j<i} I_j$




f(G_i)
f(G_j)
d(G_i,G_j)RS_f(H)
d(G_i,G_j)RS_f(H)

Suppose for contradiction that no value exists....

Can find two nonintersecting intervals I_j and I_k ($j,k < i$).



$$\frac{\left|f\left(G_j\right)-f\left(G_k\right)\right|}{d\left(G_k,G_i\right)+d\left(G_j,G_i\right)} > RS_f\left(H\right)$$

Can Find Non Intersecting Intervals

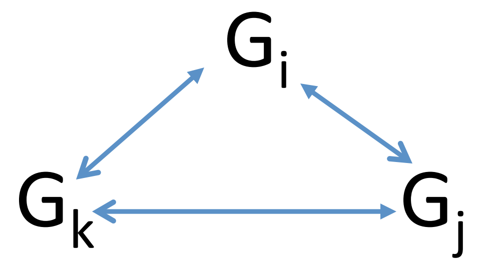
$$j = \arg\min_{t<i} \left[\max_{x \in I_t} x \right]$$

$$k = \arg\max_{t<i} \left[\min_{x \in I_t} x \right]$$

$$x \in I_j \cap I_k \Rightarrow \forall t < i, \quad x \in I_t$$

Suppose for contradiction that no value exists....

Triangle Inequality



$$\frac{\left|f\left(G_j\right)-f\left(G_k\right)\right|}{d\left(G_k,G_j\right)} \geq \frac{\left|f\left(G_j\right)-f\left(G_k\right)\right|}{d\left(G_k,G_i\right)+d\left(G_j,G_i\right)} > RS_f\left(H\right)$$

Contradiction of Inductive Hypothesis!

Suppose for contradiction that no value exists….

Then there must exist two intervals which don't intersect (j,k < i).

$$[f(G_j) - d(G_i, G_j) RS_f(H), f(G_j) + d(G_i, G_j) RS_f(H)]$$

$$[f(G_k) - d(G_i, G_k) RS_f(H), f(G_k) + d(G_i, G_k) RS_f(H)]$$

$$\frac{f(G_j) - f(G_k)}{d(G_k, G_i) + d(G_j, G_i)} > RS_f(H)$$

Suppose for contradiction that no value exists….

Triangle Inequality

$$\frac{f(G_j) - f(G_k)}{d(G_k, G_j)} \geq \frac{f(G_j) - f(G_k)}{d(G_k, G_i) + d(G_j, G_i)} > RS_f(H)$$

Contradiction of Inductive Hypothesis!

General Construction of f_H

Theorem: f_H satisfies

1. $f_H(G) = f(G)$ for all G [illegible] H
2. $GS_{f_H} = RS_f(H)$

Disclaimer: The general construction of f_H is *not* efficient.

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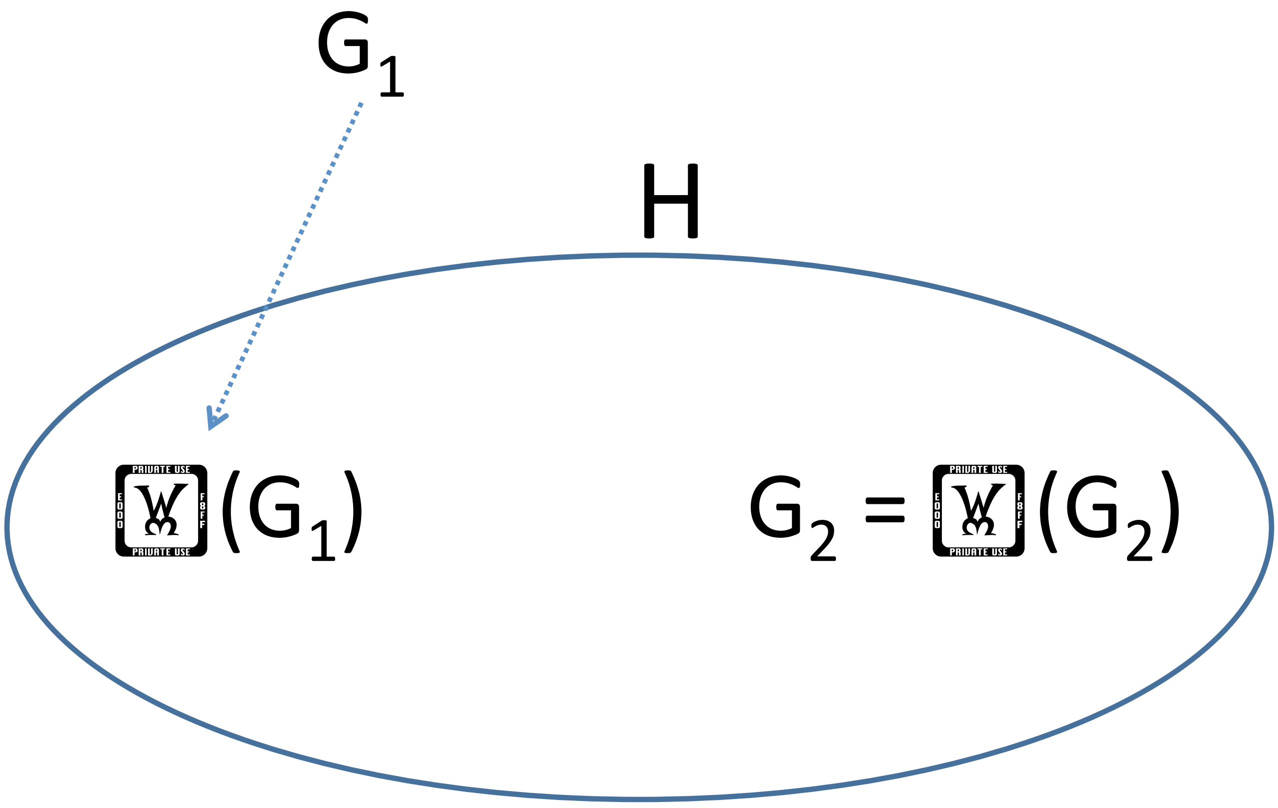
  – **Efficient Algorithms for H_k via Projections**
    - Edge Adjacency Model
    - Vertex Adjacency

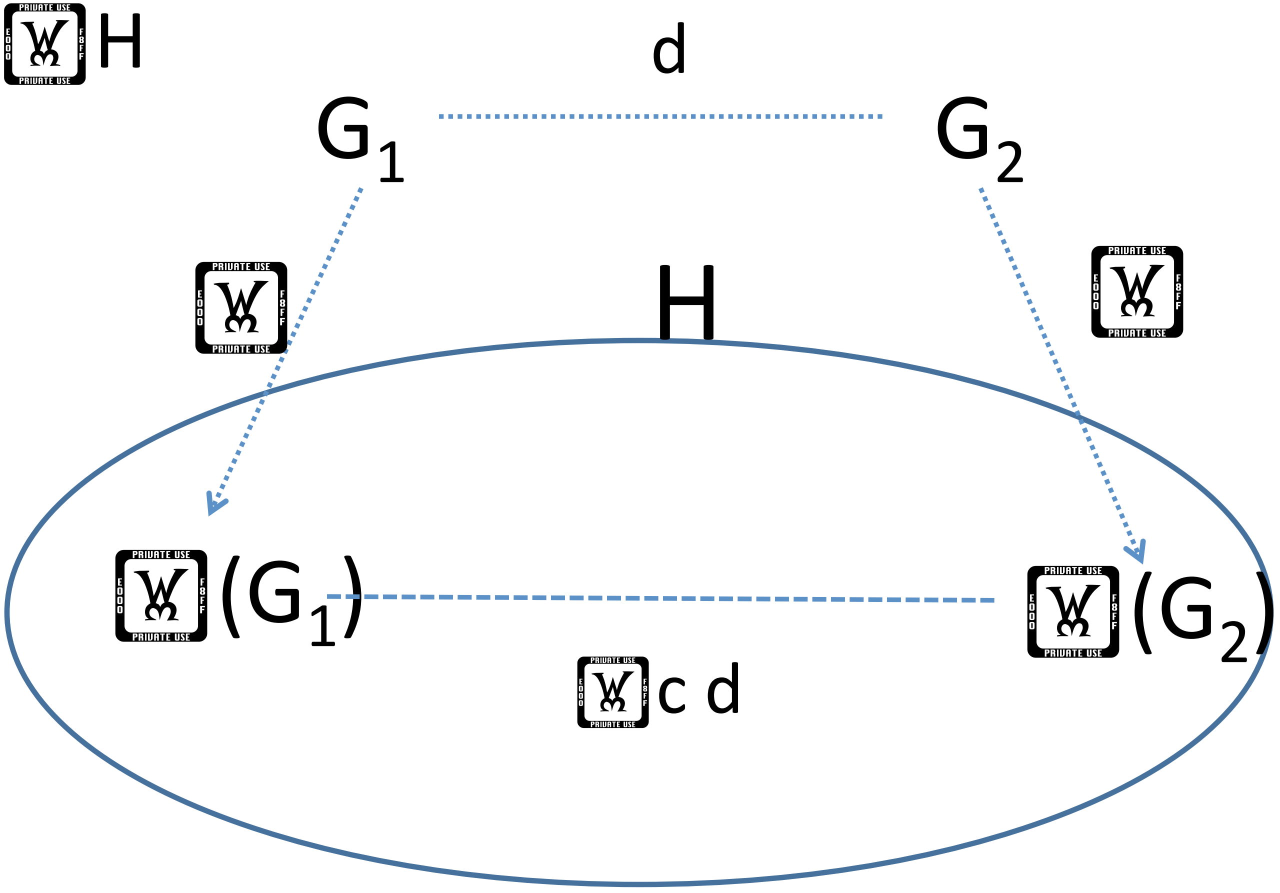
Projection onto H

: G H



c-smooth Projection

□: G □ H



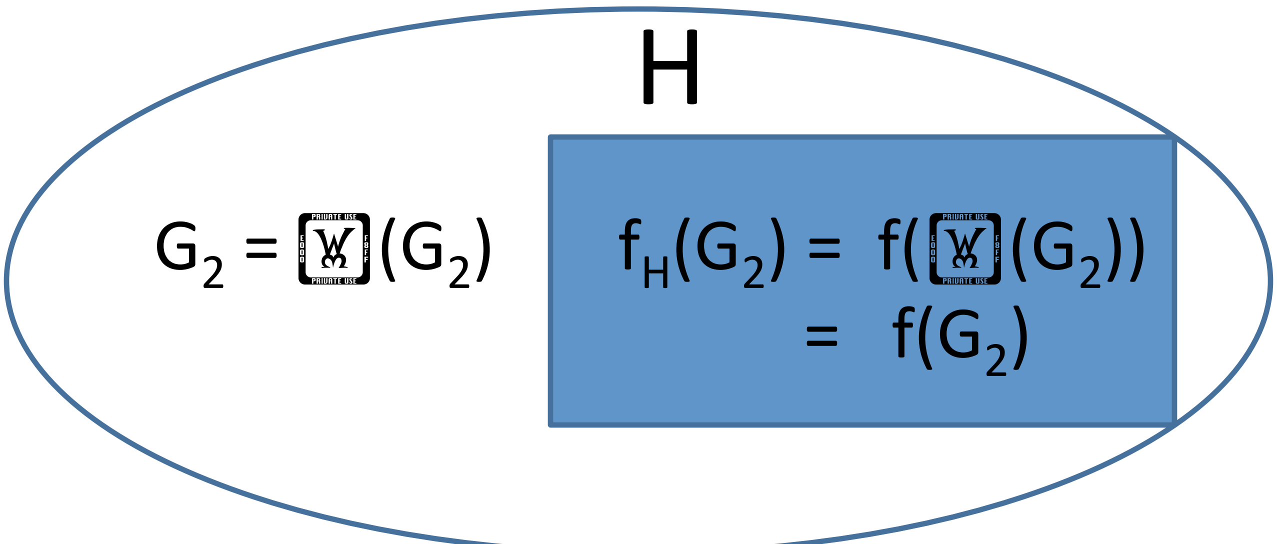
c-smooth Projection Lemma

Let : G H be a c-smooth projection then $f_H(G) = f((G))$ satisfies:

1. For G H, $f_H(G) = f(G)$, and
2. $GS_{f_H} \leq c\ RS_f(H)$.

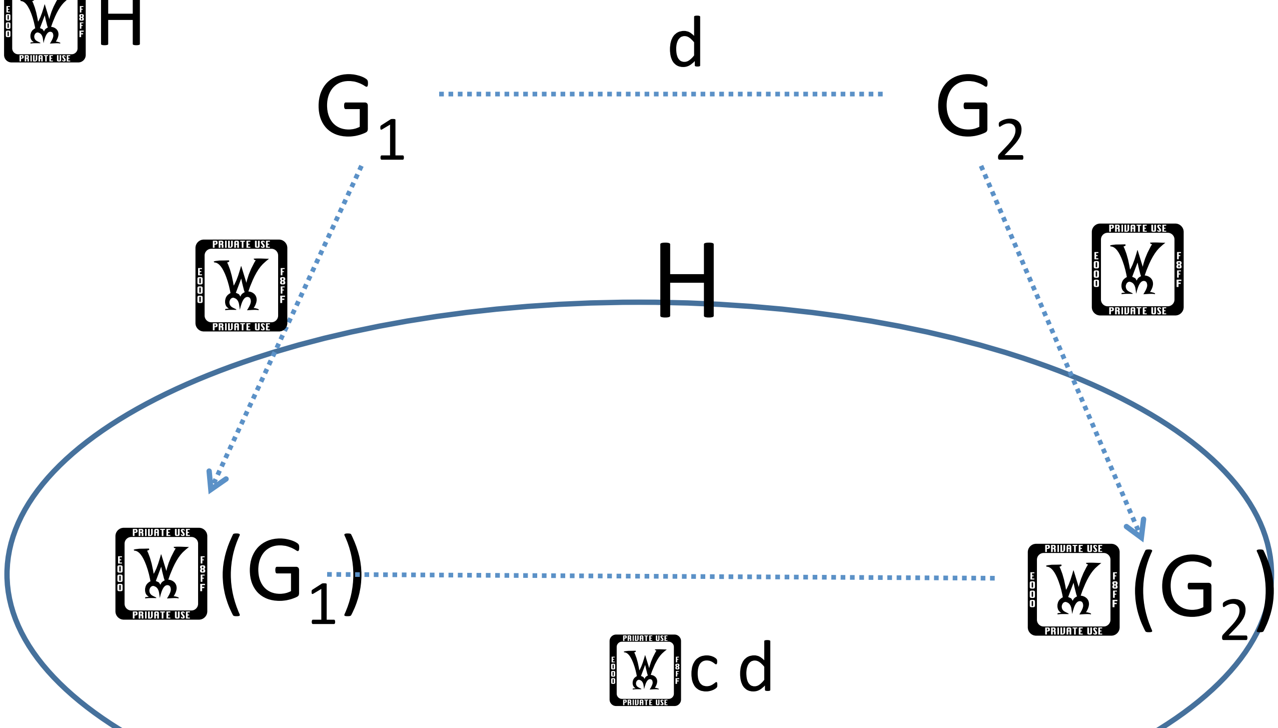
Proof of (1)

(1) For G [illegible] H, $f_H(G) = f(G)$,



Proof of (2)

: G H



$$| f_H(G_1) - f_H(G_2) | = | f(\text{}(G_1)) - f(\text{}(G_2)) |$$

$$\leq d(\text{}(G_1), \text{}(G_2))\, RS_f(H)/d$$

$$\leq c\, RS_f(H)$$

Outline

- Differential Privacy in Social Networks
- The Problem
- Restricted Sensitivity
- **Algorithms**
  – General Template
  – Possibility: A General Inefficient Algorithm
  – Efficient Algorithms for H_k via Projections
    - **Edge Adjacency Model**
    - Vertex Adjacency

Edge Adjacency (□: G □ H_k)

1. For each vertex v with deg(v) > k:
   - Let $(v,u_1),\ldots,(v, u_{deg(v)})$ denote the edges incident to v in canonical order
   - Mark the edge (v,u_i) for each i > k.
2. Delete all marked edges.
3. Call the resulting graph □(G).

Edge Adjacency (□: G □ H_k)

Claim: The efficiently computable mapping □(G) is a 3-smooth projection.

Proof:

(1) If G □ H_k then no edges are deleted so □(G)=G.

(2) WTS: For any G~G', d(□(G), □(G')) ≤ 3.

Edge Adjacency (□: G □ H_k)

(2) For any G~G’, d(□(G), □(G’)) ≤ 3.

Proof:

If E(G’) = E(G)+(u,v) then

i. □(G’) might still contain (u,v).
ii. At most one edge (u,w_1) □ E(□(G))\ E(G’).
iii. At most one edge (v,w_2) □ E(□(G))\ E(G’).

So d(□(G), □(G’)) ≤ 3.

(QED)

Outline

- Differential Privacy in Social Networks
- The Problem
- Restricted Sensitivity
- **Algorithms**
  – General Template
  – Possibility: A General Inefficient Algorithm
  – **Efficient Algorithms for H_k via Projections**

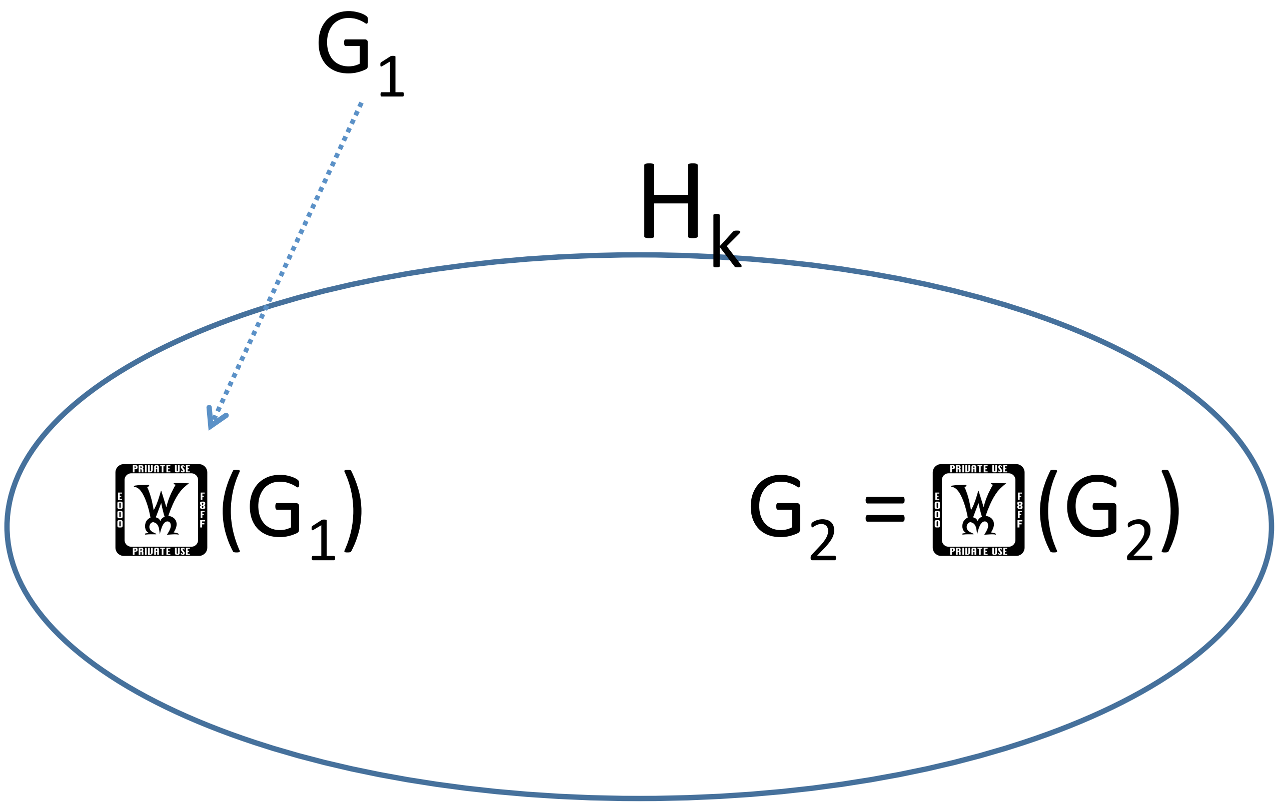
High Level Picture

- Concept: Smooth Distance Estimation
  - Project all G to H_{2k}
  - Lemma: Accuracy for G in H_k (leveraging smooth sensitivity)

- Constructing a Smooth Distance Estimator
  - LP Rounding

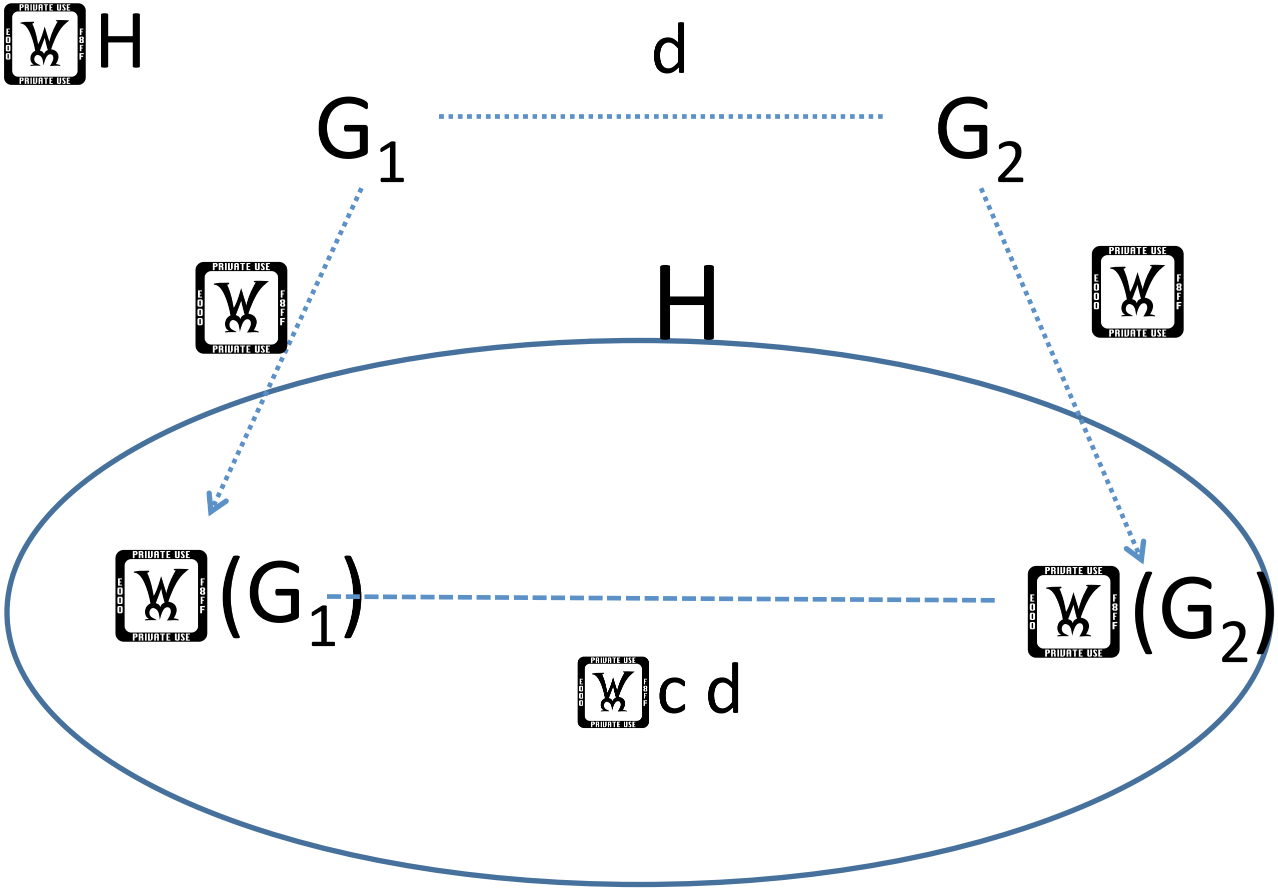
Naïve Projection

[illegible]: G [illegible] H



Map Close Graphs to Close Graphs

□: G □ H



Projection Lemma

Let □: G □H be a projection such that

1. For G □ H, □(G) = G, and
2. For G~G', d(□(G), □(G)) □ c

then $f_H(G) = f(□(G))$ satisfies:

1. For G □ H, $f_H(G) = f(G)$, and
2. $GS_{f_H} \leq c\ RS_f(H)$.

Construction is efficient in the edge adjacency model!

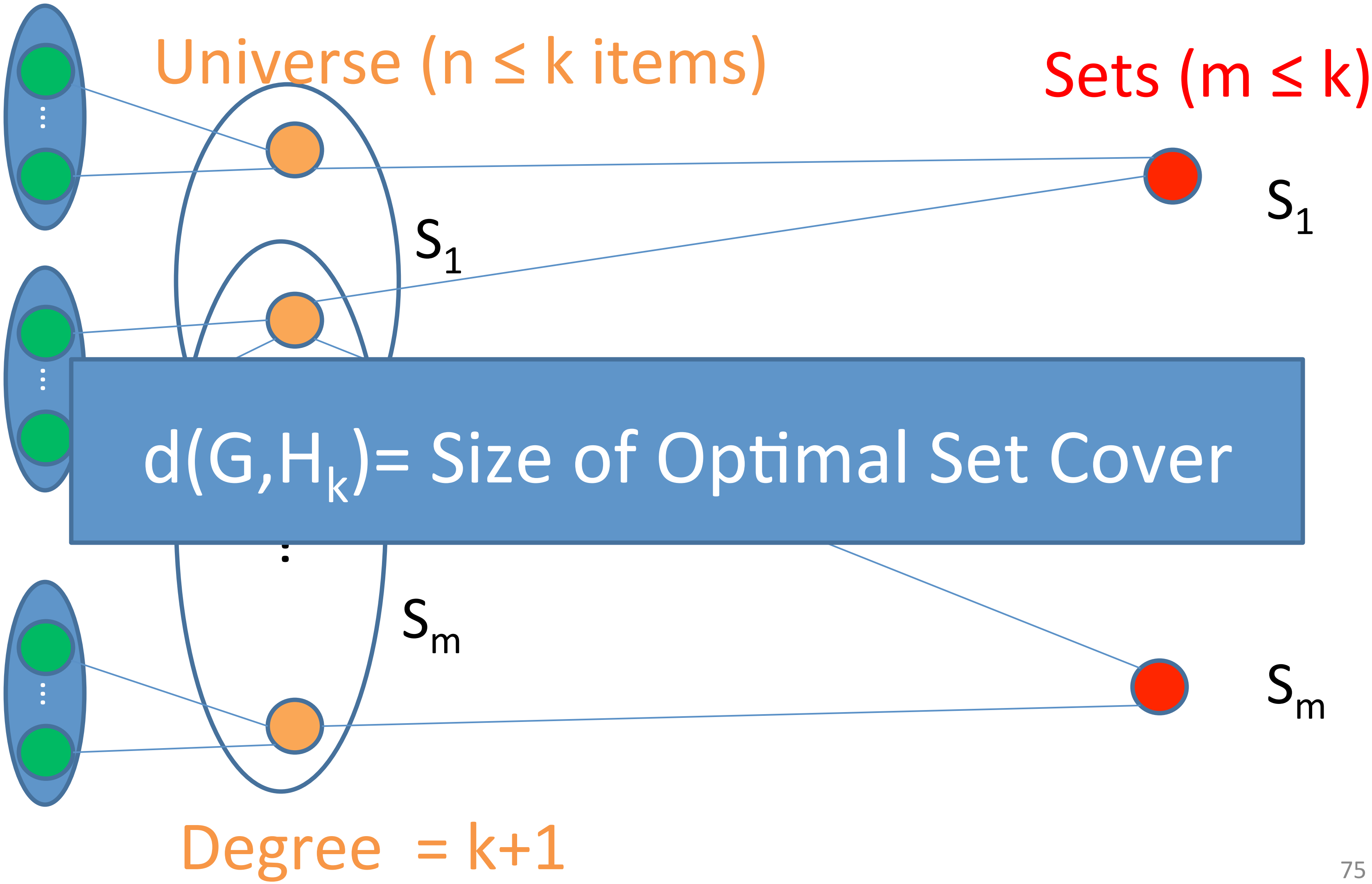
Vertex Adjacency?

- Map close graphs to close graphs in H_k?
  - Worked for Edge Adjacency Model

- Can't Work! ☹ Why?
- It would allow us to approximate $d(G,H_k)$.

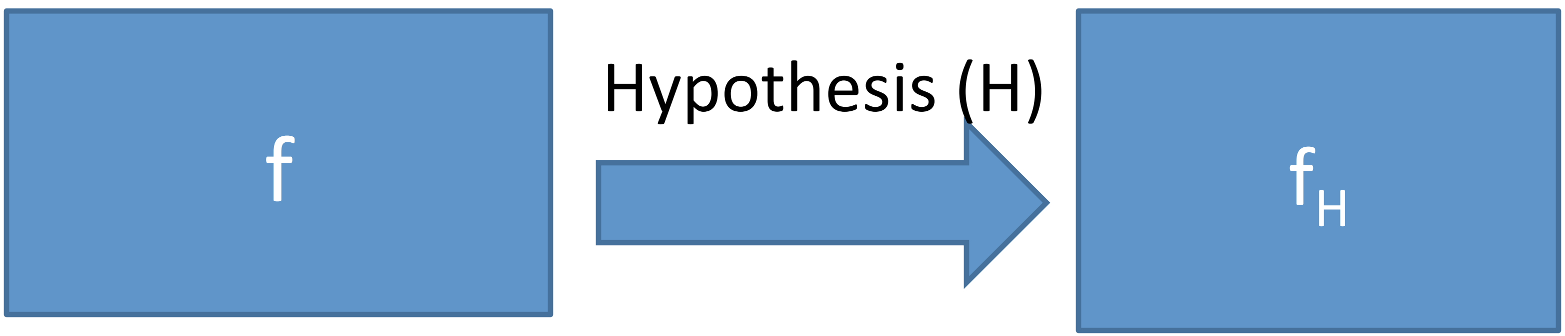
Claim: It is NP-hard to approximate $d(G,H_k)$ to within any constant factor (reduction from set cover)

Reduction from Set Cover



$d(G,H_k)$= Size of Optimal Set Cover

A New Approach



Leverage Smooth Sensitivity!

$$\forall G \in H,\ S_{f_H,\beta}(G) = O(RS_f(H))$$

$$GS_{f_H} = RS_f(H)$$

Privacy For All

Theorem: The mechanism

$$A(G) = f_{H_k}(G) + Lap\left(\frac{2S_{f_{H_k},\beta}(G)}{\varepsilon}\right)$$

preserves differential privacy for [illegible]=-[illegible]/2 ln [illegible].

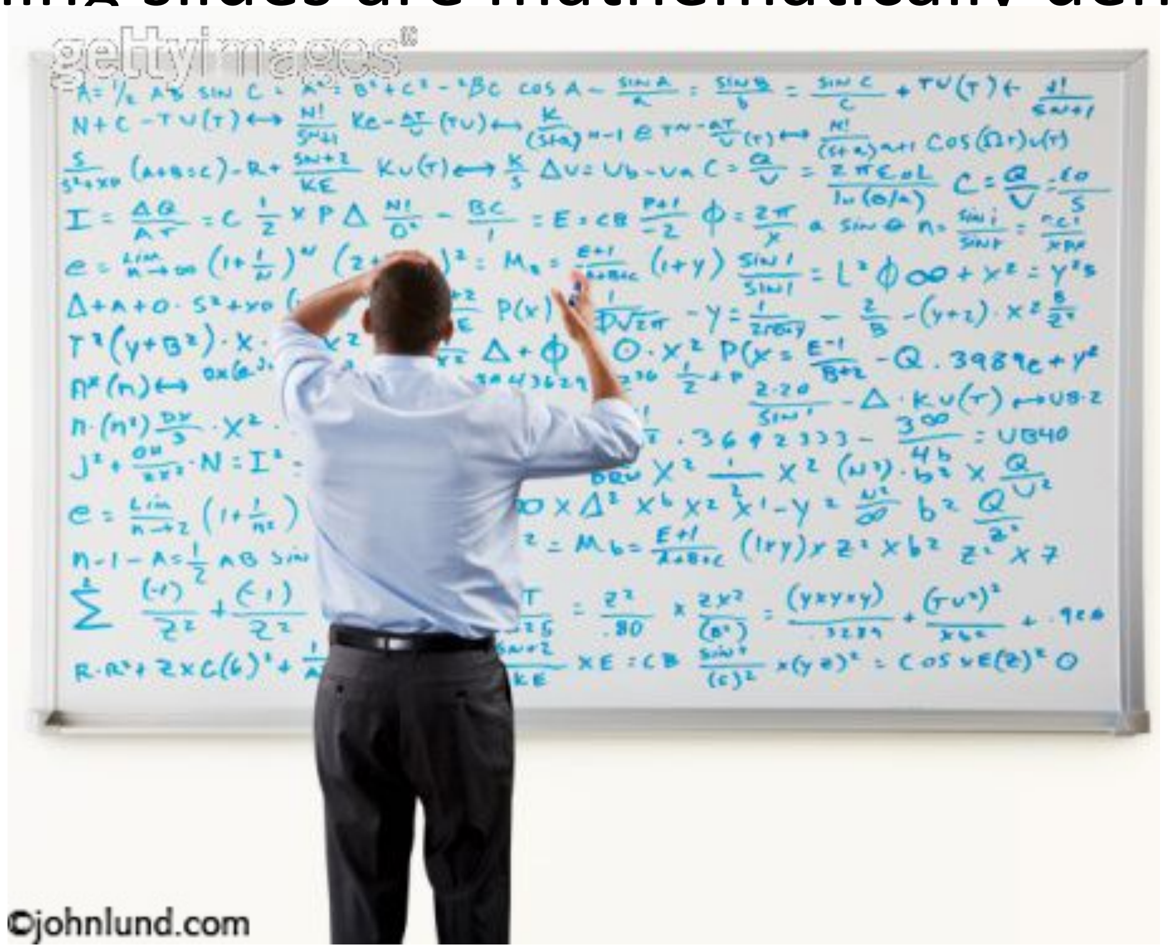
Goal: Accuracy For Some

$$\forall G \in H_k, S_{f_{H_k},\beta}(G) = O(k).$$

For Local Profile Queries

Warning!

Remaining slides are mathematically dense!



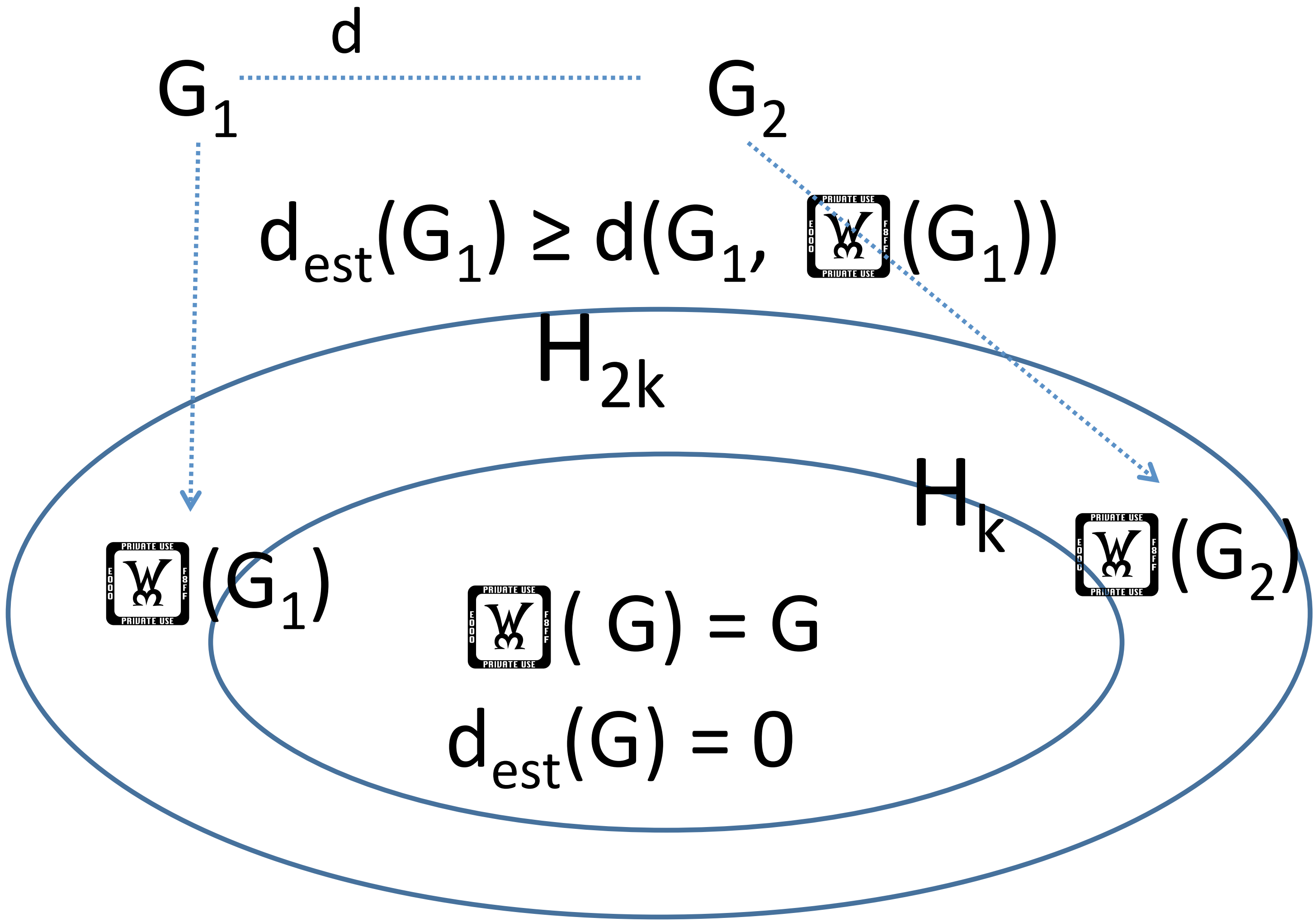
High Level Picture

- Concept: c-Smooth Distance Estimation

- Lemma
  - Accuracy for Some

- Constructing a 4-Smooth Distance Estimator
  - LP Rounding

c-Smooth Distance Estimation



C-smooth [illegible] $|d_{est}(G_1)- d_{est}(G_2)| \leq c\ d$

c-Smooth Distance Estimation

Definition: Let □: G □ H_{2k} be an efficiently computable projection and let d_{est} be an efficiently computable function which satisfies

1. For G □ H_k, $d_{est}(G) = 0$,
2. $d_{est}(G) \geq d(G,$ □$(G))$, and
3. $|d_{est}(G) - d_{est}(G')| \leq c$ for $G \sim G'$,

then d_{est} is a c-smooth distance estimator.

High Level Picture

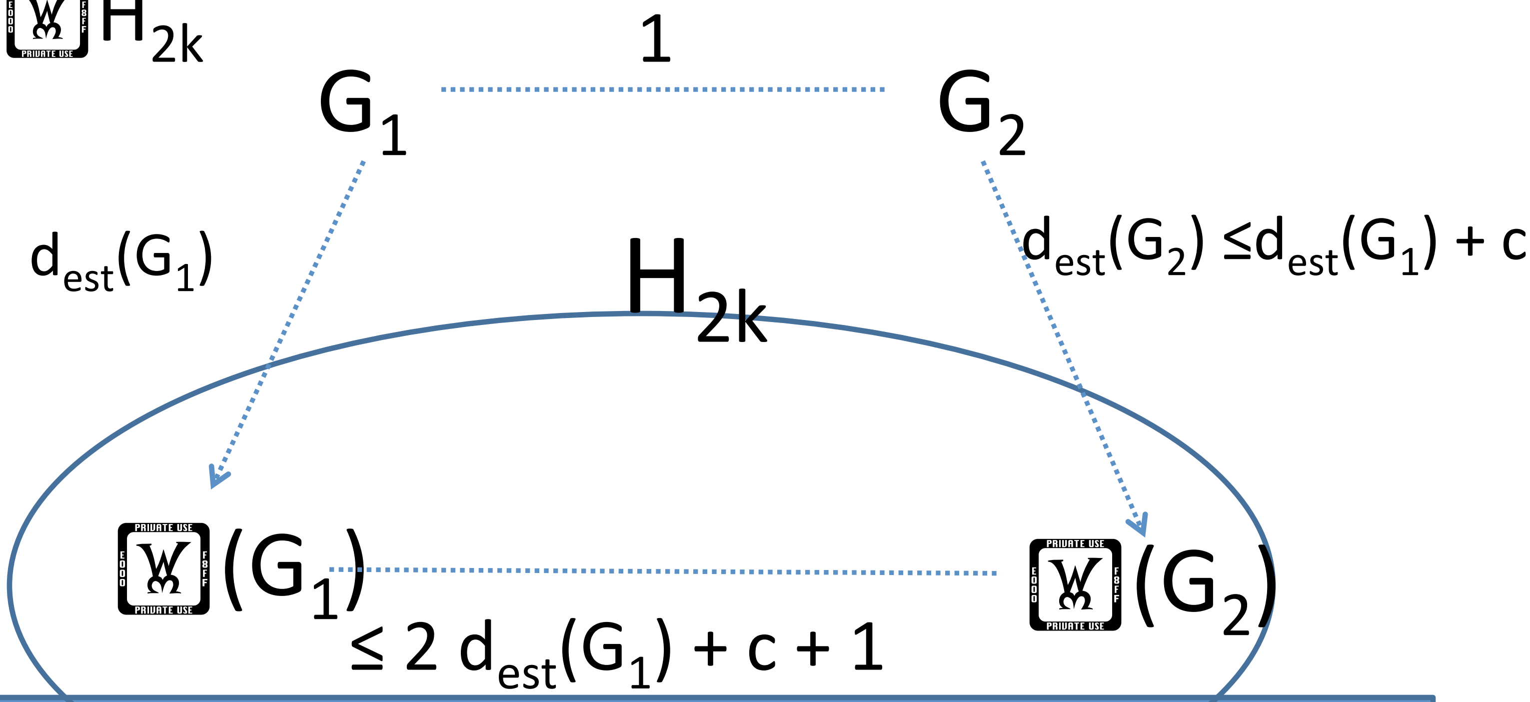
- Concept: c-Smooth Distance Estimation

- **Lemma**
  - **Privacy for All, Accuracy for Some**

- Constructing a 4-Smooth Distance Estimator
  - LP Rounding

c-Smooth Distance Estimation

: G H_{2k}



$$LS_{f_H}(G_1) \leq (2d_{est}(G_1) + c + 1) RS_f(H_{2k})$$

Smooth Sensitivity

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1)RS_f(H_{2k})$$

Fact 1: $S_{f_{H_k},\beta}(G) \geq LS_{f_{H_k}}(G)$

Fact 2: $S_{f_{H_k},\beta}(G)$ is [illegible]-smooth

Fact 3: $\forall G \in H_k, \quad S_{f_{H_k},\beta}(G) = O(k)$

c-Smooth Distance Estimation Lemma

Let [illegible]: G [illegible] H_{2k} be a projection with a c-smooth distance estimator d_{est}, and let

$$f_{H_k}(G) = f(\mu(G)).$$

Then for every G [illegible] H_k $S_{f_{H_k},\beta}(G) = O(k)$.

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1)RS_f(H_{2k})$$

Fact 1: $S_{f_{H_k},\beta}(G) \geq LS_{f_{H_k}}(G)$

$$S_{f_{H_k},\beta}(G) \geq \exp\left(-\frac{\beta}{c}\left(d_{est}(G) - d_{est}(G)\right)\right)(2d_{est}(G) + c + 1)RS_f(H_{2k})$$

$$\geq (2d_{est}(G) + c + 1)RS_f(H_{2k})$$

$$\geq (2d_{est}(G) + c + 1)\max_{G' \sim G}\left(\frac{f(\mu(G)) - f(\mu(G'))}{d(G, G')}\right)$$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1) RS_f(H_{2k})$$

Fact 1: $S_{f_{H_k},\beta}(G) \geq LS_{f_{H_k}}(G)$

$$S_{f_{H_k},\beta}(G) \geq (2d_{est}(G) + c + 1) \max_{G' \sim G}\left(\frac{f(\mu(G)) - f(\mu(G'))}{2d_{est}(G) + c + 1}\right)$$

$$\geq \max_{G' \sim G}\left(f_{H_k}(G) - f_{H_k}(G')\right)$$

$$= LS_{f_{H_k}}(G)$$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1)RS_f(H_{2k})$$

Fact 2: $S_{f_{H_k},\beta}(G)$ is [illegible]-smooth.

$$\frac{S_{f_{H_k},\beta}(G)}{S_{f_{H_k},\beta}(G')} = \frac{\max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1)RS_f(H_{2k})}{\max_{d \geq d_{est}(G')} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G')\right)\right)(2d + c + 1)RS_f(H_{2k})}$$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1)RS_f(H_{2k})$$

Fact 2: $S_{f_{H_k},\beta}(G)$ is [illegible]-smooth.

$$\frac{S_{f_{H_k},\beta}(G)}{S_{f_{H_k},\beta}(G')} = \frac{\exp\left(-\frac{\beta}{c}\left(d^* - d_{est}(G)\right)\right)(2d^* + c + 1)RS_f(H_{2k})}{\max_{d \geq d_{est}(G')} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G')\right)\right)(2d + c + 1)RS_f(H_{2k})}$$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1) RS_f(H_{2k})$$

Fact 2: $S_{f_{H_k},\beta}(G)$ is [illegible]-smooth.

$$\frac{S_{f_{H_k},\beta}(G)}{S_{f_{H_k},\beta}(G')} \leq \frac{\exp\left(-\frac{\beta}{c}\left(d^* - d_{est}(G)\right)\right)(2d^* + c + 1) RS_f(H_{2k})}{\exp\left(-\frac{\beta}{c}\left(d^* - d_{est}(G')\right)\right)(2d^* + c + 1) RS_f(H_{2k})}$$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1)RS_f(H_{2k})$$

Fact 2: $S_{f_{H_k},\beta}(G)$ is [illegible]-smooth.

$$\frac{S_{f_{H_k},\beta}(G)}{S_{f_{H_k},\beta}(G')} \leq \frac{\exp\left(-\frac{\beta}{c}\left(d^* - d_{est}(G)\right)\right)}{\exp\left(-\frac{\beta}{c}\left(d^* - d_{est}(G')\right)\right)}$$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1) RS_f\left(H_{2k}\right)$$

Fact 2: $S_{f_{H_k},\beta}(G)$ is [illegible]-smooth.

$$\frac{S_{f_{H_k},\beta}(G)}{S_{f_{H_k},\beta}(G')} \leq \exp\left(-\frac{\beta}{c}\left|d_{est}(G') - d_{est}(G)\right|\right)$$

$$\leq \exp(-\beta)$$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1) RS_f(H_{2k})$$

Fact 3: $\forall G \in H_k, \quad S_{f_{H_k},\beta}(G) = O(k)$

Calculus ➡ $S_{f_{H_k},\beta}(G) \leq g\left(\frac{\beta}{c}\right) \exp\left(\frac{\beta}{c} d_{est}(G)\right) RS_f(H_{2k})$

Where $g(x) = \begin{cases} c+1 & 0 \leq x \leq \frac{2}{c+1} \\ 2\frac{1}{x}\exp\left(-1 + \frac{c+1}{2}x\right) & x > \frac{2}{c+1} \end{cases}$

Smooth Upper Bound

$$S_{f_{H_k},\beta}(G) = \max_{d \geq d_{est}(G)} \exp\left(-\frac{\beta}{c}\left(d - d_{est}(G)\right)\right)(2d + c + 1)RS_f(H_{2k})$$

Fact 3: $\forall G \in H_k, \quad S_{f_{H_k},\beta}(G) = O(k)$

$$S_{f_{H_k},\beta}(G) \leq g\left(\frac{\beta}{c}\right)\exp\left(\frac{\beta}{c} d_{est}(G)\right) RS_f(H_{2k})$$

$$\forall G \in H_k, \quad S_{f_{H_k},\beta}(G) = g\left(\frac{\beta}{c}\right) RS_f(H_{2k}) = O(k)$$

High Level Picture

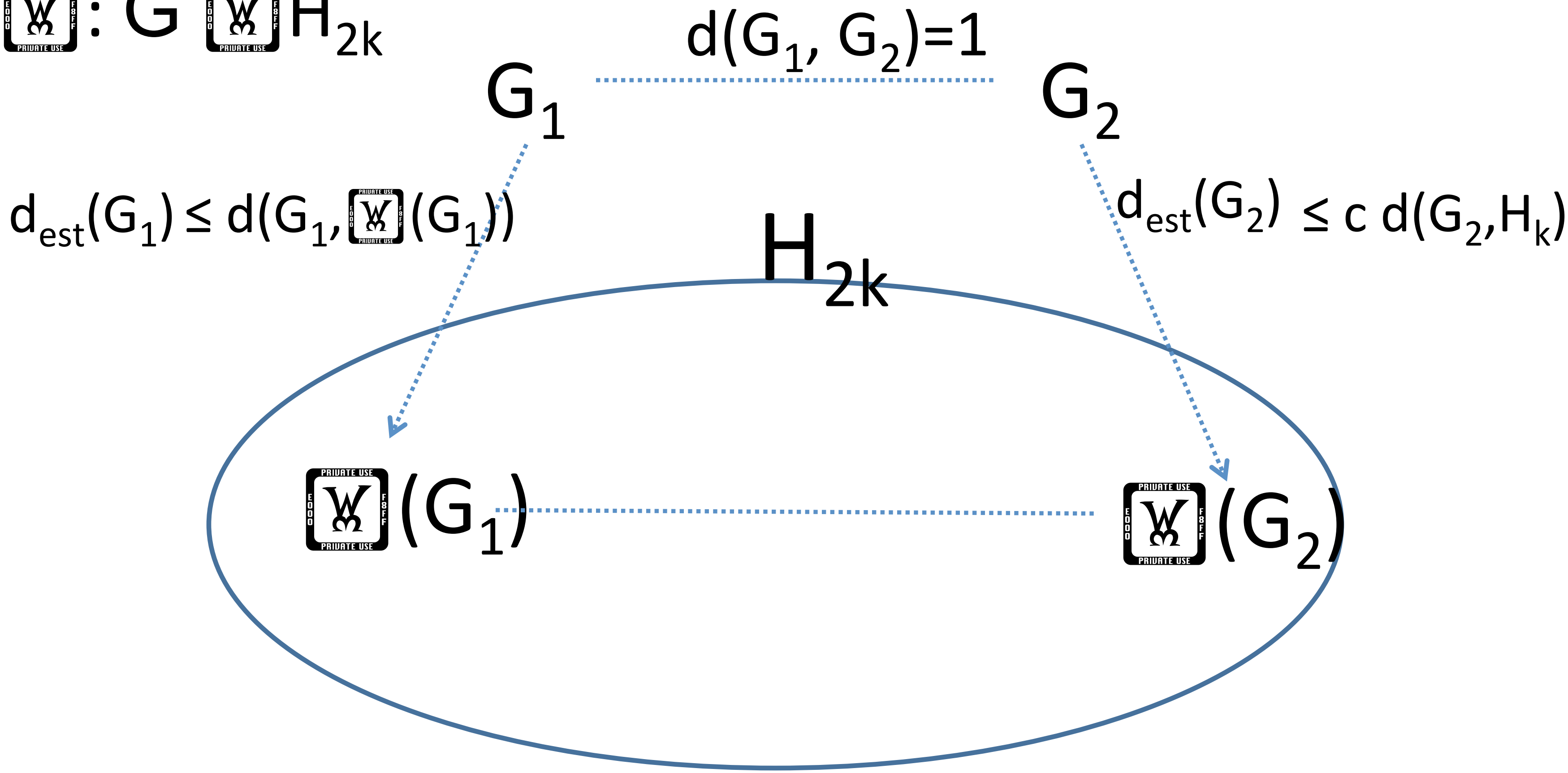
- Concept: c-Smooth Distance Estimation

- Lemma
  - Privacy for All, Accuracy for Some

- **Constructing a 4-Smooth Distance Estimator**
  - **LP Rounding**

c-Smooth Distance Estimation

□: G □ H_{2k}



C-smooth □ $|d_{est}(G_1) - d_{est}(G_2)| \leq c$

c-Smooth Distance Estimation via LP

$$\min \sum_{v \in V} x_v \quad s.t$$

Integral

Intuitions

∀v

∀u

∀(

∀u

or

Proof: Let v be the vertex such that G-v = G'-v.

1. Solve LP for G, and set
   - $x_v^* = 1$
   - $x_u^* = x_u$ for u v
2. Now x* is a feasible LP solution for G'.

$$\left| d_{est}(G) - d_{est}(G') \right| \le 4 x_v^*$$

(G) H_k

$$d_{est}(G) = 4 \sum_{v \in V} x_v$$

4-Smooth: $\left| d_{est}(G) - d_{est}(G') \right| \le 4$

Rounding the LP

$$\min \sum_{v \in V} x_v \quad s.t$$

$$\forall v, \quad 1 \geq x_v \geq 0$$

$$\forall u, v, \quad w_{u,v} \geq 0$$

$$\forall (u,v) \in E, \quad w_{u,v} \geq 1 - x_u - x_v$$

$$\forall u, \quad \sum_{v \neq u} w_{u,v} \leq k$$

$$y_v = \begin{cases} 1 & \text{If } x_v \geq 1/4 \\ 0 & \text{o.w.} \end{cases}$$

$$e_{u,v} = \begin{cases} 0 & \text{If } x_v \geq 1/4 \\ 0 & \text{If } x_u \geq 1/4 \\ 1 & \text{o.w.} \end{cases}$$

$e_{u,v}=1 \Rightarrow w_{u,v} \geq 1/2$

Rounding the LP

$$\sum_{v \in V} y_v \leq 4 \sum_{v \in V} x_v$$

$$\forall v, \quad 1 \geq y_v \geq 0$$

$$\forall u,v, \quad e_{u,v} \geq 0$$

$$\forall (u,v) \in E, \quad e_{u,v} \geq 1 - y_u - y_v$$

$$\forall u, \quad \sum_{v \neq u} e_{u,v} \leq 2k$$

$$y_v = \begin{cases} 1 & \text{If } x_v \geq ¼ \\ 0 & \text{o.w.} \end{cases}$$

$$e_{u,v} = \begin{cases} 0 & \text{If } x_v \geq ¼ \\ 0 & \text{If } x_u \geq ¼ \\ 1 & \text{o.w.} \end{cases}$$

$e_{u,v}=1$ ⇒ $w_{u,v} \geq ½$

Keep edge ⇔ $e_{u,v} = 1$

Call the resulting graph (G).

Rounding Facts

$$d\left(G,\mu(G)\right)\leq\sum_{v\in V}y_v\leq 4\sum_{v\in V}x_v=d_{est}(G)$$

$\forall G,$

So d_{est} is 4-smooth distance estimator for !

$$\forall G\in H_k,\quad \mu(G)=G$$

$$\forall G\in H_k,\quad d_{est}(G)=0$$

Reminder

Let □: G □ H_{2k} be a projection with a c-smooth distance estimator d_{est}, and let

$$f_{H_k}(G) = f(\mu(G)).$$

Then for every G □ H_k $S_{f_{H_k},\beta}(G) = O(k).$

Furthermore, $S_{f_{H_k},\beta}(G), \mu$ are both efficiently computable.

Goal Met

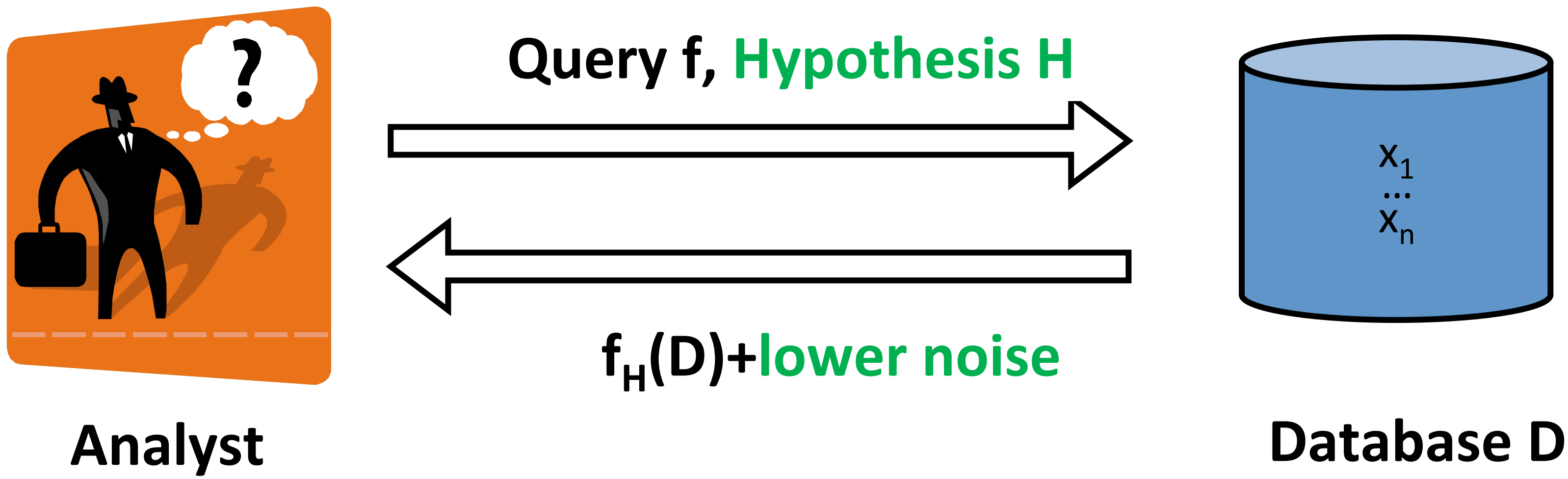
Theorem: The efficiently computable mechanism

$$A(G) = f_{H_k}(G) + Lap\left(\frac{2S_{f_{H_k},\beta}(G)}{\varepsilon}\right)$$

preserves differential privacy for [illegible]=-[illegible]/2 ln [illegible].

$$\forall G \in H_{k,} A(G) = f(G) + Lap\left(\frac{O(k)}{\varepsilon}\right).$$

Differential Privacy via Restricted Sensitivity



- Accurate <u>for D in H</u>
- Differential Privacy

Summary

	Adjacency	Hypothesis	Efficient?	Sensitivity of f_H
Alg 1	Any	Any	No	$RS_f(H)$
Alg 2	Edge	H_k	Yes	$3\ RS_f(H_k)$
Alg 3	Vertex	H_k	Yes	$O(RS_f(H_{2k}))$

	Local Profile Query		Subgraph Counting Query (P)	
Adjacency	**Smooth**	**Restricted**	**Smooth**	**Restricted**
Vertex	n	2k+1	$O(n^{\|P\|-1})$	$O(\|P\|\ k^{\|P\|-1})$

Open Questions

- Restricted sensitivity: Other relevant hypotheses H and associated constructions

- Social network privacy: Alternative to vertex adjacency
  - Too weak: Information about node A also in node B *influenced* by A

Thanks for Listening!

Questions?