Analog Sequential Linear Programming Solver

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Product Pitch

Our design is an **analog computer** that implements the Sequential Linear Programming (SLP) algorithm, which solves continuous Nonlinear Programming (NLP) problems. We aim to solve these problems faster and more efficiently than current digital solutions.

We demonstrate that our design can solve Nonlinear Model Predictive Control (NMPC) problems, which is a class of NLP problems. In particular, our analog computer swings up a damped inverted pendulum with optimal cost by solving a sequence of NMPC problems.

System Architecture

Our system features a heterogeneous architecture that consists of a digital computer and an analog computer. The NMPC problem is first converted to a form solvable by SLP on the digital computer, which then invokes the analog computer to solve Linear Programming (LP) subproblems repeatedly. For each subproblem, the digital computer configures the analog computer accordingly and measures its state to recover the solution. This is repeated until the NMPC problem is solved.

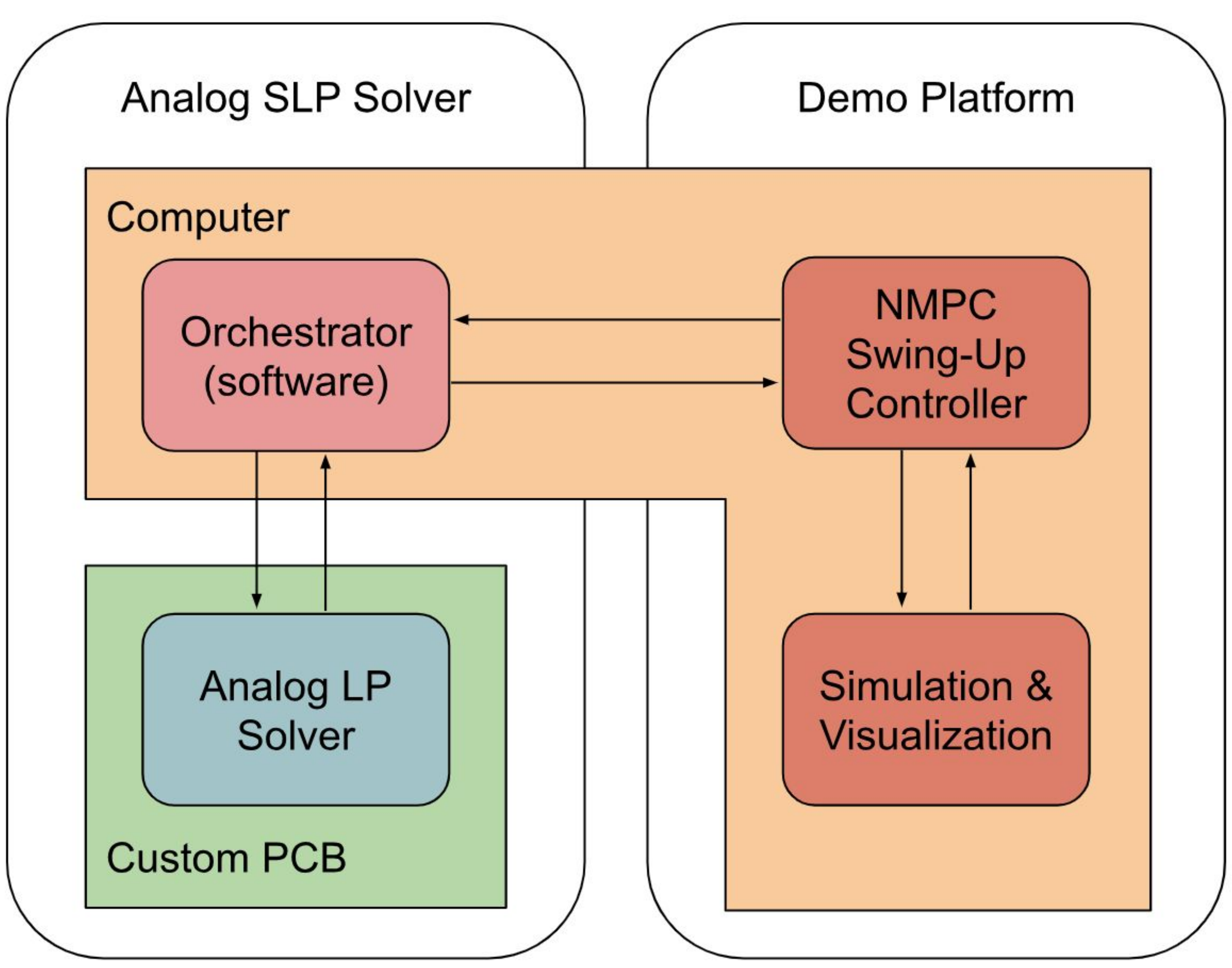

Figure 1. High-level system architecture

Conclusions & Additional Information

Our results demonstrate that an analog SLP solver can outperform its digital counterpart, highlighting the potential of analog computers in mathematical optimization. This design can be extended to implement Sequential Quadratic Programming (SQP) for better convergence and robustness.

Lessons Learned:

- Verify previous work done by others before expanding it.
- Decouple the project and perform risk management.
- Share responsibilities and work at a maintainable pace.

* Equal contribution, listed in alphabetical order

System Description

Our analog computer solves LPs by acting as the **physical analogue of the mathematical problem**. Instead of solving the LP arithmetically, we configure the analog circuit such that its steady state corresponds to the LP's solution.

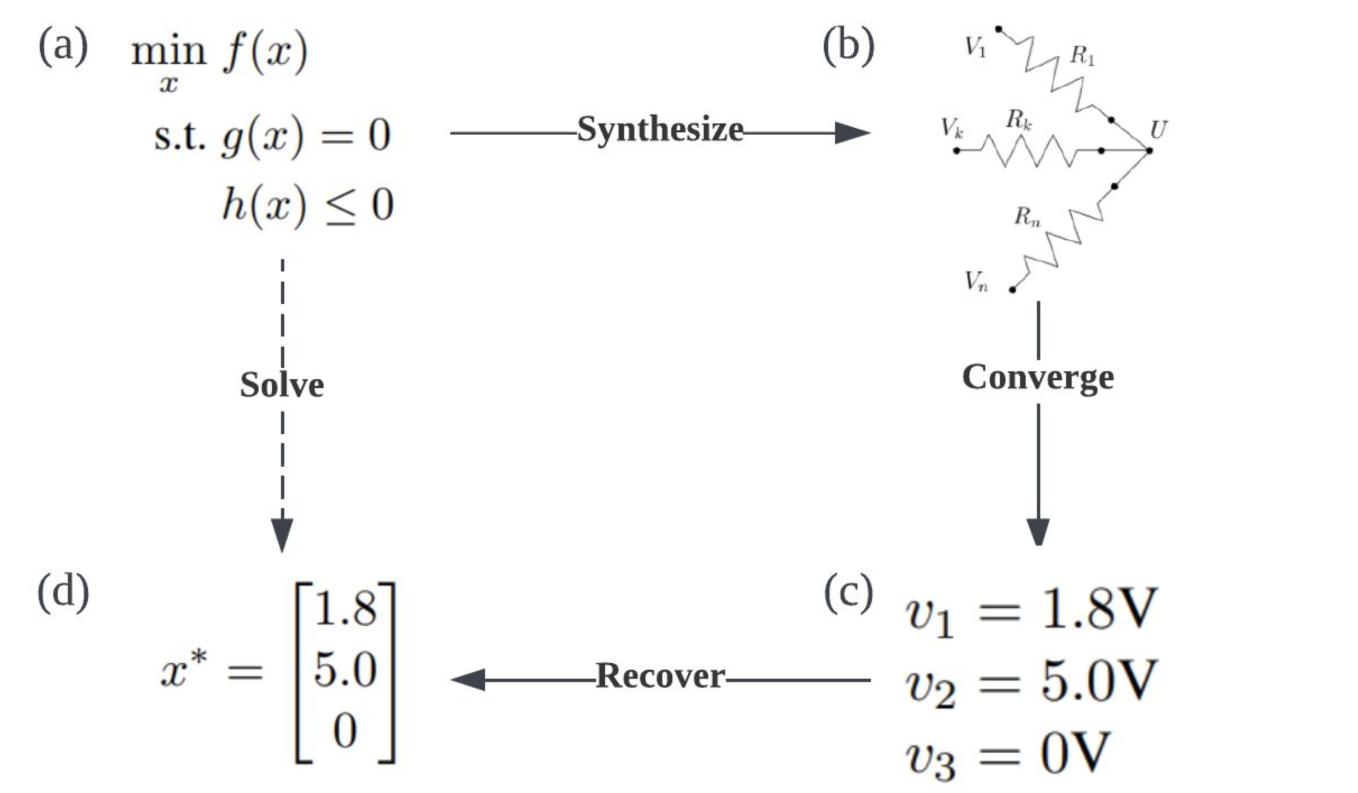

Figure 2. Procedure of solving optimization problems with the analog computer

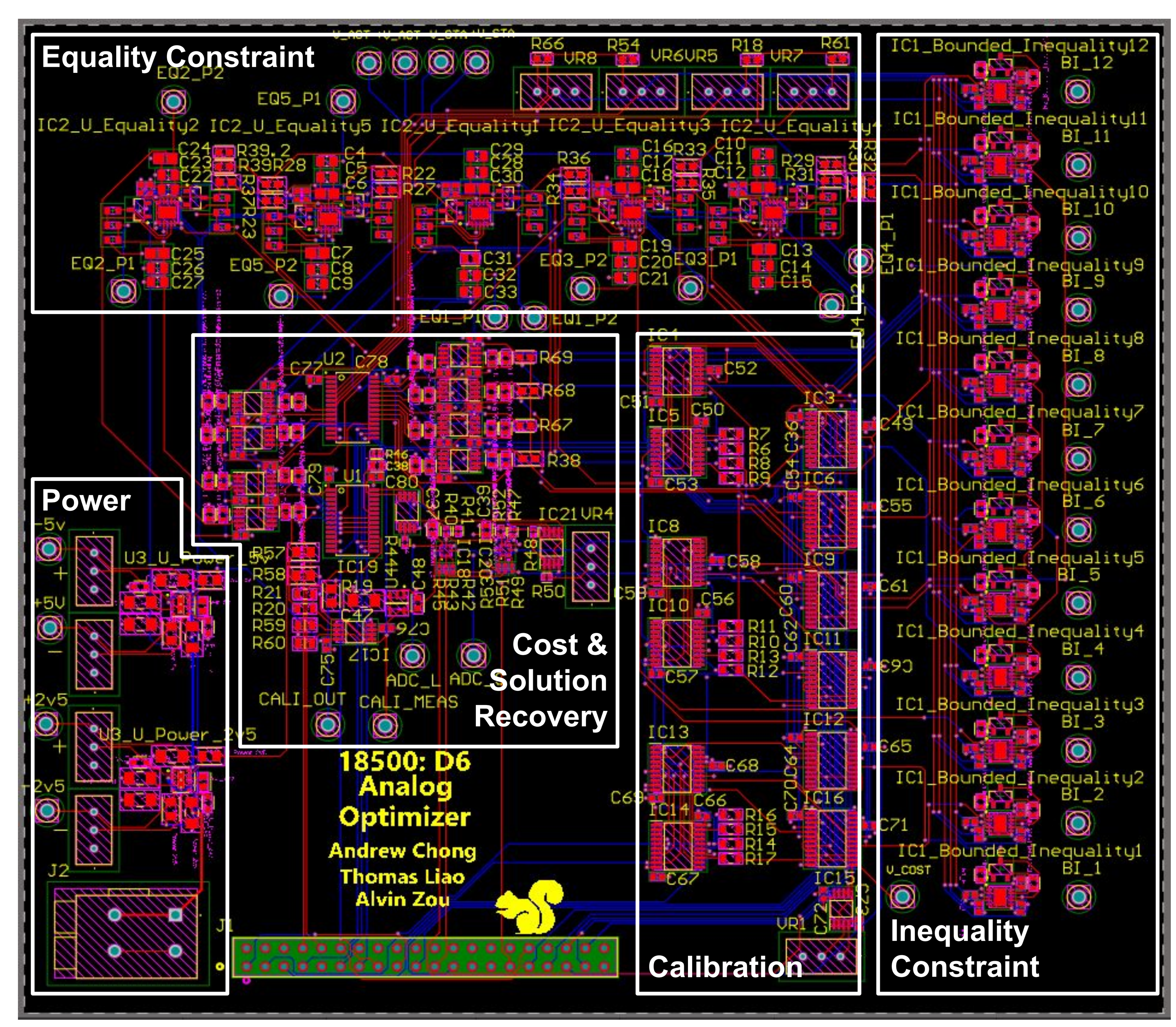

Figure 3. PCB layout of the analog computer

System Evaluation

For our analog circuit, we verified that the results produced by the simulation matched the results produced by the digital solver. We also verified that it could solve LPs **47.02% faster**.

variable name	x_1	x_2	x_3	x_4	x_5	x_6	Average	Valid region left	Valid region right
Solution (mV)	-0.5562	-3.706	-2.6191	-9.9999	-3.333	-2.338		-70mV	-420mV
Solution (original LP units)	-0.033372	-0.22236	-0.157146	-0.599994	-0.19998	-0.14028			
Software solution (original LP units)	-0.0332327	-0.221551	-0.156465	-0.6	-0.2	-0.143625			
Error	-0.0001393	-0.000809	-0.000681	0.000006	0.00002	0.003345			
Error %	0.4191654605	0.3651529445	0.4352411082	0.001	0.01	2.328981723	0.5932568727		

First LP

variable name	x_1	x_2	x_3	x_4	x_5	x_6	Average	Valid region left	Valid region right
Solution (mV)	-2.74	-18.43	-13.02	-49.99	-16.61	-11.88		-300mV	-800mV
Solution (original LP units)	-0.03288	-0.22116	-0.15624	-0.59988	-0.19932	-0.14256			
Software solution (original LP units)	-0.0332327	-0.221551	-0.156465	-0.6	-0.2	-0.143625			
Error	0.0003527	0.000391	0.000225	0.00012	0.00068	0.001065			
Error %	1.061304077	0.1764830671	0.1438021283	0.02	0.34	0.7415143603	0.4138506054		

Last LP

Table 1. Accuracy of the solution produced by the simulated analog computer

With our software, we verified that the SLP solver can correctly solve the stated problem. We also verified its accuracy by comparing it against a reference solver.

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