Team D6 – Analog SQP Solver

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What?

An **analog computer** that solves a specific class of **optimization problems**

Why?

Many engineering problems can be formulated as **optimization problems**, which needs to be solved **quickly and power-efficiently**

How?

Implement a state-of-the-art **optimization algorithm** with an **analog circuit**, and building a **demonstrable** system

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Use Case

Customer: A high-performance autonomous vehicle company that requires solving NMPC problems in real time and with low power

NMPC: Nonlinear Model Predictive Control

Need: A proof-of-concept solver that can solve NMPC problems faster and more efficiently than existing digital solutions

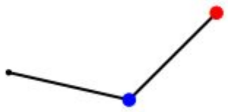
Source: https://www.digitaltrends.com/cars/roborace-devbot/

Use Case Requirements (Functional)

FR1: Complexity

- The solver must be able to solve at least one NMPC problem with at least **9 variables** and **3 constraints** with the SQP algorithm

SQP: Sequential Quadratic Programming



Source:
https://scipython.com/blog/the-double-pendulum/

FR2: Demonstrability

- The functionality of the solver should be demonstrated by visualization

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Use Case Requirements (Non-Functional)

NR1: Performance

- The solver should demonstrate a **10%** increase in performance over at least one modern, digital solver like Casadi

NR2: Efficiency

The solver should demonstrate a **10%** increase in energy efficiency over at least one modern, digital solver like Casadi

NR3: Accuracy

- The the solution produced by the system must be within ±**10%** of the optimal solution

NR4: Cost

- The total cost of the project should not exceed **$600**

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Technical Challenges

1. Adapt the circuit to solve the problem with the appropriate structure. **[FR1, FR2]**
2. Derive dynamics of system and synthesize a NMPC controller to formulate optimization problem **[FR1, FR2]**
3. Create analog circuit that can solve QP subproblems for SQP with sufficient speed and low enough power consumption **[NR1, NR2]**

QP: Quadratic Program

4. Ensuring analog circuit is correct and precise **[NR3]**
5. Visualize results through a live demo **[FR2]**

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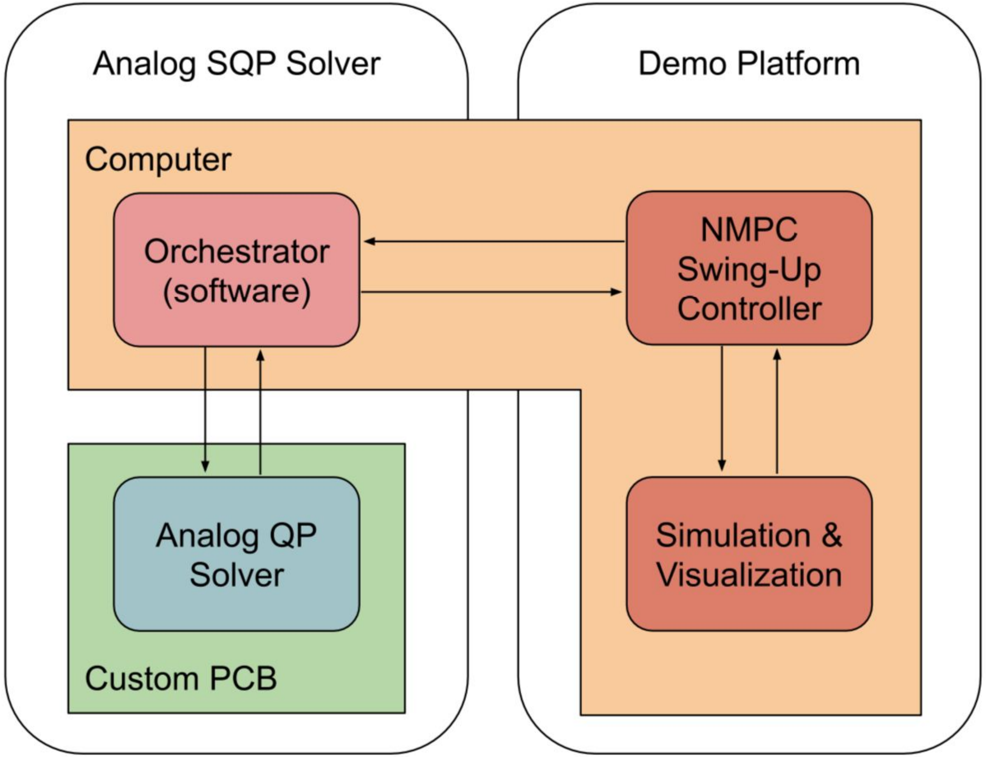
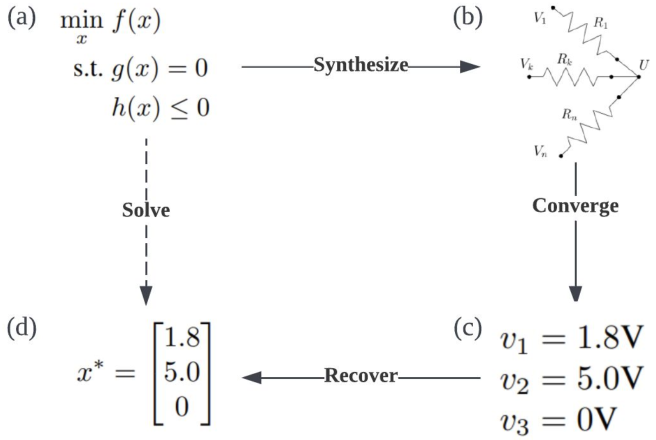
Solution Approach





Our system is a **physical analog** of the **mathematical problem**

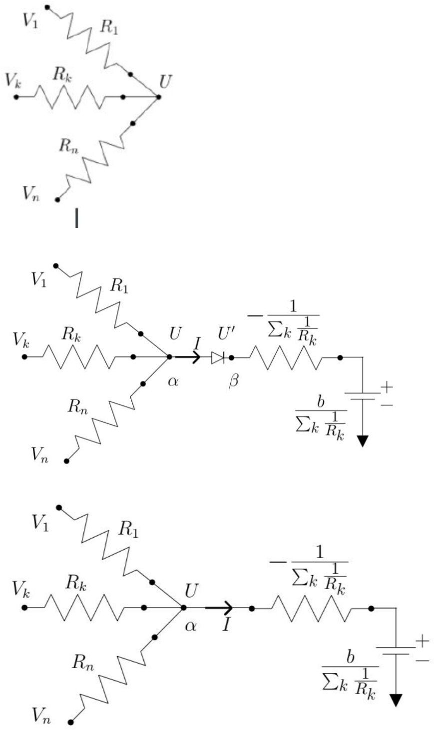
Solution Approach

Analog QP Solver

Cost Function: $\min_{x} f(x)$

Equality Constraint: $\text{s.t. } g(x) = 0$

Inequality Constraint: $h(x) \leq 0$



The **lowest-energy state** of the analog circuit is equivalent to the **optimal solution** to the QP subproblem

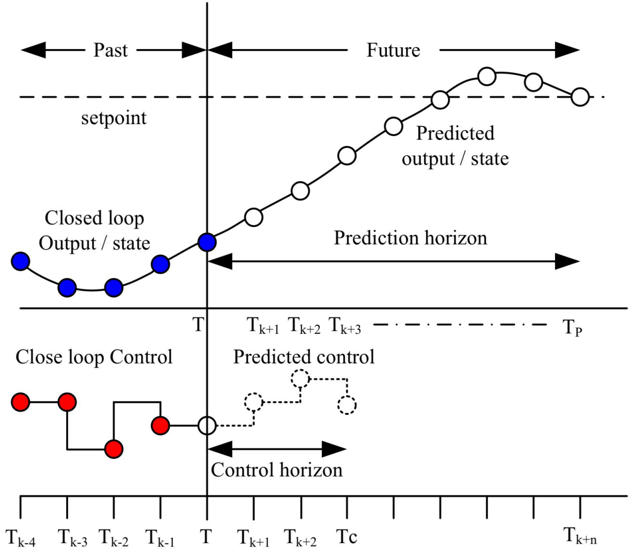
Source: https://doi.org/10.1016/j.compchemeng.2014.01.011

Solution Approach

Swinging up an inverted double-pendulum can be **performed** by an NMPC controller

NMPC can be **mathematically solved** by the SQP algorithm

The SQP algorithm is **implemented** by our system, using an **analog circuit** to **solve** its QP subproblems



Source: http://dx.doi.org/10.7763/IJIEE.2012.V2.167

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Testing, Verification and Metrics

Prototype	Description	Goals
P1	A circuit simulation of the analog QP solver	Verify that the circuit design solves the desired problem
P2	A visualized digital implementation with a software QP solver in place of the analog QP solver	Verify that the the software components are functional
P3	A manufactured PCB that implements the analog QP solver	Verify that the PCB is functional
P4	A fully-integrated analog SQP solver with all hardware and software	Verify that the entire system is functional after integration

Testing, Verification and Metrics

T1: Run circuit simulation 30 times and compare solution against a software solver

- **T1.1:** Ensure speed is 10% faster **[NR1]**
- **T1.2:** Ensure error margin is within 10% **[NR3]**

T2: Run digital implementation 30 times and verify it solves the problem at least 90% of the time **[FR2]**

T3: Run T1 using PCB **[NR1, NR3]**

- **T3.3:** Ensure power consumption is 10% lower **[NR2]**

T4: Run T1, T2, and T3.3 using the fully integrated solver **[FR2, NR1, NR2, NR3]**

Tasks and Division of Labor

Project Management: Weekly rotation of Project Manager

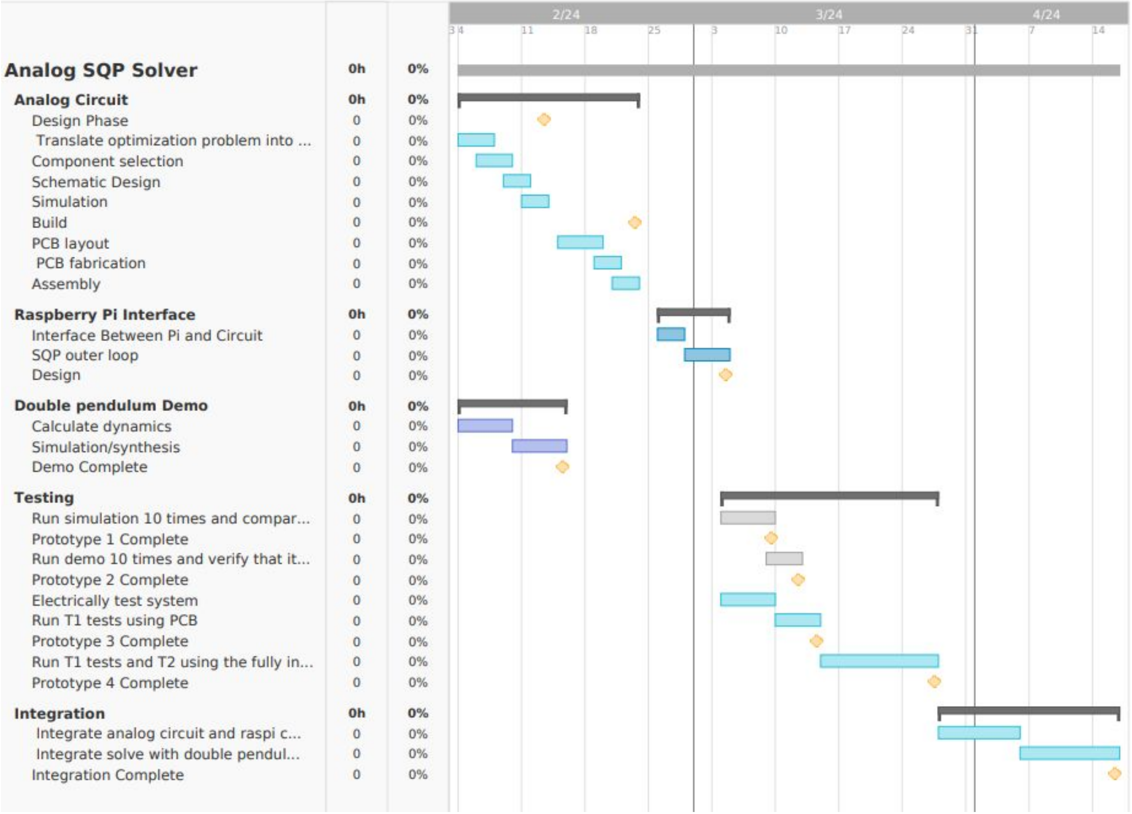
Andrew Chong: Circuit design, Simulation, PCB fabrication, Hardware + Software Integration

Thomas Liao: SQP algorithm development, QP subproblem formulation, NMPC synthesis for double-pendulum swing-up

Alvin Zou: Dynamics equation derivation, Casadi implementation

Equal contribution, listed in alphabetical order

Schedule



MVP:

Swing up an inverted single pendulum

Reach Goal:

Swing up inverted double pendulum, which satisfies all FRs and NRs

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